# Preparation of multiparty entangled states using pairwise perfectly efficient single-probe photon four-wave mixing

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We propose a scheme to achieve multiparty entanglement with perfectly efficient, ultraslow, multichannel pairwise four-wave mixing (FWM). A cold atomic medium is illuminated with an $N$-mode continuous-wave (cw) control laser to produce coherent mixtures of excited states. An ultraslowly propagating, single-photon quantum probe field completes multichannel, pairwise FWM, creating a depth dependent entanglement of $N$ Fock states. We show explicitly that this scheme can be utilized to realize an $N$-party entangled state of ultraslowly propagating quantized fields. In particular, we give the explicit analytical expression of a three-party W-state propagating at an ultraslow group velocity, and the numerical results of a multiparty W state of $2M+1$ Fock states.

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## I. INTRODUCTION

Entanglement of the quantum states of separate particles is at the heart of quantum-information sciences (QIS) and has been intensively studied [1–5]. Recently, a novel and efficient scheme to create the entanglement of two Fock states with a single (or a few) ultra-slow photons via four-wave mixing (FWM) [1] has been proposed. The scheme has been shown to have several advantages over the previous Fock state entanglement scheme using cavity QED techniques [2]. In particular, it has been shown that such a maximally entangled state can be efficiently generated, stored in the medium and later retrieved with its full entanglement properties recovered. These latter two capabilities have significant implications in QIS. Ultimately, it would be desirable to be able to entangle a large number of Fock states efficiently, store the entangled state for a long period of time, and retrieve it without losing important entanglement properties because $N$-party entangled states ($N\geqslant 3$) are generally superior to biparty entangled states in testing the foundations of quantum mechanics [2–4]. In addition, since entanglement is a key resource for quantum computation and quantum information, the ability to entangle a large number of states or particles represents a measure of being able to execute more complex quantum computations. For these reasons, seeking efficient schemes to realize the entanglement of a large number of quantum states or particles is an important and challenging task.

In this paper, we explore the possibility of entangling a large number of Fock states using a perfect efficient, pairwise FWM technique. Specifically, we propose a scheme using a perfectly efficient, ultraslow, multichannel pairwise four-wave mixing (FWM) technique as shown in Fig. 1. Here a cold atomic medium is illuminated with an $N$-mode continuous-wave (cw) control laser to produce coherent mixtures of excited states. An ultraslowly propagating, single-photon quantum probe field completes multichannel, pairwise FWM, creating a depth dependent entanglement of $N$ Fock states. In Sec. II, we describe the system and present the general solution describing the evolution of the probe fields. The general solution provides a convenient basis for the investigation of an $N$-party entangled state of ultraslowly propagating quantized probe fields. In Sec. III, we give the analytical expression of a three-party W-state propagating with an ultraslow group velocity. In Sec. IV, we present numerical results showing an entanglement of $2M+1$ Fock states, and we conclude the paper with a summary in Sec. V.

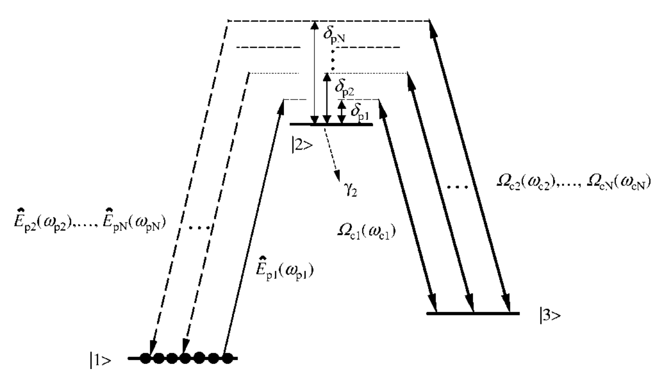


FIG. 1. A three-level lifetime broadened atomic system couples with a quantized probe field $\hat{E}_{p1}(\omega_{p1})$ and a strong classically treated continuous-wave (cw) $N$-mode control field $(\Omega_{c1},\omega_{c1};\Omega_{c2},\omega_{c2};\ldots;\Omega_{cN},\omega_{cN})$. The probe field frequency $\omega_{p1}$ and the control field frequency $\omega_{c1}$ are such that the exact two-photon resonance between atomic levels $|1\rangle$ and $|3\rangle$ is achieved. The complex detuning are defined as $d_{pj}=\delta_{pj}+i\gamma(j=1,2,\ldots,N)$ where $\delta_{pj}$ and $\gamma_2=2\gamma$ are the detunings from level $|2\rangle$ and the decay rate of level $|2\rangle$, respectively.

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## II. GOVERNING EQUATIONS AND GENERAL SOLUTION

We use the interaction picture to calculate the atomic dynamics and the probe photon field, but treat the strong $N$-mode control fields and their interactions with the medium classically. Taking the standard plane wave and slowly-varying-phase-and-amplitude approximations, we derive the following equations of motion for the complex amplitude of the quantized probe fields

$$\frac{\partial \hat{E}_{pj}^{(+)}}{\partial z}+\frac{1}{c}\frac{\partial \hat{E}_{pj}^{(+)}}{\partial t}=\frac{i\kappa_{12}\hbar}{D_{12}}\hat{S}_{pj} \quad (j=1,2,\ldots,N), \tag{1}$$

where $\hat{S}_{pj}$ are the polarization operators, $\kappa_{12}=2\pi N\omega_{12}|D_{12}|^2/(\hbar c)$, and $N$ and $D_{12}$ are the density of atoms and the dipole moment for the transition $|1\rangle\to|2\rangle$, respectively.

Under the conditions where $|d_{pj}\tau|\gg 1$, $|d_{pj}|\gg|\Omega_{cj}|$, and $|\Sigma_{j=1}^{N}|\Omega_{cj}|^2/d_{pj}|\gg 1/\tau$, the validity of the adiabatic following approximation for $\hat{S}_{pj}$ is well satisfied, and we have

$$\hat{S}_{pj}=-\frac{D_{12}}{\hbar d_{pj}}\hat{E}_{pj}^{(+)}-\frac{\Omega_{cj}^{*}}{d_{pj}}\hat{R}_3, \tag{2}$$

where $\hat{R}_3$ obeys

$$\frac{\partial \hat{R}_3}{\partial t}=-iD_{12}\sum_{j=1}^{N}\frac{\Omega_{cj}}{\hbar d_{pj}}\hat{E}_{pj}^{(+)}-i\left(\sum_{j=1}^{N}\frac{|\Omega_{cj}|^2}{d_{pj}}\right)\hat{R}_3. \tag{3}$$

Taking the Fourier transformation of Eqs. (1)–(3), we obtain

$$\frac{\partial \hat{Z}_{pj}^{(+)}}{\partial z}-i\frac{\omega}{c}\hat{Z}_{pj}^{(+)}=i\kappa_{12}\frac{[\omega d-D+(d|\Omega_{cj}|^2/d_{pj})]\hat{Z}_{pj}^{(+)}+d\sum_{k\neq j}(\Omega_{cj}^{*}\Omega_{ck}/d_{pk})\hat{Z}_{pk}^{(+)}}{(D-\omega d)d_{pj}}, \tag{4}$$

where $d\equiv\Pi_{k=1}^{N}d_{pk}$, $D=d\Sigma_{j=1}^{N}(|\Omega_{cj}|^2/d_{pj})$, and is the Fourier transformation of $\hat{E}_{pj}^{(+)}$ with $\omega$ being the transformation variable.

We now consider the case where all $N$ control fields have the same intensity, i.e., $\Omega_{c1}=\Omega_{c2}=\cdots=\Omega_{cN}\equiv\Omega_c$. In this case, Eq. (4) can be put into a concise matrix form $-i\partial\hat{Z}/\partial\eta=G\hat{Z}$ with a solution

$$\hat{Z}=e^{i\eta G}\hat{Z}_0, \tag{5}$$

where $\eta=z\kappa_{12}|\Omega_c|^N/(D-\omega d)\equiv z(\kappa_{12}/|\Omega_c|)/(\tilde{D}-\tilde{\omega}\tilde{d})$, $d_j=d_{pj}/|\Omega_c|$, $\tilde{d}=d/|\Omega_c|^N\equiv\Pi_{k=1}^{N}d_k$, $\tilde{D}=D/|\Omega_c|^{N+1}=\Sigma_{j=1}^{N}(\tilde{d}/d_j)$ and $\tilde{\omega}=\omega/|\Omega_c|$. $\hat{Z}=\exp(-i\omega z/c)(\hat{Z}_{p1}^{(+)},\hat{Z}_{p2}^{(+)},\cdots,\hat{Z}_{pN}^{(+)})^T$ is a column vector of $N$ components, $\hat{Z}_0=\hat{Z}|_{z=0}$, and $G$ is a $N\times N$ matrix with elements $G_{ij}=[(1/d_j)(\tilde{\omega}\tilde{d}-\tilde{D}+\tilde{d}/d_j)-\tilde{d}/d_j^2]\delta_{ij}+\tilde{d}/(d_id_j)$.

In order to obtain the explicit expressions of $N$ quantities $(\hat{Z}_{pj}^{(+)})$, we need to first find the eigenvalues $\xi$ of the matrix $G$ by solving the equation $\det(G-\xi I)=0$. It is readily shown that this equation can be reduced to the following form:

$$\sum_{j=1}^{N}\frac{\tilde{d}}{\xi d_j^2+d_j(\tilde{D}-\tilde{\omega}\tilde{d})}=1. \tag{6}$$

Once the eigenvalues $\xi$ are obtained, the coefficient matrix for diagonalizing the matrix $G$ can be trivially obtained from Eq. (5), yielding explicit expressions of the $N$ quantities $\hat{Z}_{pj}^{(+)}$.

In order to carry out the inverse Fourier transformation analytically, thereby gaining important physical insight, we follow the approximation in Ref. [1] by neglecting $\omega$ dependent terms in the coefficients of the solution, and keeping only terms that are linear in $\omega$ in the exponents, i.e., taking $\xi=\xi^{(0)}+\tilde{\omega}\xi^{(1)}+O(\tilde{\omega}^2)$. This approximation is justified because the assumptions made previously lead to a well behaved adiabatic following solution of Eq. (2)[1]. We thus have

$$\hat{E}_{pj}^{(+)}(z,t)=\sum_{k=1}^{N}\hat{a}_{pk}\sum_{n=1}^{N}c_{jk}^{(n)}e^{iz\sigma_n}P\left(t-\frac{z}{V_{gn}}\right) \quad (j=1,2,\ldots,N), \tag{7}$$

where $P(t)$ is the pulse shape function for the initial photon wave packet for the first probe field of frequency $\omega_{p1}$, $\hat{a}_{pk}$ is the annihilation operator for the $k$th probe field, , $\sigma_n=\kappa_{12}\xi_n^{(0)}/(|\Omega_c|\tilde{D})$, and the group velocities are given by

$$\frac{1}{V_{gn}}=\frac{1}{c}+\frac{\kappa_{12}}{|\Omega_c|^2}\mathrm{Re}\left[\frac{1}{\sum_{j=1}^{N}(1+d_j\xi_n^{(0)}/\tilde{D})^{-2}}\right]. \tag{8}$$

Here $(n=1,2,\ldots,N)$ are the roots of the following equation for $\xi^{(0)}$ [6]:

$$\sum_{j=1}^{N}\frac{\xi^{(0)}}{\xi^{(0)}d_j+\tilde{D}}=0. \tag{9}$$

Equations (7)–(9) indicate that in general the wave packet will break into $N$ parts that travel with different group velocities but all of them *retain* a pulse shape identical to that of the input probe field. Under the initial condition of only one photon wave packet in the probe field $E_{p1}$ and none in others, the state vector of the probe waves in the Schrödinger picture can be expressed as

$$|\Psi\rangle = \sum_{k,n=1}^{N} [c_{1k}^{(n)} e^{iz\sigma_n}]^* P\left(t - \frac{z}{V_{gn}}\right) \hat{a}_{pk}^{\dagger} |\text{vac}\rangle. \tag{10}$$

Here $|\text{vac}\rangle = |0_{\omega_{p1}}\rangle |0_{\omega_{p2}}\rangle \ldots |0_{\omega_{pN}}\rangle$ denotes the vacuum state of the probe waves, and hence $\hat{a}_{pk}^{\dagger}|\text{vac}\rangle = |0_{\omega_{p1}}\rangle \ldots |1_{\omega_{pk}}\rangle \ldots |0_{\omega_{pN}}\rangle$ is the state with one photon in the $k$th probe wave and none in the others. Equation (10) represents a linear combination of $N$ Fock states and it is an entangled state at specific space-time points $(z,t)$.

In order to obtain the $N$-party entangled state for the quantized probe fields, and more importantly to be able to store and retrieve the entangled state with its full entanglement properties recovered, it is desirable to have all $N$ parts of the wave packet traveling with closely matched group velocities. This is particularly important when slow propagation velocity is an essential feature of the problem because under ultraslow propagation conditions the longitudinal spatial distribution of the wave packet is very small and any significant mismatch will result in significant reduction of efficiency and will render the storage and recovery incomplete. In the case of $N$-channel entanglement this is a formidable task since one must choose appropriate values for $N$ complex detunings $d_{pj} = \delta_{pj} + i\gamma (j=1,2,\ldots,N)$ so that the $N$ group velocities $V_{gj}$ have nearly identical values. In the following, we show that it is indeed possible to realize efficient entanglement of $N$ Fock states propagating with the same group velocity by considering two specific situations.

## III. THE THREE-PARTY W STATES

In this section, we show that in the case of $N=3$, we can choose the parameters so as to achieve the same ultraslow group velocity for three probe fields, and show explicitly and analytically that the entanglement of three Fock states can indeed be realized.

It follows from Eq. (9) that with $N=3$ the three $\xi^{(0)}$'s are (with $\xi_-^{(0)} \equiv \xi_2^{(0)}$ and $\xi_+^{(0)} \equiv \xi_3^{(0)}$)

$$\xi_1^{(0)} = 0, \quad \xi_{\pm}^{(0)} = -\sum_{k=1}^{3} d_k \pm \sqrt{\sum_{k=1}^{3} d_k^2 - \tilde{D}}. \tag{11}$$

Substituting these $\xi^{(0)}$'s into Eq. (8) and taking $\delta_{p1}=0$, $\delta_{p2} = -\delta_{p3} = \Delta$, we readily obtain

$$\frac{1}{V_{g1}} = \frac{1}{c} + \frac{\kappa_{12}}{3|\Omega_c|^2}, \tag{12a}$$

$$\frac{1}{V_{g\pm}} = \frac{1}{c} + \frac{\kappa_{12}}{3|\Omega_c|^2} \mathrm{Re}\left[\frac{(\Delta^2 - 8\gamma^2) \pm 3\sqrt{3} i\gamma |\Delta|}{\Delta^2 + \gamma^2}\right]. \tag{12b}$$

Therefore, under the condition $\gamma \ll |\Delta|$, the group velocities can be closely matched. We thus have

$$\hat{E}_{pj}^{(+)}(z,t) = CP\left(t - \frac{z}{V_{g1}}\right) \sum_{k=1}^{3} \hat{a}_{pk} M_{jk}(z), \tag{13}$$

where $C$ is an appropriate constant, $M_{jk}(z) = M_{kj}(z)$ $(j,k = 1,2,3$ but $j \neq k)$, $M_{33}(z) = M_{22}^*(z)$, and

$$M_{11}(z) = [1 + 2e^{-Bz}\cos(Az)], \tag{14a}$$

$$M_{12}(z) = [1 - e^{-Bz}\cos(Az) + i\sqrt{3}e^{-Bz}\sin(Az)], \tag{14b}$$

$$M_{13}(z) = [1 - e^{-Bz}\cos(Az) - i\sqrt{3}e^{-Bz}\sin(Az)], \tag{14c}$$

$$M_{23}(z) = 1 - e^{-Bz}\cos(Az), \tag{14d}$$

$$M_{22}(z) = 1 + 2e^{-Bz}\cos(Az) - i\sqrt{3}e^{-Bz}\sin(Az). \tag{14e}$$

In the above equations $B = \mathrm{Im}(\sigma_\pm) = 3\kappa_{12}\gamma/(\Delta^2 + \gamma^2) \approx 3\kappa_{12}\gamma/\Delta^2$, $A = \mathrm{Re}(\sigma_-) = \kappa_{12}\sqrt{3}|\Delta|/(\Delta^2+\gamma^2) \approx \kappa_{12}\sqrt{3}/|\Delta|$ under the condition $\gamma \ll |\Delta|$, $\sigma_\pm = \kappa_{12}(\mp\sqrt{3}|\Delta| + 3i\gamma)/(\Delta^2+\gamma^2)$, and $\sigma_1 = 0$. Consequently, the state vector of the probe wave is given as [see Eq. (10)]

$$|\Psi\rangle = C^*[M_{11}^*(z)|1_{\omega_{p1}}\rangle|0_{\omega_{p2}}\rangle|0_{\omega_{p3}}\rangle + M_{12}^*(z)|0_{\omega_{p1}}\rangle|1_{\omega_{p2}}\rangle|0_{\omega_{p3}}\rangle + M_{13}^*(z)|0_{\omega_{p1}}\rangle|0_{\omega_{p2}}\rangle|1_{\omega_{p3}}\rangle]P\left(t - \frac{z}{V_{g1}}\right). \tag{15}$$

Equations (14) and (15) indicate that under the condition of $\gamma \ll |\Delta|$ (so that $B \approx 0$) and when the distance $z$ satisfies $\cos(Az) = (\sqrt{3}-1)/2$, we have

$$|\Psi\rangle = \sqrt{3}C^*[|1_{\omega_{p1}}\rangle|0_{\omega_{p2}}\rangle|0_{\omega_{p3}}\rangle + e^{-i\theta}|0_{\omega_{p1}}\rangle|1_{\omega_{p2}}\rangle|0_{\omega_{p3}}\rangle + e^{i\theta}|0_{\omega_{p1}}\rangle|0_{\omega_{p2}}\rangle|1_{\omega_{p3}}\rangle]P\left(t - \frac{z}{V_{g1}}\right), \tag{16}$$

where $e^{i\theta} = \pm\sqrt{2\sqrt{3}}/(\sqrt{3}+1)$. This state is equivalent to the standard three-party W-state. We note that unitary transformations $\hat{a}_{p3}^{\dagger} \to e^{-i\theta}\hat{a}_{p3}^{\dagger}$ and $\hat{a}_{p2}^{\dagger} \to e^{i\theta}\hat{a}_{p2}^{\dagger}$ transform Eq. (16) into

$$|\Psi\rangle = \sqrt{3}C^*[|1_{\omega_{p1}}\rangle|0_{\omega_{p2}}\rangle|0_{\omega_{p3}}\rangle + |0_{\omega_{p1}}\rangle|\tilde{1}_{\omega_{p2}}\rangle|0_{\omega_{p3}}\rangle + |0_{\omega_{p1}}\rangle \times |0_{\omega_{p2}}\rangle|\tilde{1}_{\omega_{p3}}\rangle]P\left(t - \frac{z}{V_{g1}}\right), \tag{17}$$

which is the standard three-party W-state [5] propagating with an ultraslow group velocity. We emphasize that unlike most of the previous schemes that are probabilistic (with a very small probability), the three-photon W-state obtained in our scheme is nonprobabilistic in nature.

It is interesting to note that a maximally entangled state of the last two probe waves can also be produced. For instance, when the distance $z$ satisfies $\cos(Az) = -1/2$ (again assuming $\gamma \ll |\Delta|$ so that $B \approx 0$), Eq. (15) becomes

$$|\Psi\rangle = s|0_{\omega_{p1}}\rangle(|1_{\omega_{p2}}\rangle|0_{\omega_{p3}}\rangle \pm i|0_{\omega_{p2}}\rangle|1_{\omega_{p3}}\rangle)P\left(t - \frac{z}{V_{g1}}\right), \tag{18}$$

where $s = 3(1 \pm i)C^*/2$ is a constant. This is a maximally entangled state of the second and third probe waves.

## IV. W STATE OF 2*M*+1 FOCK STATES

In this section, we show numerically that in the case of $N=2M+1$, we can choose the parameters so as to achieve

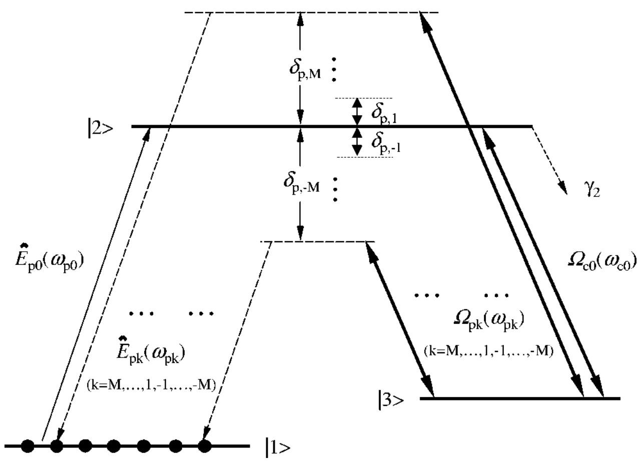


FIG. 2. A three-level lifetime broadened atomic system couples with a quantized probe field $\hat{E}_{p0}(\omega_{p0})$ and $2M+1$ strong, classically treated continuous-wave (cw) control fields $[\Omega_{cM}(\omega_{cM}),\ldots,\Omega_{c,-M}(\omega_{c,-M})]$. The probe field frequency $\omega_{p0}$ and corresponding control field frequency $\omega_{c0}$ are such that exact one- and two-photon resonance between atomic levels $|1\rangle$ and $|2\rangle$ $|1\rangle$ and $|3\rangle$ are achieved.

identical group velocities for the $2M+1$ probe fields. For this purpose it is convenient to relabel the probe and control fields as shown in Fig. 2.

It is obvious from Eq. (8) that when a control field in the three-state system depicted in Fig. 2 is sufficiently intense, the propagation velocities of the probe and FWM fields are very close to speed of light in vacuum. Consequently, modifications to the group velocities of the probe and FWM fields can be neglected [7]. We now assume that the $(2M+1)$ channels are equally spaced. Therefore, $d_{pj}=j\Delta+i\gamma$ with $j=\pm 1,\ldots,\pm M$. We further require that the $2M$ control field Rabi frequencies satisfy $\Omega_{cj}=|j|\Omega_c$ $(j\neq 0)$. The reason behind this choice of control fields is the consideration of achieving the same *effective* FWM production rate for all channels. This is critically important to the realization of a multiparty W-state. As we will demonstrate, different choices of Rabi frequencies and propagation distances will allow one to access a subset of multiparty W-states but only the above selection permits an all-channel-participating entangled state. We note that the $j=0$ channel is an important key element in achieving our goal. Indeed, the single-photon wave packet must be injected through this channel in order to obtain ideal results. For this channel, we have $d_{p0}=i\gamma$ and we take $\Omega_{c0}=\Omega_c$. Under these assumptions Eq. (5) becomes

$$\hat{Z}=e^{i\kappa_{12}zG}\hat{Z}_0. \tag{19}$$

Here, $\hat{Z}=\exp(-i\omega z/c)(\hat{Z}_{pM}^{(+)},\cdots,\hat{Z}_{p,-M}^{(+)})^T$ is a column vector of $(2M+1)$ components, $\hat{Z}_0=\hat{Z}|_{z=0}$, and $G$ is a $(2M+1)\times(2M+1)$ matrix with its elements given by

$$g_{00}=\frac{X-1}{i\gamma},\quad g_{0n}=g_{n0}=X\frac{|n|}{(n+i\delta)\Delta}, \tag{20a}$$

$$g_{nk}=iX\delta\frac{|n||k|}{(n+i\delta)(k+i\delta)\Delta}(1-\delta_{nk})-\frac{1}{(n+i\delta)\Delta}\left[1-iX\delta\frac{n^2}{(n+i\delta)}\right]\delta_{nk}, \tag{20b}$$

where $\delta_{nk}$ is the Kronecker $\delta$ function, $n,k=\pm 1,\pm 2,\ldots,\pm M$ and

$$X=\frac{1}{1+X_0},\quad X_0=2\delta^2\sum_{l=1}^{M}\frac{l^2}{l^2+\delta^2},\quad \delta=\frac{\gamma}{\Delta}.$$

In general, the coefficient matrix $e^{i\kappa_{12}zG}$ cannot be calculated analytically. In the following, we will demonstrate by numerical calculation that a $(2M+1)$-party W-state can indeed be generated with 100% efficiency, provided that a single probe photon wave packet is injected in the $j=0$ channel. Before doing that, however, we will discuss some features of this novel FWM scheme.

When a single probe photon is injected into the $j=0$ channel where the control field $\Omega_{c0}=\Omega_c$ maintains an electromagnetically induced transparency channel for the probe field, the single photon wave packet propagates nearly freely and is eventually absorbed via a two-photon process. Because of the existence of $2M$ control fields, this single probe photon wave packet is simultaneously converted into $2M$ FWM fields projected into the $2M$ exit channels. Since the control fields are chosen such that the *effective* FWM production rate is identical for all $2M$ channels, the projection or production of the $2M$ FWM fields occurs with *equal* probability. Consequently, as the initial probe photon is being absorbed, $2M$ FWM fields are produced in $2M$ channels with equal amplitudes, creating a perfect $2M$-mode FWM field with 100% conversion efficiency, and thereby generating a multiparty W-state of $2M$ Fock states. In Fig. 3 we plotted the absolute values of the probability amplitude of each product-type state obtained from Eqs. (19) and (20) as a function of $\kappa_{12}z/\Delta$. In this case, we take $M=4$ and we expect to generate a maximum entanglement of 9 Fock states [8]. The dashed curve starting at 1.0 represents the absolute value of the probability amplitude of the initial input channel. As the propagation distance $z$ increases, the input single-photon wave packet is absorbed and the production of all $2M=8$ FWM fields increases at the same rate (within our approximation). When the initial probe photon wave packet is fully absorbed, the dashed line reaches zero amplitude and all $2M=8$ mode FWM fields have reached the *same* amplitude. At this depth inside the medium, the following entangled state has been generated,

$$|\Psi\rangle=CP\left(t-\frac{z}{c}\right)|0_{\omega_{p0}}\rangle\sum_{k\neq 0,k=-M}^{M}\hat{a}_{pk}^{\dagger}|\mathrm{vac}\rangle, \tag{21}$$

where $C$ is a complex *c-number* function of propagation distance, $P(t)$ is the pulse shape function for the initial photon wave packet, $|\mathrm{vac}\rangle=|0_{\omega_{pM}}\rangle\ldots|0_{\omega_{p,1}}\rangle|0_{\omega_{p,-1}}\rangle\ldots|0_{\omega_{p,-M}}\rangle$ denotes the vacuum state of the $2M$ FWM waves (i.e., without the probe wave $\omega_{p0}$), and hence $|0_{\omega_{p0}}\rangle(\hat{a}_{pk}^{\dagger}|\mathrm{vac}\rangle)=|0_{\omega_{p0}}\rangle|0_{\omega_{pM}}\rangle\ldots|1_{\omega_{pk}}\rangle\ldots|0_{\omega_{p,-M}}\rangle$ is the state with one

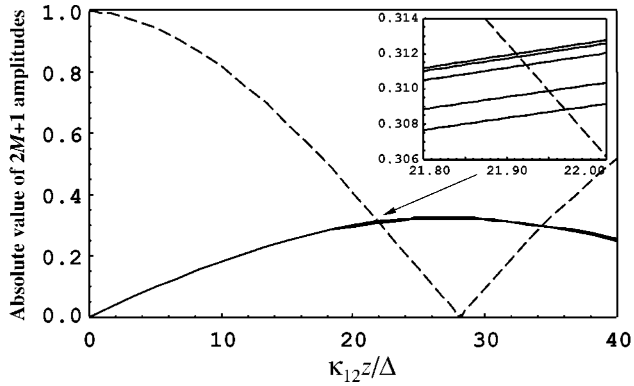


FIG. 3. Absolute values of the $2M+1=9$ coefficients of the nine product type states as shown in Eq. (21) and (22) as a function of $\kappa_{12}z/\Delta$. There are nine curves in this plot and we have taken $\delta=\gamma/\Delta=0.05$. We focus on the first period of oscillation as it is the most important. The dashed curve represents the absolute probability amplitude of the input photon wave packet in the $j=0$ channel. As the wave packet propagates in the medium, simultaneous generation of $2M$ quantum FWM field occurs, creating a deterministic, perfectly efficient multiparty W-state of $2M$ Fock states. The solid line contains eight (almost identical) coefficients of the eight product-type states (see inset, each solid curve contains several indistinguishable curves). When the dashed curve reaches zero, the initial probe photon is fully absorbed and the amplitude of the multiparty entangled state of $2M$ Fock states reaches a maximum. Prior to this point, a region of $\kappa_{12}z/\Delta$ can be found [see inset and Eq. (22)] so that all $2M+1$ Fock states are perfectly entangled (within our approximation), giving a 100% efficient, deterministic multiparty $W$ state.

photon in the $k$th probe (FWM) channel and none in the others. Equation (21) represents a multiparty W-state of $2M$ Fock states (without the initial probe photon state) at the given propagation depth.

We note that a multiparty W-state of $(2M+1)$ Fock states can be generated at a suitable propagation distance $z$ [9]. In this region of $z$, all $(2M+1)$ product type states have nearly the same amplitude. This represents a multiparty W-state of $(2M+1)$ Fock states (see the inset of Fig. 3). At this depth inside the medium, we have

$$|\Psi\rangle = C'P\left(t-\frac{z}{c}\right)\sum_{k=-M}^{M}\hat{a}_{pk}^{\dagger}|\mathrm{vac}'\rangle, \qquad (22)$$

where $C'$ is a new propagation distance dependent $c$-number function and $|\mathrm{vac}'\rangle \equiv |0_{\omega_{p0}}\rangle|\mathrm{vac}\rangle$. Equation (22) demonstrates that a deterministic, multiparty W-state of a large number of Fock states can be achieved with this highly efficient, multichannel, pairwise FWM scheme.

## V. CONCLUSIONS AND SUMMARY

In summary, we have proposed a scheme to achieve maximum entanglement of a large number of Fock states using a perfectly efficient ultraslowly propagating, multichannel, pairwise FWM technique. We have presented solutions for the $N$ quantized probe waves and have shown that an ultraslowly propagating, single-photon quantum probe field via FWM can create a depth dependent entanglement of $N$ quantized probe waves. We have demonstrated analytically that the three-party W-state can indeed be achieved for the initial condition of one photon in the first probe wave and none in the others. We have also shown numerically there exists the multiparty W-state of $2M+1$ Fock states.

The perfectly efficient, multichannel, pairwise FWM technique discussed here may have profound importance in producing controllable entangled states. By adjusting the individual control field Rabi frequency, it is possible to access different entanglement subspaces, giving an interesting tunability to the scheme. Indeed, it is possible to null at several channels yet maintain efficient entanglement of other channels. Such ability of being able to access entangled subspaces may have important applications in QIS. We emphasize that the most important feature of the scheme presented here is the deterministic and perfectly efficient generation of a multiparty W-state of $N$ Fock states. This is to be contrasted with most of the previous schemes where the probability of entangling $N>3$ states is very small. The framework presented here may also serve as a useful basis for entangling multiple photons. Experimental demonstration of the scheme discussed in the present study and possible extension to multiphoton entanglement are currently under way. Further theoretical studies on efficient access to desirable entanglement subspaces and multiphoton entanglement have also been pursued.

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[6] We notice the trivial root of Eq. (9) is $\xi_1^{(0)}=0$ which gives a group velocity of $1/V_{g1}=1/c+\kappa_{12}/(N|\Omega_c|^2)$ for any $N$.
[7] Neglecting $\omega d$ terms in Eq. (4) implies a well behaved adia-

batic solution of Eq. (2) and all waves travel with the same group velocity very close to $c$ so that differences in propagation velocities are not important. It should be noted that Eq. (4) with both $\omega d$ terms kept can still be easily calculated numerically. By evaluating Eq. (4) in the region of $-\epsilon \leqslant \omega \leqslant \epsilon$ where $\epsilon \ll 1$ one can further examine the possible corrections due to velocity differences. This can provide useful insights on the possible extension of the present scheme to the ultraslow propagation regime using a very short medium so that peak separations between all FWM fields are still small.

[8] Similar results have been obtained with $M=3$ to 10, i.e., 7 to 21 channels.

[9] Experimentally, the length of the medium is usually fixed. Therefore, the tuning parameter will be the concentration (see the definition of $\kappa_{12}$).