Sub-Doppler and subnatural narrowing of an absorption line induced by interacting dark resonances in a tripod system

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A method is presented for obtaining sub-Doppler and subnatural narrowing and increased absorption of a spectral line. The ultranarrow spectral line is confined between two closely spaced electromagnetically induced transparency windows in a nearly degenerate tripod atomic system formed by an $F_g=1 \rightarrow F_e=0$ transition, split by a magnetic field. The system is driven by a σ-polarized pump and probed by a tunable π-polarized laser. It can be used to measure small magnetic fields and also as a magneto-optic switch.

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In recent years, it has been shown for a variety of four-level systems driven by two pumps that the probe absorption spectrum is characterized by a double dark resonance, consisting of two electromagnetically induced transparency (EIT) windows, separated by a sharp absorption peak [1–9]. In this paper, we show that in contrast to other four-level systems, the linewidth of the sharp absorption peak that appears in the probe absorption spectrum of a nearly degenerate tripod system can be made *simultaneously* subnatural and sub-Doppler for copropagating lasers, provided the absolute value of the pump detunings from resonance is smaller than the natural linewidth. The tripod configuration can be realized by interacting a tunable π-polarized probe with an $F_g=1 \rightarrow F_e=0$ atomic transition, driven by symmetrically detuned σ_+- and σ_--polarized pumps with equal Rabi frequencies [see Fig. 1(a)]. Alternatively, an analogous situation can be achieved by lifting the degeneracy of the $F_g=1$ state with a weak magnetic field and driving the system with a single σ-polarized pump at the resonance frequency of the degenerate $F_g=1 \rightarrow F_e=0$ transition [see Fig. 1(b)]. The nearly degenerate tripod system is thus uniquely suitable for high-resolution spectroscopy. In addition, the setup of Fig. 1(b) can be used to measure small magnetic fields and also as a magneto-optic switch.

The generic four-level system discussed by Lukin *et al.* [2] is based on a three-level Λ EIT system consisting of two low-lying states and an excited state, interacting with a strong pump on one transition and a weak probe on the other transition. The lower state of the pumped transition is coupled by an additional pump to another low-lying state. The absorption spectrum is similar to that of the tripod system when both pumps are resonant [2,6]. However, in contrast to the tripod system, Doppler broadening eliminates the central peak in the generic four-level system [10]. Only by choosing a special wave-vector configuration, which strongly limits the interaction region, can Doppler broadening be successfully overcome in experiments involving the generic system [4,7]. When the additional pump couples the lower state of the pumped transition to a higher rather than a lower-lying state [3,8], the Doppler-free spectrum again resembles that of the tripod system and, as before, Doppler broadening eliminates the central peak, leading to the observation of only a small change in the EIT spectrum for copropagating fields [3]. Suprisingly, however, a sharp peak with sub-Doppler *but not subnatural* linewidth has been observed for counter-propagating pumps [8].

Here, we investigate a tripod system consisting of three lower levels g_{1-3} and an excited level e, interacting with two pump lasers of frequency $\omega_{1,3}$ with equal Rabi frequencies $2V_1=2V_3=2V$, and a weak probe laser of frequency ω_2 with Rabi frequency $2V_2$, where $2V_i=\mu_{eg_i}E_i/\hbar$, $i=1-3$. The detunings of these lasers from their respective transitions are $\Delta_{1-3}=\omega_{eg_{1-3}}-\omega_{1-3}$ and, in the absence of the fields, the population is equally distributed amongst the Zeeman ground sublevels.

We now write the equations of motion for the interaction of the probe with the pumped tripod. We express the off-diagonal density-matrix elements ρ_{eg_2} and $\rho_{g_ig_j}$ in terms of

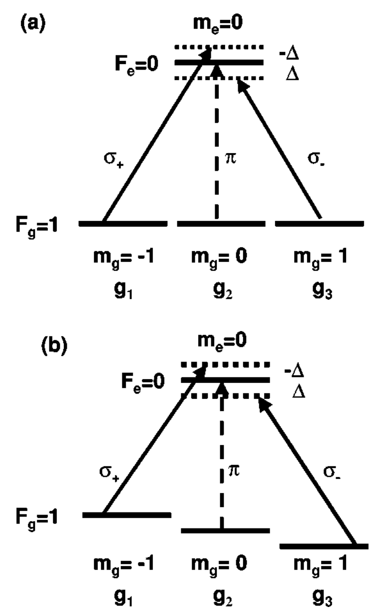

FIG. 1. (a) Degenerate $F_g=1 \rightarrow F_e=0$ transition interacting with symmetrically detuned σ_+- and σ_--polarized pumps and a π-polarized probe. (b) $F_g=1 \rightarrow F_e=0$ transition whose degeneracy is lifted by applying a magnetic field, interacting with a single σ-polarized pump and a π-polarized probe.

their Fourier amplitudes as $\rho_{eg_2}=\rho_{eg_2}(\omega_2)\exp(-i\omega_2 t)$, $\rho_{g_1g_2}=\rho_{g_1g_2}(\omega_2-\omega_1)\exp[-i(\omega_2-\omega_1)t]$, and $\rho_{g_3g_2}=\rho_{g_3g_2}(\omega_2-\omega_3)\exp[-i(\omega_2-\omega_3)t]$ and obtain the following set of linear equations for the Fourier amplitudes:

$$\dot{\rho}_{eg_2}(\omega_2) = -[i\Delta_2+\gamma_{eg_2}]\rho_{eg_2}(\omega_2) + i[V\rho_{g_1g_2}(\omega_2-\omega_1) + V\rho_{g_3g_2}(\omega_2-\omega_3) + V_2(\rho^0_{g_2g_2}-\rho^0_{ee})], \quad (1)$$

$$\dot{\rho}_{g_1g_2}(\omega_2-\omega_1) = -[i(\Delta_2-\Delta_1)+\gamma_{g_1g_2}]\rho_{g_1g_2}(\omega_2-\omega_1) + iV\rho_{eg_2}(\omega_2) - iV_2\rho^0_{g_1e}(-\omega_1), \quad (2)$$

$$\dot{\rho}_{g_3g_2}(\omega_2-\omega_3) = -[i(\Delta_2-\Delta_3)+\gamma_{g_3g_2}]\rho_{g_3g_2}(\omega_2-\omega_3) + iV\rho_{eg_2}(\omega_2) - iV_2\rho^0_{g_3e}(-\omega_3), \quad (3)$$

where $\rho^0_{g_2g_2}$ and ρ^0_{ee}, the population of the states g_2 and e, and $\rho^0_{eg_1}(\omega_1)$ and $\rho^0_{eg_3}(\omega_3)$ whose real and imaginary parts are proportional to the refraction and absorption of the pumps at frequencies ω_1 and ω_3, are calculated to all orders in the pump and to zero order in the probe. The complete equations for the interaction of the symmetrically detuned pumps ($\Delta_1=-\Delta_3$) with the $F_g=1\rightarrow F_e=0$ transition, assuming that $\gamma_{eg_i}=\gamma_{eg}$ ($i=1$–3) and $\gamma_{g_1g_2}=\gamma_{g_3g_2}=\gamma_{gg}$, where γ_{ij} is the transverse decay rate from state i to j, are given in Ref. [11] (see, also, [12]). Analytical solutions for $\rho^0_{eg_1}(\omega_1)$ and $\rho^0_{eg_3}(\omega_3)$ are given in terms of the pump-induced populations in Ref. [13]. We note that $\rho^0_{eg_1}(\omega_1)=-\rho^{0*}_{eg_3}(\omega_3)$ for symmetrically detuned pumps. As we are only interested in the steady-state results in this paper, we set the time derivatives of the Fourier amplitudes to zero in Eqs. (1)–(3).

The probe susceptibility $\chi(\omega_2)$ is given by [14] $\chi(\omega_2)=(N|\mu_{eg_2}|^2/\hbar V_2)\rho_{eg_2}(\omega_2)$, where N is the atomic number density, and the probe refraction and absorption are proportional to the real and imaginary parts of $\chi(\omega_2)$. By inserting Eqs. (2) and (3) into Eq. (1) we obtain an analytical expression for $\rho_{eg_2}(\omega_2)$, to all orders in V and to first order in V_2:

$$\rho_{eg_2}(\omega_2) = \frac{V_2(\rho^0_{g_2g_2}-\rho^0_{ee}) + VV_2\left[\dfrac{\rho^0_{g_1e}(-\omega_1)}{\Delta_1-\Delta_2+i\gamma_{gg}} + \dfrac{\rho^0_{g_3e}(-\omega_3)}{\Delta_3-\Delta_2+i\gamma_{gg}}\right]}{(\Delta_2-i\gamma_{eg}) + \dfrac{V^2}{\Delta_1-\Delta_2+i\gamma_{gg}} + \dfrac{V^2}{\Delta_3-\Delta_2+i\gamma_{gg}}}. \quad (4)$$

When the second term in the numerator can be neglected, we see from the denominator that when $\Delta_1=-\Delta_3=\Delta$, the probe absorption has minima at $\Delta_2=\pm\Delta$ (EIT windows) and maxima at $\Delta_2=0$ (central peak) and $\Delta_2=\pm(2V^2+\Delta^2)^{1/2}$ (Autler-Townes peaks). If, in addition, $\rho^0_{g_2g_2}=1$, Eq. (4) reduces to the expression obtained by Paspalakis and Knight [1].

We first consider a case where V is of the same order of magnitude as γ_{eg}, the natural linewidth, and $\Delta/\gamma_{eg_2}<1$. In Fig. 2(a), we use Eq. (4) to plot the probe absorption spectrum as a function of the probe detuning Δ_2/γ_{eg}, for two values of V/γ_{eg}. In Fig. 2(a), we see that the pump-induced broadening of the EIT windows results in *narrowing* of the central absorption line of the tripod. Since the position of the EIT windows is constant for a particular value of Δ, the half-width of the central peak must always be smaller than Δ which can be chosen such that $\Delta/\gamma_{eg}<1$, leading to a subnatural width [see Fig. 2(a)]. It should be pointed out that in Fig. 2(a), the absorption spectrum is 3 times more intense in the presence of the pumps than in their absence. This is due to pumping of all the population into the $m_g=0$ state which also leads to the second term in the numerator of Eq. (4) being negligible. For these parameters, γ_{gg} can safely be neglected. The pump-induced narrowing of the central peak can then be explained by considering the second term in the denominator of Eq.(4) which contains the sum of the two-photon coherences. This term is zero when $\Delta_2=0$, leading to a maximum in the absorption, and increases with increasing V as Δ_2 moves away from zero, leading to pump-induced narrowing of the central peak. If we describe the tripod system as the sum of two Λ systems sharing a common transition, these results are consistent with those of Vemuri *et al.* [15] for a Λ system interacting with a detuned pump.

We have seen from Fig. 2(a) that the width of the central peak decreases with increasing V or decreasing Δ. The question now arises as to how far this process can be continued before (a) complete optical pumping to the $m_g=0$ state, as assumed in Ref. [1], ceases to occur, (b) the second term in the numerator, ignored in Ref. [1], becomes important, and and (c) the central peak disappears and only a single EIT window remains as shown in Fig. 2(b). In order to answer these questions, we analyze Eq. (4) numerically for the case of very small detuning $\Delta=5\gamma_{gg}$ with $\gamma_{gg}/\gamma_{eg}=10^{-5}$. We obtain a well-resolved spectrum, similar to Fig. 2(a), provided V lies in the range $50\Delta-150\Delta$. In this range, the population in the $m_g=0$ state varies between 0.6 and 0.74 (with a maximum of 0.75 at 120Δ), due to insufficent pumping at low V and coherent population trapping (CPT) in the $m_g=\pm1$ states at high V, so that the overall absorption is reduced. In addition, the second term in the numerator is significant, due to insufficient pumping out of the $m_g=\pm1$ states, and has the effect of deepening the EIT dips and decreasing the central peak. When V increases beyond this range, the absorption peak decreases and is replaced by a very small and narrow

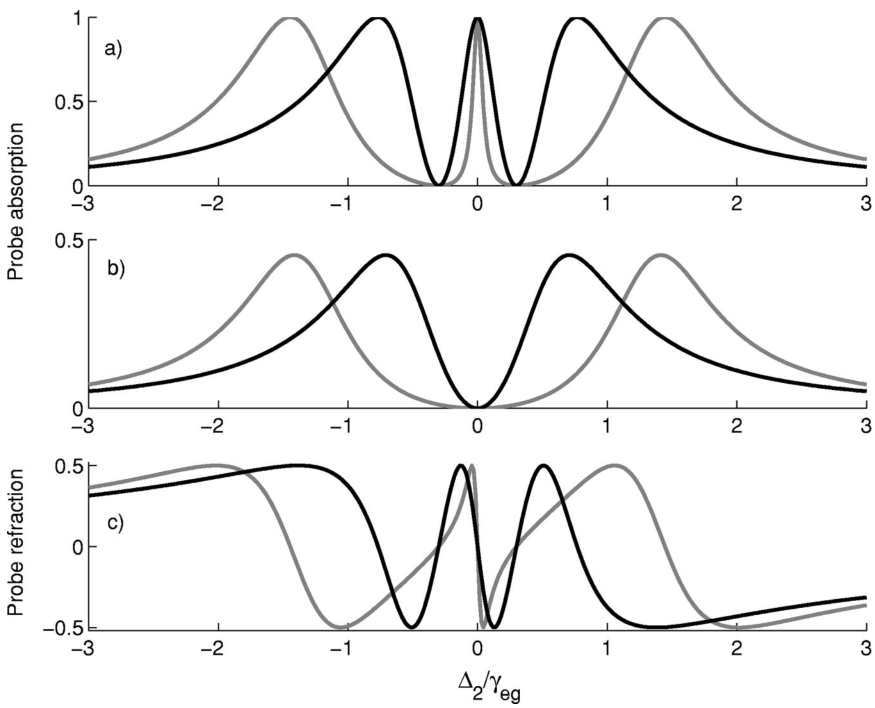

FIG. 2. (a), (b) Plot of $\mathrm{Im}\,\rho_{eg_2}(\omega_2)/(V_2/\gamma_{eg})$, which is proportional to the probe absorption, versus the probe detuning Δ_2/γ_{eg_2} for $V/\gamma_{eg}=0.5$ (black) and $V/\gamma_{eg}=1$ (grey). In (a) $\Delta/\gamma_{eg}=0.3$ and in (b) $\Delta/\gamma_{eg}=0$. Note that the two EIT windows separated by an absorption peak at $\Delta_2/\gamma_{eg}=0$ in (a) are replaced by a single EIT window with a minimum at $\Delta_2/\gamma_{eg}=0$ in (b). (c) Plot of $\mathrm{Re}\,\rho_{eg_2}(\omega_2)/(V_2/\gamma_{eg})$ which is proportional to the probe refraction versus Δ_2/γ_{eg} for the same parameters as in (a). In all plots $\gamma_{gg}/\gamma_{eg}=10^{-5}$.

stimulated emission peak when $V\simeq 10^3\Delta$. This emission peak derives from the second term in the numerator. For even higher pump Rabi frequencies, Δ/V becomes very small and the two EIT windows fuse into a single one, as in Fig. 2(b).

The probe refraction is shown in Fig. 2(c). The dispersion is negative at $\Delta_2=0$ and becomes steeper as the absorption peak becomes narrower. This effect is similar to the steep negative dispersion observed in systems that exhibit EIA [12,16]. As expected [17,18], the dispersion is positive in the EIT windows and becomes less steep with increasing V, in line with the increase in width of the EIT window.

For the system of Fig. 1(b) which involves a single pump laser, the degeneracy is lifted by applying a magnetic field such that $\Delta=-\Delta_1=\Delta_3=g\mu_B B$. Measuring the distance between the two minima in the absorption, 2Δ, gives a straightforward optical method for determining the magnetic field strength. When the magnetic field is switched off, the absorption is given by Eq. (4) with $\Delta=0$ which leads to EIT as shown in Fig. 2(b). Maximum transparency now occurs at

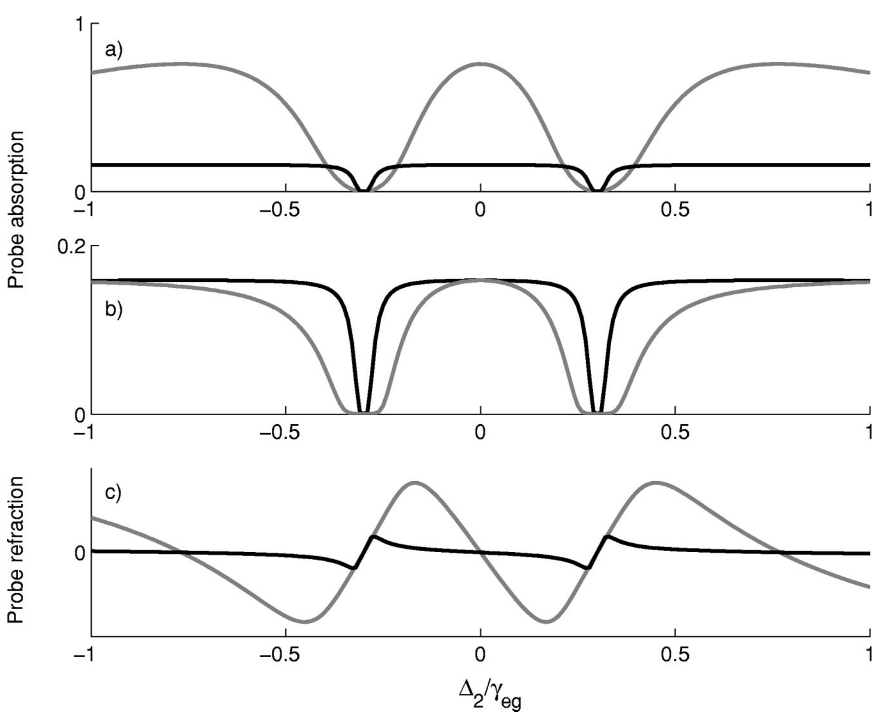

FIG. 3. (a), (b) Doppler-broadened version of probe absorption spectrum of Fig. 1(a) for $\Delta/\gamma_{eg}=0.3$. In (a), $V/\gamma_{eg}=0.5$, $D/\gamma_{eg}=1$ (grey) and $D/\gamma_{eg}=10$ (black). In (b), $D/\gamma_{eg}=10$, $V/\gamma_{eg}=0.5$ (black) and $V/\gamma_{eg}=1$ (grey). (c) Doppler-broadened version of probe refraction of Fig. 1(c) for same parameters as in (a). In all plots $\gamma_{gg}/\gamma_{eg}=10^{-5}$.

$\Delta_2=0$ instead of the maximum absorption shown in Fig. 2(a). Thus, optimal magnetic switching from absorption to transmission is obtained when $\omega_1=\omega_2=\omega_3$, showing that an efficient magneto-optic switching device can be achieved in a degenerate tripod system using a single laser, together with appropriate polarizers.

We now consider the effect of Doppler broadening on the probe absorption and refraction. The probe absorption spectrum is shown in Fig. 3(a) for two values of the Doppler width D. The first point to notice is that, as is the case for a Λ system [15,17–21], the positions of the EIT windows do not change with Doppler broadening but their widths become narrower, leading to an increase in width of the central absorption peak. However, by choosing the pump detuning $\Delta/\gamma_{eg_2}<1$, the subnatural and sub-Doppler width of the central peak will be preserved for all values of D. In Fig. 3(b), we plot the probe absorption spectrum for two values of V/γ_{eg} and the same value of D. We see that the phenomenon of pump-induced line narrowing persists even in the presence of Doppler broadening, although with a larger width.

The Doppler broadened refraction is shown in Fig. 3(c) for the same parameters as in Figs. 3(a). We see that, as for the Λ system, Doppler broadening does not change the dispersion in the EIT windows [17,18], but does shorten the range of probe detuning over which the dispersion is constant, in line with the narrowing of the EIT windows. The negative dispersion at line center becomes less steep as the Doppler broadening increases.

In certain respects, the central peak in the tripod system is similar to the electromagnetically induced absorption peak observed in the probe absorption spectrum of $F_g\rightarrow F_e=F_g+1$, $F_g>0$, degenerate two-level systems [12,16]. Both are amplified and narrow with respect to the unpumped system, are accompanied by steep negative dispersion [22], and occur when the probe is at resonance with the transition. However, the tripod central line is narrowed because of the closely spaced EIT windows whereas the EIA is created and narrowed by transfer of coherence [23]. Finally, the absorption peak in the tripod is narrowed with increasing pump intensity and broadened with increasing Doppler width, whereas the EIA peak is broadened with increasing power intensity and narrowed with increasing Doppler width [10,23].

In conclusion, we have demonstrated that in a simple nearly degenerate tripod system interacting with a σ-polarized pump and a π-polarized probe in the presence of a small magnetic field, a strong and narrow absorption feature appears exactly at line center, flanked by two EIT windows. The absorption feature becomes narrower with increasing pump power and can be made subnatural even in the presence of Doppler broadening. Since the distance between the EIT minima is determined by the applied magnetic field, the system can be used as a magnetometer. In the absence of the magnetic field, the narrow absorption peak disappears and the system becomes transparent at line center. It can thus be used as a magneto-optic switch.

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