Relativistic electronic dressing in laser-assisted ionization of atomic hydrogen by electron impact

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Within the framework of the coplanar binary geometry where it is justified to use plane wave solutions for the study of the $(e,2e)$ reaction and in the presence of a circularly polarized laser field, we introduce as a first step the Dirac-Volkov plane wave Born approximation 1 where we take into account only the relativistic dressing of the incident and scattered electrons. Then, we introduce the Dirac-Volkov plane wave Born approximation 2 where we take totally into account the relativistic dressing of the incident, scattered, and ejected electrons. We then compare the corresponding triple differential cross sections for laser-assisted ionization of atomic hydrogen by electron impact both for the nonrelativistic and the relativistic regime.

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I. INTRODUCTION

Ehrhardt *et al.* (1969) [1] were the first to conduct electron impact ionization experiments in which the two outgoing electrons are detected in coincidence after angular and energy analysis. The first theoreticians who proposed such type of experiment were Smirnov and Neudachin (1966) [2]. These experiments now are commonly referred to as $(e,2e)$. Since then, the $(e,2e)$ reaction has been studied extensively in the nonrelativistic kinematic domain [Camilloni *et al.* (1972) [3], Weigold *et al.* (1973) [4], van der Wiel (1973) [5], and Brion (1975) [6]]. All these studies show that the nonrelativistic $(e,2e)$ reaction is a very sensitive test of the target electronic structure and the electron impact ionization reaction mechanism. For the relativistic domain, the first electron impact ionization experiments were conducted by Dangerfield and Spicer (1975) [7], then by Hoffman *et al.* (1979) [8] and by Anholt (1979) [9]. Total cross sections with relativistic electrons were measured for the K and L shells of heavy elements. Some theoreticians [Scoffield (1978) [10], and Moiseiwitsch and Stockman (1980) [11]] proposed models for total ionization cross sections. Finally, Fuss *et al.* (1982) [12] proposed a model of an $(e,2e)$ reaction they called a binary $(e,2e)$ reaction in which the maximum momentum transfer occurs, that is, a reaction where the outgoing electrons have equal energy. This type of reaction has been since then the most successful for probing atomic, molecular, and solid state structure. In a report devoted both to experimental and theoretical developments in the study of relativistic $(e,2e)$ processes, Nakel and Whelan (1999) [13] reviewed the goals of these investigations aimed at gaining a better understanding of the innershell ionization process by relativistic electrons up to the highest atomic numbers and probing the quantum mechanical Coulomb problem in the regime of high energies (up to 500 keV) and strong fields. With the advent of the laser field, many theoretical models have been proposed [14] mainly in the nonrelativistic domain whereas in the relativistic domain we can only quote the work of Reiss (1990) [15] and that of Crawford and Reiss (1994,1998) [16] who studied the relativistic ionization of hydrogen (without electron impact) by a linearly polarized light. They focused their work on the calculations of the differential transition rates and have shown that strong field atomic stabilization is enhanced by relativistic effects.

In this paper, we present a theoretical model for the relativistic electronic dressing in laser-assisted ionization of atomic hydrogen by electron impact with a circularly polarized laser field and in order to check the consistency of our calculations, we begin our study in the absence of the laser field. As we devote this analysis to atomic hydrogen, all distortion effects mentioned in [13] need not be addressed since the atomic number we deal with is that of atomic hydrogen, that is $Z=1$. It is checked first that, in the absence of the laser field and working in the coplanar binary geometry where the kinetic energies of the scattered electron and the ejected electron are nearly the same, it is justified to use plane wave solutions for the study of the $(e,2e)$ reaction. Indeed, this particular geometry is such that the ejected electron does not feel the Coulomb influence of the atomic target and can be described by a plane wave in the nonrelativistic as well as in the relativistic domain. Then, in the presence of a circularly polarized laser field, we introduce as a first step the DVPWBA1 (Dirac-Volkov plane wave Born approximation 1) where we take into account only the relativistic dressing of the incident and scattered electrons. It is shown that this approximation introduces an asymmetry in the description of the scattering process since it does not allow photon exchange between the laser field and the ejected electron and its domain of validity is only restricted to very weak fields and nonrelativistic electron kinetic energy. Then, we introduce the DVPWBA2 (Dirac-Volkov plane wave Born approximation 2) where we take totally into account the relativistic dressing of the incident, scattered, and ejected electrons.

The organization of this paper is as follows: In Sec. II we present the relativistic formalism of the $(e,2e)$ reaction in the

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absence of the laser field RPWBA (relativistic plane wave Born approximation) and we compare it in the nonrelativistic domain with the NRPWBA (nonrelativistic plane wave Born approximation) as well as the NRCBA (nonrelativistic Coulomb-Born approximation). In Sec. III we introduce the DVPWBA1. This approximation is introduced as a first step. In Sec. IV we introduce the DVPWBA2 in which we take full account of the relativistic electronic dressing of the incident, scattered, and ejected electrons. This approximation is more founded on physical grounds since the ejected electron can also exchange photons (absorption or emission) with the laser field. This more complete description of the incoming and outgoing electrons allows us to investigate the relativistic domain. In Sec. V we discuss the results we have obtained and we end by a brief conclusion in Sec. VI. Throughout this work, atomic units (a.u.) are used ($\hbar = m_e = e = 1$) where m_e is the electron mass and TDCS stands for triple differential cross section.

II. THE TDCS IN THE ABSENCE OF THE LASER FIELD

The transition matrix element for the direct channel (we neglect exchange effects) is given by

$$S_{fi} = -\frac{i}{c}\int_{-\infty}^{+\infty} dx^0 \langle \psi_{p_f}(x_1)\phi_f(x_2)|V_d|\psi_{p_i}(x_1)\phi_i(x_2)\rangle, \quad (1)$$

where $V_d = 1/r_{12} - 1/r_1$ is the direct interaction potential ($t_1 = t_2 = t \Rightarrow x_1^0 = x_2^0 = x^0$) and in the RPWBA, $\psi_{p_f}(x_1)$ is the wave function describing the scattered electron

$$\psi_{p_f}(x_1) = \frac{u(p_f,s_f)}{\sqrt{2E_f V}} e^{-ip_f x_1}, \quad (2)$$

given by a free Dirac solution normalized to the volume V. For the incident electron, we use

$$\psi_{p_i}(x_1) = \frac{u(p_i,s_i)}{\sqrt{2E_i V}} e^{-ip_i x_1}. \quad (3)$$

For the atomic target, $\phi_i(x_2) = \phi_i(t,\mathbf{r}_2)$ is the relativistic wave function of atomic hydrogen in its ground state. For the ejected electron, we use again a free Dirac solution normalized to the volume V and $\phi_f(x_2)$ is given by

$$\phi_f(x_2) = \psi_{p_B}(x_2) = \frac{u(p_B,s_B)}{\sqrt{2E_B V}} e^{-ip_B x_2}. \quad (4)$$

The free spinor $u(p,s)$ is such that $\overline{u}(p,s)u(p,s) = 2c^2$ and $u^\dagger(p,s)u(p,s) = 2E$. Using the standard methods of QED [17], we obtain for the unpolarized TDCS

$$\begin{aligned}\frac{d\overline{\sigma}}{dE_B d\Omega_B d\Omega_f} &= \frac{|\mathbf{p}_f||\mathbf{p}_B|}{|\mathbf{p}_i|c^4}\left(\sum_{s_B}|\overline{u}(p_B,s_B)\gamma^0|^2\right) \\ &\times \frac{(2E_iE_f/c^2 - p_ip_f + c^2)}{|\mathbf{p}_f - \mathbf{p}_i|^4} \\ &\times |\Phi_{1,1/2,1/2}(\mathbf{q}_1 = \boldsymbol{\Delta} - \mathbf{p}_B) \\ &- \Phi_{1,1/2,1/2}(\mathbf{q}_0 = -\mathbf{p}_B)|^2. \end{aligned} \quad (5)$$

The sum over the spins of the ejected electron gives

$$\sum_{s_B}|\overline{u}(p_B,s_B)\gamma^0|^2 = 4E_B. \quad (6)$$

The functions $\Phi_{1,1/2,1/2}(\mathbf{q})$ are the Fourier transforms of the relativistic atomic hydrogen wave functions

$$\Phi_{1,1/2,1/2}(\mathbf{q}) = (2\pi)^{(-3/2)}\int d\mathbf{r}_2 e^{i\mathbf{q}\cdot\mathbf{r}_2}\Psi_{n=1,j=1/2,m=1/2}(\mathbf{r}_2), \quad (7)$$

and $\boldsymbol{\Delta} = \mathbf{p}_i - \mathbf{p}_f$ is the momentum transfer. This TDCS is to be compared with the corresponding one in the NRPWBA (nonrelativistic plane wave Born approximation)

$$\frac{d\overline{\sigma}}{dE_B d\Omega_B d\Omega_f} = \frac{2^7}{(2\pi)^2}\frac{|\mathbf{p}_f||\mathbf{p}_B|}{|\mathbf{p}_i|}\frac{1}{|\boldsymbol{\Delta}|^4}\left\{\frac{1}{(\mathbf{q}_1^2+1)^2} - \frac{1}{(\mathbf{q}_0^2+1)^2}\right\}^2, \quad (8)$$

where $\mathbf{q}_1 = \boldsymbol{\Delta} - \mathbf{p}_B$ and $\mathbf{q}_0 = -\mathbf{p}_B$ to the TDCS in the NRCBA (nonrelativistic Coulomb-Born approximation)

$$\frac{d\sigma^{CB}}{dE_B d\Omega_B d\Omega_f} = \frac{|\mathbf{p}_f||\mathbf{p}_B|}{|\mathbf{p}_i|}|f_{ion}^{CB1}|^2, \quad (9)$$

where f_{ion}^{CB1} is the first Coulomb-Born amplitude corresponding to the ionization of atomic hydrogen by electron impact [18]

$$f_{ion}^{CB1} = -\frac{2}{|\boldsymbol{\Delta}|^2}M_{1s}(\boldsymbol{\Delta},\mathbf{p}_B). \quad (10)$$

The quantity $M_{1s}(\boldsymbol{\Delta},\mathbf{p}_B)$ is easily deduced from the Nordsieck integral [19]

$$\begin{aligned} I(\lambda) &= \int e^{-\lambda r}\frac{e^{i\mathbf{q}\cdot\mathbf{r}}}{r}\,{}_1F_1\left(\frac{i}{p_B},1,i(p_Br + \mathbf{p}_B\cdot\mathbf{r})\right)d\mathbf{r} \\ &= \frac{4\pi}{q^2+\lambda^2}\left[\frac{q^2+\lambda^2+2\mathbf{q}\cdot\mathbf{p}_B - 2i\lambda p_B}{q^2+\lambda^2}\right]^{-i/p_B}, \end{aligned}$$

giving the well-known result

$$M_{1s}(\boldsymbol{\Delta},\mathbf{p}_B) = \frac{e^{\pi/2p_B}}{2\sqrt{2}\pi^2}\Gamma\left(1 - \frac{i}{p_B}\right)\left(-\frac{dI(\lambda)}{d\lambda}\right)_{\lambda=1}. \quad (11)$$

III. THE TDCS IN THE PRESENCE OF THE LASER FIELD: THE DVPWBA1

We begin our study of the $(e,2e)$ reaction by considering first the DVPWBA1 where we only take into account the dressing of the incident and the scattered electron. The laser field is circularly polarized. Again, the transition matrix element for the direct channel is

$$S_{fi} = -\frac{i}{c}\int_{-\infty}^{+\infty} dx^0 \langle \psi_{q_f}(x_1)\phi_f(x_2)|V_d|\psi_{q_i}(x_1)\phi_i(x_2)\rangle, \quad (12)$$

where ($t_1 = t_2 = t \Rightarrow x_1^0 = x_2^0 = x^0$) and in the DVPWBA1, $\psi_{q_f}(x_1)$ is the Dirac-Volkov wave function normalized to the volume V describing the scattered electron

$$\psi_{q_f}(x_1)=\left[1+\frac{\not k \not A_{(1)}}{2c(kp_f)}\right]\frac{u(p_f,s_f)}{\sqrt{2Q_fV}}e^{is_f(x_1)}, \qquad (13)$$

where $A_{(1)}=a_1\cos(\phi_1)+a_2\sin(\phi_1)$ is the four potential of the laser field, $\phi_1=kx_1=k_0x_1^0-\mathbf{k}\cdot\mathbf{x}_1=wt-\mathbf{k}\cdot\mathbf{x}_1$ is the phase of the laser field, and w its frequency. Q is the total energy acquired by the electron in the presence of a laser field and is given by

$$Q=E-\frac{a^2w}{2c^2(kp)}. \qquad (14)$$

The phase $s_f(x_1)$ is given by

$$s_f(x_1)=-q_fx_1-\frac{a_1p_f}{c(kp_f)}\sin(\phi_1)+\frac{a_2p_f}{c(kp_f)}\cos(\phi_1). \qquad (15)$$

For the incident electron, we use

$$\psi_{q_i}(x_1)=\left[1+\frac{\not k \not A_{(1)}}{2c(kp_i)}\right]\frac{u(p_i,s_i)}{\sqrt{2Q_iV}}e^{is_i(x_1)}, \qquad (16)$$

with the phase $s_i(x_1)$ given by

$$s_i(x_1)=-q_ix_1-\frac{a_1p_i}{c(kp_i)}\sin(\phi_1)+\frac{a_2p_i}{c(kp_i)}\cos(\phi_1), \qquad (17)$$

where the four vector q^μ is such that

$$q^\mu=p^\mu-\frac{a^2}{2c^2(kp)}k^\mu, \qquad (18)$$

and $a^2=a^\mu a_\mu=a_1^2=a_2^2$. For the atomic target, $\phi_i(x_2)=\phi_i(t,\mathbf{r}_2)=e^{-i\varepsilon_bt}\phi_i(\mathbf{r}_2)$ is the relativistic wave function of atomic hydrogen in its ground state and $\varepsilon_b=c^2(\sqrt{1-\alpha^2}-1)$ is the binding energy of the ground state of atomic hydrogen with $\alpha=1/c$ the fine structure constant. For the ejected electron, we use a free Dirac solution normalized to the volume V and $\phi_f(x_2)$

$$\phi_f(x_2)=\psi_{p_B}(x_2)=\frac{u(p_B,s_B)}{\sqrt{2E_BV}}e^{-ip_Bx_2}. \qquad (19)$$

Using the standard methods of QED, we have for the unpolarized TDCS

$$\frac{d\overline{\sigma}}{dE_Bd\Omega_Bd\Omega_f}=\sum_{s=-\infty}^{+\infty}\left.\frac{d\overline{\sigma}^{(s)}}{dE_Bd\Omega_Bd\Omega_f}\right|_{Q_f=Q_i+sw+\varepsilon_b-E_B}, \qquad (20)$$

where the expression of $d\overline{\sigma}^{(s)}/dE_Bd\Omega_Bd\Omega_f$ is

$$\frac{d\overline{\sigma}^{(s)}}{dE_Bd\Omega_Bd\Omega_f}=\frac{1}{2}\frac{|\mathbf{q}_f||\mathbf{p}_B|}{|\mathbf{q}_i|c^6}\frac{\left(\sum_{s_i,s_f}|M_{fi}^{(s)}|^2/2\right)}{|\mathbf{q}_f-\mathbf{q}_i-s\mathbf{k}|^4}\times(4E_B)|\Phi_{1,1/2,1/2}(\mathbf{q}=\boldsymbol{\Delta}_s-\mathbf{p}_B)-\Phi_{1,1/2,1/2}(\mathbf{q}=-\mathbf{p}_B)|^2. \qquad (21)$$

The sum $(\sum_{s_i,s_f}|M_{fi}^{(s)}|^2/2)$ has already been evaluated in a previous work [20] and $\boldsymbol{\Delta}_s=\mathbf{q}_i-\mathbf{q}_f+s\mathbf{k}$ is the momentum transfer in presence of the laser field. This TDCS is compared to the corresponding TDCSs in the nonrelativistic regime. On the one hand, the calculations within the framework of the NRPWBA1 (where the incident and scattered electrons are described by nonrelativistic Volkov plane waves whereas the ejected electron is described by a nonrelativistic free plane wave) give

$$\frac{d\overline{\sigma}^{\rm NRPWBA1}}{dE_Bd\Omega_Bd\Omega_f}\sum_{s=-\infty}^{+\infty}\left.\frac{d\overline{\sigma}^{(s)}}{dE_Bd\Omega_Bd\Omega_f}\right|_{E_f=E_i+sw+\varepsilon_{1s}-E_B}, \qquad (22)$$

with

$$\frac{d\overline{\sigma}^{(s)}}{dE_Bd\Omega_Bd\Omega_f}=\frac{2^7}{(2\pi)^2}\frac{|\mathbf{p}_f||\mathbf{p}_B|}{|\mathbf{p}_i|}\frac{J_s^2(z_{NR})}{|\mathbf{p}_f-\mathbf{p}_i-s\mathbf{k}|^4}\times\left\{\frac{1}{(\mathbf{q}_{1s}^2+1)^2}-\frac{1}{(\mathbf{q}_{0s}^2+1)^2}\right\}^2, \qquad (23)$$

where $\varepsilon_{1s}=-0.5$ a.u. is the nonrelativistic binding energy of atomic hydrogen in its ground state, $\mathbf{q}_{1s}=\mathbf{p}_i+s\mathbf{k}-\mathbf{p}_f-\mathbf{p}_B=\boldsymbol{\Delta}_s^{NR}-\mathbf{p}_B$ and $\mathbf{q}_{0s}=-\mathbf{p}_B$. On the other hand, the calculations within the framework of the NRCBA1 (where the incident and scattered electrons are described by nonrelativistic Volkov plane waves whereas the ejected electron is described by a Coulomb wave function) give

$$\frac{d\overline{\sigma}^{\rm NRCBA1}}{dE_Bd\Omega_Bd\Omega_f}=\sum_{s=-\infty}^{+\infty}\left.\frac{d\overline{\sigma}^{(s)}}{dE_Bd\Omega_Bd\Omega_f}\right|_{E_f=E_i+sw+\varepsilon_{1s}-E_B}, \qquad (24)$$

with

$$\frac{d\overline{\sigma}^{(s)}}{dE_Bd\Omega_Bd\Omega_f}=\frac{1}{2\pi^4}\frac{|\mathbf{p}_f||\mathbf{p}_B|}{|\mathbf{p}_i|}\frac{J_s^2(z_{NR})}{|\mathbf{p}_f-\mathbf{p}_i-s\mathbf{k}|^4}e^{\pi/p_B}\times\left|\Gamma\left(1-\frac{i}{p_B}\right)\right|^2|I(\mathbf{q}_s=\mathbf{p}_i-\mathbf{p}_f+s\mathbf{k}-\mathbf{p}_B)|^2. \qquad (25)$$

Note that we may write $\mathbf{q}_s=\mathbf{p}_i-\mathbf{p}_f+s\mathbf{k}-\mathbf{p}_B=\boldsymbol{\Delta}_s^{NR}-\mathbf{p}_B$. The result for $I(\mathbf{q}_s=\boldsymbol{\Delta}_s^{NR}-\mathbf{p}_B)$ is

$$I(\mathbf{q}_s=\mathbf{p}_i-\mathbf{p}_f+s\mathbf{k}-\mathbf{p}_B)=\frac{16\pi}{(\mathbf{q}_s^2+1)^{2-i/p_B}}\frac{\boldsymbol{\Delta}_s^{NR}[\boldsymbol{\Delta}_s^{NR}-\mathbf{p}_B(1+i/p_B)]}{[(\boldsymbol{\Delta}_s^{NR})^2-(p_B+i)^2]^{1+i/p_B}}. \qquad (26)$$

In the expressions of the last two nonrelativistic TDCSs, the argument of the ordinary Bessel functions is given by

$$z_{NR}=\frac{|a|}{cw}|\boldsymbol{\Delta}_s^{NR}|. \qquad (27)$$

IV. THE TDCS IN THE PRESENCE OF THE LASER FIELD: THE DVPWBA2

We now take into account the electronic relativistic dressing of all electrons which are described by Dirac-Volkov plane waves normalized to the volume V. This will give rise to a new trace to be calculated but it will turn out that taking into account the relativistic electronic dressing of the ejected electron amounts simply to introduce a new sum on the l_B photons that can be exchanged with the laser field. The transition amplitude in the DVPWBA2 is now given by

$$S_{fi} = -\frac{i}{c}\int_{-\infty}^{+\infty} dx^0 \langle \psi_{q_f}(x_1)\phi_f(x_2)|V_d|\psi_{q_i}(x_1)\phi_i(x_2)\rangle. \quad (28)$$

The difference between DVPWBA1 and DVPWBA2 is related to the way we choose $\phi_f(x_2)$. Now, the Dirac-Volkov wave function for the ejected electron is such that

$$\phi_f(x_2) = \psi_{q_B}(x_2) = \left[1 + \frac{\not{k}\not{A}_{(2)}}{2c(kp_B)}\right]\frac{u(p_B,s_B)}{\sqrt{2Q_B V}}e^{is_B(x_2)}, \quad (29)$$

where $A_{(2)}=a_1\cos(\phi_2)+a_2\sin(\phi_2)$ is the four potential of the laser field felt by the ejected electron, $\phi_2=kx_2=k_0x_2^0-\mathbf{k}\cdot\mathbf{x}_2=wt-\mathbf{k}\cdot\mathbf{x}_2$ is the phase of the laser field, and w its frequency. Proceeding along the same line as before, we get for the unpolarized TDCS

$$\frac{d\overline{\sigma}}{dE_B d\Omega_B d\Omega_f} = \sum_{s,l_B=-\infty}^{+\infty} \left.\frac{d\overline{\sigma}^{(s,l_B)}}{dE_B d\Omega_B d\Omega_f}\right|_{Q_f=Q_i+(s+l_B)w+\varepsilon_b-Q_B}, \quad (30)$$

with

$$\frac{d\overline{\sigma}^{(s,l_B)}}{dE_B d\Omega_B d\Omega_f} = \frac{1}{2}\frac{|\mathbf{q}_f||\mathbf{q}_B|}{|\mathbf{q}_i|c^6}\frac{(\sum_{s_i,s_f}|M_{fi}^{(s)}|^2/2)}{|\mathbf{q}_f-\mathbf{q}_i-s\mathbf{k}|^4}\sum_{s_B}|\overline{u}(p_B,s_B)\Gamma_{l_B}\gamma^0|^2|\Phi_{1,1/2,1/2}(\mathbf{q}=\mathbf{\Delta}_{s+l_B}-\mathbf{q}_B) - \Phi_{1,1/2,1/2}(\mathbf{q}=-\mathbf{q}_B+l_B\mathbf{k})|^2. \quad (31)$$

The quantity $\mathbf{\Delta}_{s+l_B}$ is simply given by $\mathbf{\Delta}_{s+l_B}=\mathbf{q}_i-\mathbf{q}_f+(s+l_B)\mathbf{k}$. Introducing the factor $c(p_B)=1/[2c(kp_B)]$, the symbol Γ_{l_B} is defined as

$$\Gamma_{l_B} = B_{l_B}(z_B) + c(p_B)[\not{a}_1\not{k}B_{1l_B}(z_B) + \not{a}_2\not{k}B_{2l_B}(z_B)], \quad (32)$$

where the three quantities $B_{l_B}(z_B)$, $B_{1l_B}(z_B)$, and $B_{2l_B}(z_B)$ are, respectively, given by

$$B_{l_B}(z_B) = J_{l_B}(z_B)e^{il_B\phi_{0B}},$$

$$B_{1l_B}(z_B) = \{J_{l_B+1}(z_B)e^{i(l_B+1)\phi_{0B}} + J_{l_B-1}(z_B)e^{i(l_B-1)\phi_{0B}}\}/2,$$

$$B_{2l_B}(z_B) = \{J_{l_B+1}(z_B)e^{i(l_B+1)\phi_{0B}} - J_{l_B-1}(z_B)e^{i(l_B-1)\phi_{0B}}\}/2i, \quad (33)$$

where $z_B=[|a|/c(k\cdot p_B)]\sqrt{(\hat{\mathbf{y}}\cdot\mathbf{p}_B)^2+(\hat{\mathbf{x}}\cdot\mathbf{p}_B)^2}$ is the argument of the ordinary Bessel functions that will appear in the calculations and the phase ϕ_{0B} is defined by

$$\phi_{0B} = \arctan[(\hat{\mathbf{y}}\cdot\mathbf{p}_B)/(\hat{\mathbf{x}}\cdot\mathbf{p}_B)]. \quad (34)$$

The sum over the spins of the ejected electron can be transformed to traces of gamma matrices. Using REDUCE [21], we find

$$\sum_{s_B}|\overline{u}(p_B,s_B)\Gamma_{l_B}\gamma^0|^2 = 4\{E_BJ_{l_B}^2(z_B) - wc(p_B)[\cos(\phi_{0B})(a_1p_B) + \sin(\phi_{0B})(a_2p_B)]J_{l_B}(z_B)[J_{l_B+1}(z_B) + J_{l_B-1}(z_B)] - a^2w(kp_B)[c(p_B)]^2 \times[J_{l_B+1}^2(z_B) + J_{l_B-1}^2(z_B)]\}. \quad (35)$$

In the absence of the laser field, only the term $4E_BJ_{l_B}^2(z_B=0)\delta_{l_B,0}=4E_B$ contributes, which was to be expected. Once again, one encounters terms proportional to $\cos(\phi_{0B})$ as well as to $\sin(\phi_{0B})$ which contributes to the sum over the spins of the ejected electron. We compare this TDCS with the corresponding cross sections in the framework of the NRPWBA2 (where the incident, scattered, and ejected electrons are described by nonrelativistic Volkov plane waves).

V. RESULTS AND DISCUSSION

A. In the absence of the laser field

We begin our discussion by the kinematics of the problem. In the absence of the laser field, there is no dressing of angular coordinates [22] and we choose a geometry where $\mathbf{p}_i$ is along the Oz axis ($\theta_i=\phi_i=0$). For the scattered electron, we choose ($\theta_f=45°$, $\phi_f=0$) and for the ejected electron we choose $\phi_B=180°$ and the angle θ_B varies from 0° to 360°. This is an angular situation where we have a coplanar geometry. Spin effects are fully included and we use an exact relativistic description of the electrons and the atomic target. In order to check the validity of the coplanar binary geometry, we begin first by comparing the three TDCSs (RPWBA, NRPWBA, and NRCBA) in the nonrelativistic domain. In the expression of the Fourier transforms $\Phi_{1,1/2,1/2}(\mathbf{q}=\mathbf{\Delta}-\mathbf{p}_B)$ and $\Phi_{1,1/2,1/2}(\mathbf{q}=-\mathbf{p}_B)$, one has to determine the angle between $\mathbf{\Delta}-\mathbf{p}_B$ and $\mathbf{p}_i$ on the one hand and the angle between $-\mathbf{p}_B$ and $\mathbf{p}_i$ on the other hand. Keeping in mind that $\mathbf{p}_i$ is along the Oz axis, one finds

$$\cos(\widehat{\mathbf{\Delta}-\mathbf{p}_B,\mathbf{p}_i}) = \frac{[p_i - p_f\cos\theta_f - p_B\cos(\theta_B)]}{|\mathbf{\Delta}-\mathbf{p}_B|}, \quad (36)$$

while $\cos(\widehat{-\mathbf{p}_B,\mathbf{p}_i})=-\cos(\theta_B)$ from which one deduces the corresponding angles. To have an idea of how the ejected electron loses its Coulombian behavior, we begin with a process where the incident electron kinetic energy is $T_i=1350$ eV, and the ejected electron kinetic energy is $T_B=574.5$ eV. In Fig. 1, we see that both RPWBA and NRPWBA give nearly the same results whereas NRCBA gives a higher TDCS due to the fact that the ejected electron still feels the influence of the atomic field. Increasing this energy of the ejected electron from 574.5 to 674.5 eV gives rise to almost three indistinguishable curves. This situation is shown in Fig. 2. As the incident electron kinetic energy is increased to 2700 eV which corresponds to a relativistic parameter $\gamma=(1-v^2/c^2)^{-1/2}=1.0053$, that of the ejected electron is increased to 1349.5 eV; we have very good agreement between the three TDCSs. This is a crucial test for the models we will develop in the presence of a laser field since the

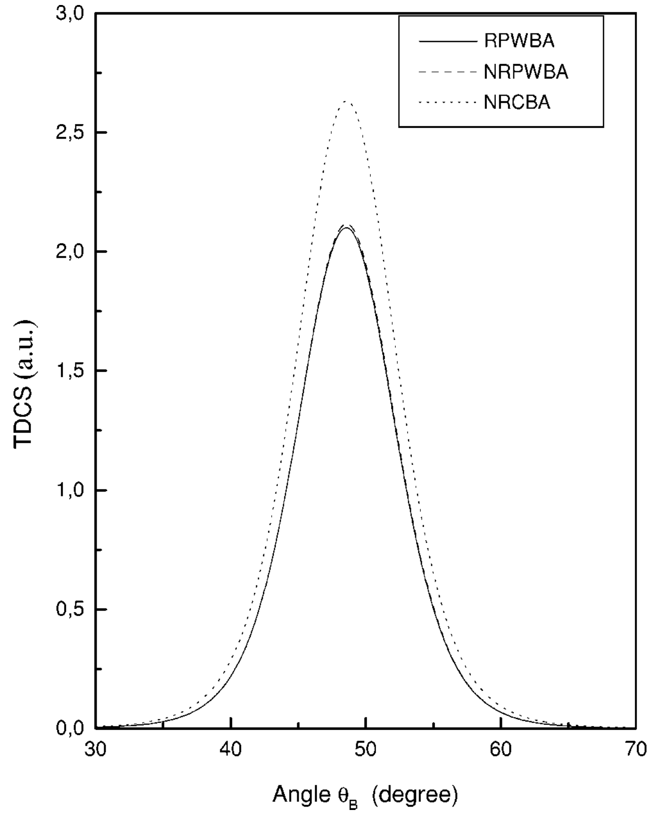

FIG. 1. The three TDCSs scaled in units of 10^{-3} a.u. The solid line represents the nonrelativistic TDCS in the Coulomb-Born approximation, the long-dashed line represents the corresponding TDCS in the plane wave approximation, and the dotted line sketches the relativistic TDCS in the plane wave approximation. The incident electron kinetic energy is $T_i=1350$ eV and the ejected electron kinetic energy is $T_B=574.5$ eV.

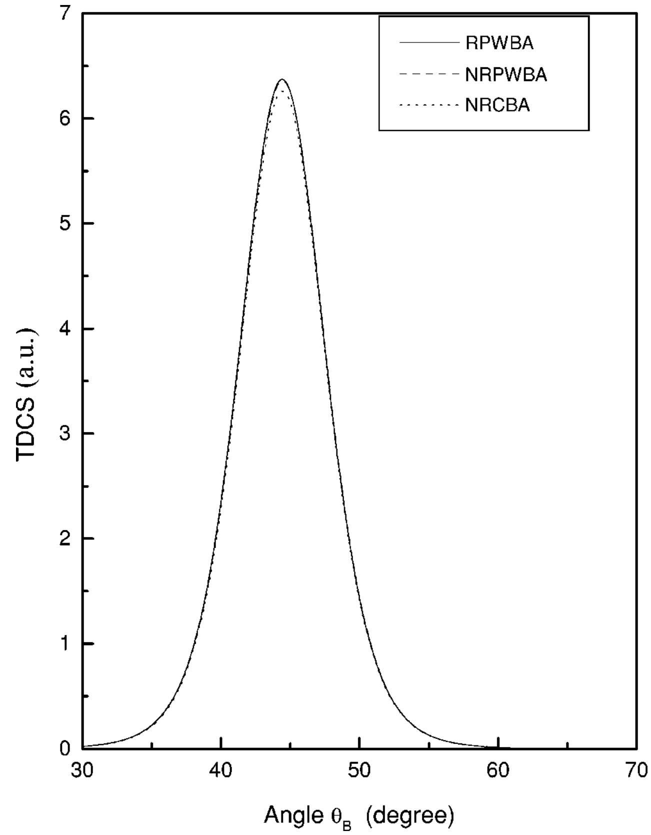

FIG. 2. Same as in Fig. 1 but for $T_i=1350$ eV and $T_B=674.5$ eV.

adjunction of the latter cannot be done without a judicious choice of a geometry. It must be borne in mind that this coplanar binary geometry is not well suited for the study of the domain of low kinetic energy for the ejected electron. The agreement between these three approaches remains good up to $T_i=15$ keV from which the RPWBA gives results that are a little lower than NRPWBA and NRCBA because relativistic and spin effects can no longer be ignored. Due to the relative simplicity of the model (even if the calculations are far from being obvious) and the exact nondressed relativistic description of the target, it is remarkable that such an agreement should be reached for these energies. When the laser field is introduced, the dressing of angular coordinates is not important for the nonrelativistic regime ($\gamma=1.0053$, $\mathcal{E}=0.05$ a.u.) but becomes noticeable for the relativistic regime ($\gamma=2.0$, $\mathcal{E}=1.00$ a.u.) where $\mathcal{E}$ is the electric field strength. The unit of electric field strength in atomic units is $\mathcal{E}=5.142\,25\times10^{9}$ V/cm.

B. In the presence of the laser field

1. The nonrelativistic regime ($\gamma=1.0053$, $\mathcal{E}=0.05$ a.u.)

The first check to be done is to take a zero electric field strength in order to recover all the results in the absence of the laser field. We have done these checks for all approximations. It has been shown [23] that for a laser frequency $w=0.043$ a.u., which corresponds to a laser photon energy of 1.17 eV, dressing effects due to the atomic target are not very important. A complete and exact relativistic treatment of the ejected electron is not analytically possible since the nonrelativistic wave equation for continuum states in a Coulomb field is separable in parabolic coordinates, but the corresponding Dirac equation is not. In other words, a decomposition of the relativistic continuum wave function into partial waves is not as straightforward as for the nonrelativistic case and the quantum numbers of each partial wave have to be taken into account very carefully. However, a tedious numerical construction of the first few partial waves is possible.

We first compare the results obtained within the three approximations (DVPWBA1, NRPWBA1, and NRCBA1) where it is expected on physical grounds that these cannot be used to study the relativistic regime. The three summed TDCSs are all peaked around $\theta_B=45°$, $\phi_B=180°$ which was to be expected for the case of the geometry chosen since for the scattered electron, the choice we have made is $\theta_f=45°$, $\phi_f=0°$ and in the xOy plane, this amounts to a scattered electron and an ejected electron having an opposite value of θ. Even with no photon exchange and for an electric field strength of 0.05 a.u., the presence of the laser field reduces considerably the magnitude of the TDCSs. The NRPWBA1 and NRCBA1 TDCSs are nearly indistinguishable whereas the DVPWBA1 TDCS is lower than the former ones in the vicinity of the maximum for $\theta_B=45°$. For this angle, we have

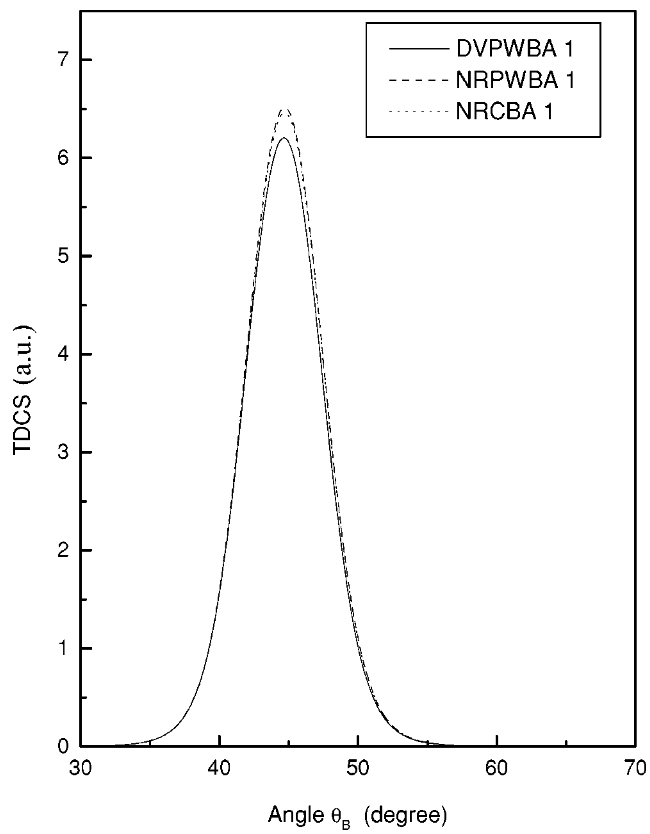

FIG. 3. The summed TDCSs for an exchange of ±100 photons in a nonrelativistic regime scaled in units of 10^{-4} a.u.

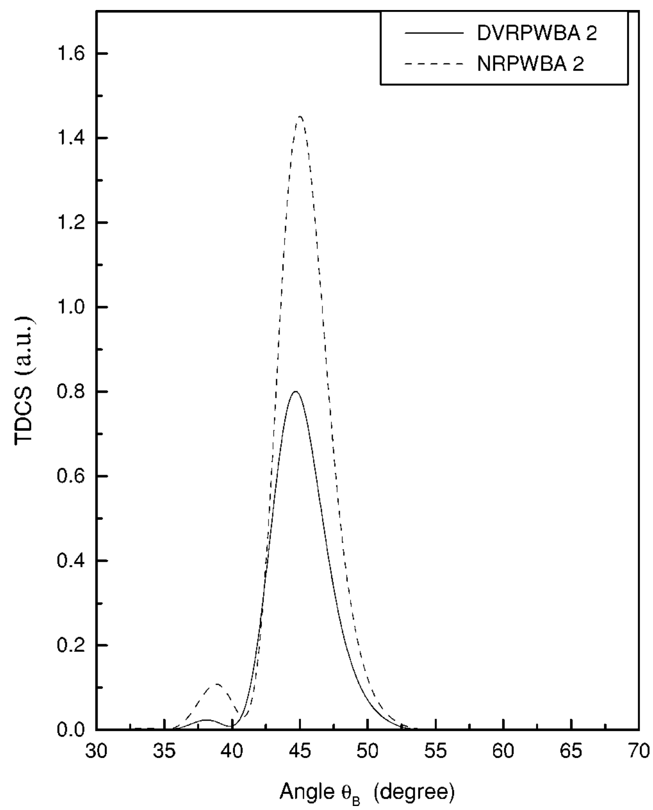

FIG. 4. The two TDCSs for $s=1$ and $l_B=-1$ in the nonrelativistic regime scaled in units of 10^{-8} a.u. We obtain the same figure for $s=-1$ and $l_B=1$.

TDCS(NRPWBA1)$\simeq 0.717\times 10^{-5}$ a.u., TDCS(NRCBA1) $\simeq 0.7115\times 10^{-5}$ a.u., and TDCS(DVPWBA1)$\simeq 0.472 \times 10^{-5}$ a.u. Two interesting cases are those corresponding to the absorption and emission of one photon. We have shown in a previous work [24] that when all electrons are described by Dirac-Volkov planes, the corresponding differential cross sections for the absorption and emission (of one photon) processes are identical. It is not the case for these three approximations since the ejected electron is described by a free Dirac plane wave. The DVPWBA1 TDCS is larger than the two other nonrelativistic TDCSs by a factor of 4 at the maximum for $\theta_B=45°$ for the absorption process and the emission process but these relativistic TDCSs are not identical. The TDCS for the emission of one photon is smaller than the corresponding one for the absorption of one photon by a factor of 2 at the same maximum. These remarks are not without interest since a crucial test of our next model (all electrons are described by Dirac-Volkov plane waves) will be to compare the two TDCSs within the framework of DVPWBA2 for these two processes. It will be a sound consistency check of our calculations. In Fig. 3 we show the three summed TDCSs for an exchange of ±100 photons and we obtain close curves for the nonrelativistic plane wave and Coulomb-Born results with the relativistic plane wave TDCS a little lower than the former ones in the vicinity of the peak at $\theta_B=45°$. For angles lower or larger than $\theta_B=45°$, the three TDCSs give nearly the same results. The dressing of angular coordinates being negligible, the maximum is maintained for the above-mentioned value of θ_B but in the relativistic regime this maximum is shifted. Also, we have compared the relativistic TDCS for different numbers of photons exchanged, typically ±50, ±100, and ±150 photons. The TDCSs increase when the number of the photons exchanged increases, and finally, we have compared the relativistic TDCS without laser field with these summed relativistic TDCSs in order to obtain a check of the well known pseudo sum-rule [25]. We have obtained results that converge to the relativistic TDCS without laser field but complete convergence is not reached since the ejected electron is not properly described. Now, we discuss the results obtained within the framework of the three more accurate approximations (DVPWBA2, NRPWBA2, and NRCBA2) in the same nonrelativistic regime. The description of the ejected electron by a Dirac-Volkov plane wave is more accurate and necessary on physical grounds (there is no constraint that forbids the ejected electron to exchange photons with the laser field). We have first investigated the case where no photon is exchanged at all ($s=0$, $l_B=0$). There is a shift of the location for the maximum corresponding to the relativistic TDCS while the two nonrelativistic TDCSs are nearly the same. The magnitude of the TDCSs is also considerably reduced. We have TDCS(DVPWBA2) $\simeq 0.347\times 10^{-8}$ a.u. for $\theta_B=41°$ while TDCS(NRPWBA2) $\simeq$ TDCS(NRCBA2) $\simeq 0.3326 \times 10^{-8}$ a.u. for $\theta_B=43°$. There are three small secondary peaks for the relativistic TDCS and two secondary peaks for the two nonrelativistic TDCSs. This behavior stems in the relativistic description from the contribution of the sum over the spins of the ejected electron. This sum shows a narrow

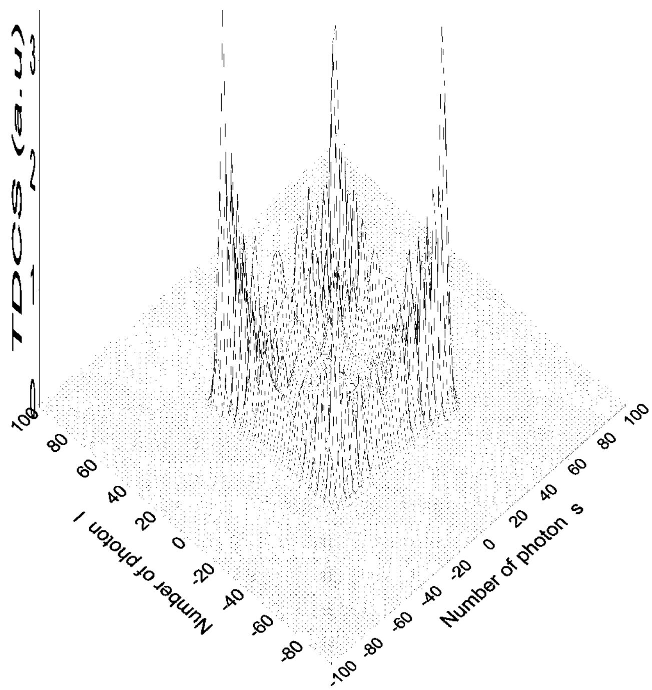

FIG. 5. The envelope of the TDCS scaled in 10^{-6} as function of the photon energy transfer in the nonrelativistic regime and for an electric field strength $\mathcal{E}=0.01$ a.u. and $\theta_B=45°$.

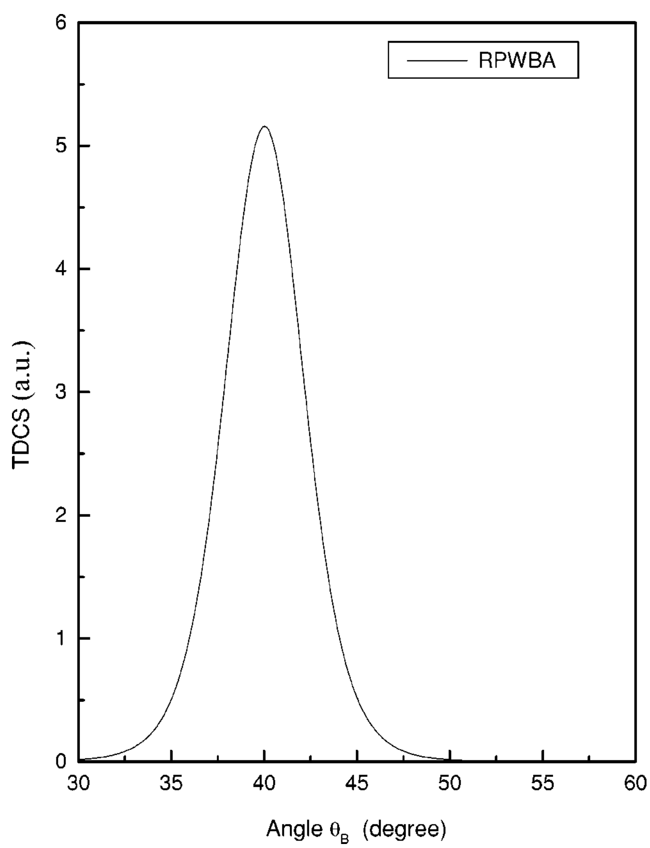

FIG. 6. The RPWBA TDCS without laser field in the relativistic regime, scaled in 10^{-16} a.u.

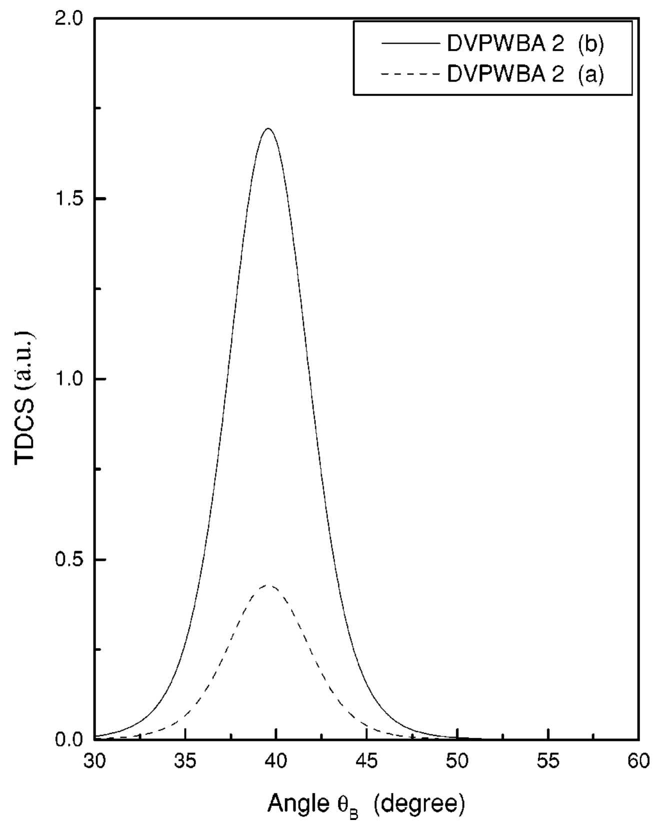

FIG. 7. The summed TDCS (a) for an exchange of $s=\pm 50$, $l_B=\pm 50$ and (b) for an exchange of $s=\pm 100$, $l_B=\pm 100$ in the relativistic regime, scaled in 10^{-22} a.u.

peak for $\theta_B=41°$ and presents two minima for $\theta_B=45°$ and $\theta_B=50°$. The other peaks can also be traced back to the behavior of this sum. Relativistic and spin effects begin at this stage to become noticeable since the shift as well as the magnitude of the relativistic TDCS with respect to the nonrelativistic TDCSs are clear signatures even in the nonrelativistic regime. Moreover, the most crucial test of our model is the complete symmetry between the emission and absorption processes. In Fig. 4, we show the relativistic and nonrelativistic TDCSs for $s=1$ and $l_B=-1$. We have obtained the same curve for the case $s=-1$ and $l_B=1$. For these two curves, the maximum shifts back to the value $\theta_B=45°$. As the number of exchanged photons s is linked both to the incident and scattered electron, we simulated a process where the number $s=2l_B$ to check the corresponding influence on the TDCSs and on the location of the maximum. The summed TDCSs for ($s=\pm 50$, $l_B=\pm 25$) and for ($s=\pm 100$, $l_B=\pm 50$) are almost halved when compared to the corresponding TDCSs for $s=l_B$. All curves are peaked around a maximum angle due to the behaviour of the square of the Fourier transform of the relativistic atomic hydrogen wave functions that falls off rapidly to zero in a small region around this maximum. Another interesting remark concerns once again the behavior of the sum over the spins of the ejected electron with the number of photons exchanged. As $l \geqslant \pm 500$, this sum is almost zero and contributes also to the rapid falloff of the corresponding relativistic TDCS. To illustrate the complexity of the location of the visual cutoff of the relativistic

TDCS, we consider an electric field strength $\mathcal{E}=0.01$ a.u. where it is expected that the numbers s and l of photons exchanged will not be high. The envelope of photon energy transfer obtained is a curve in three dimensions. As in the case of the process of Mott scattering in a strong laser field, we observe in Fig. 5 a rapid falloff of the relativistic TDCS for $s \simeq l \simeq \pm 40$ where the absolute value of the indices of the ordinary Bessel functions are close to their arguments. Also, as a side result, we see clearly in this figure that there is a complete symmetry between s and l. To obtain a converging envelope, one has to sum over the same numbers s and l of photons exchanged.

2. The relativistic regime ($\gamma=2$, $\mathcal{E}=1.00$ a.u.)

In the relativistic regime, dressing of angular coordinates is not negligible and we indeed observed as in the case of excitation a shift from $\theta_B=45°$ to lower values. The relativistic parameter $\gamma=2$ corresponds to an incident electron kinetic energy of c^2 in atomic units or the rest mass of the electron (0.511 MeV). In Fig. 6 we show the behavior of the relativistic TDCS in absence of a laser field in order to have an idea about the reduction of the corresponding TDCS when the laser field is introduced. The maximum is well located at $\theta_B=40°$ and at this value we have for the corresponding TDCS(RPWBA) $\simeq 0.54 \times 10^{-15}$ a.u. For this regime and in the presence of a strong laser field, the nonrelativistic TDCSs are no longer reliable and we will focus instead on the discussion of the results obtained within the DVPWBA1 and DVPWBA2. For the DVPWBA1, we first analyzed what happens when no photon is exchanged. There is a drastic reduction of the TDCS with a maximum shifted for θ_B lower than 40° and at this value we have for the corresponding TDCS(DVPWBA1) $\simeq 0.5 \times 10^{-21}$ a.u. Once again, there is an asymmetry between the absorption and emission processes of one photon. Due to a lack of high speed computing facilities we cannot check the pseudo sum-rule but we have a TDCS that increases when the number of photons exchanged increases. For the DVPWBA2, there is also a considerable reduction of the TDCS for $s=l_B=0$. In Fig. 7 we compare the summed relativistic TDCS for ($s=\pm 50$, $l_B=\pm 50$) and for ($s=\pm 100$, $l_B=\pm 100$) where the shift of the maximum is clearly visible. We still have complete symmetry between the absorption and emission processes.

VI. CONCLUSION

In this work, we have investigated the contribution of the relativistic electronic dressing in laser-assisted ionization of atomic hydrogen by electronic impact using the Dirac-Volkov plane wave solutions to describe the incoming and the two outgoing electrons. We have worked in the binary coplanar geometry where the description of the ejected electron by a relativistic Coulomb wave function is not necessary. The influence of the laser field is taken into account to all orders in the Dirac-Volkov description of electrons and the description of the atomic target used is the analytical relativistic atomic hydrogen wave function. It turns out that all the TDCSs are well peaked around a maximum angle due to the behavior of the Fourier transforms of the relativistic atomic hydrogen wave functions. Symmetry between the absorption and emission processes is obtained when all electrons are described by Dirac-Volkov plane waves both for the nonrelativistic and relativistic regime.

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