Relativistic electronic dressing in laser-assisted electron-hydrogen elastic collisions

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We study the effects of the relativistic electronic dressing in laser-assisted electron-hydrogen atom elastic collisions. We begin by considering the case when no radiation is present. This is necessary in order to check the consistency of our calculations and we then carry out the calculations using the relativistic Dirac-Volkov states. It turns out that a simple formal analogy links the analytical expressions of the unpolarized differential cross section without laser and the unpolarized differential cross section in the presence of a laser field.

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I. INTRODUCTION

Recently, the study of relativistic aspects of laser-induced processes has proved necessary, particularly as a result of very paramount breakthrough in laser technology which is capable now of attaining considerable ultrahigh intensities which could never have been dreamt of three or four decades ago. Many experiments that have shown a relativistic signature have been recently reported. To name some, the transition between Thomson and Compton scattering inside a very strong laser field was investigated by Moore, Knauer, and Meyerhofer [1]. Bula *et al.* [2] performed experiments on nonlinear Compton scattering at SLAC. Also, there are many other types of laser-assisted processes in which relativistic effects may be important. For instance, the process of emission of very energetic electrons and ions from atomic clusters which are submitted to ultrastrong infrared laser pulses [3]. It is now obvious that the whole apparatus and formalism of the nonrelativistic quantum collision theory [4] have to be revisited in order to extend known nonrelativistic results to the relativistic domain. Many theoretical studies of laser-assisted electron-atom collision have been mainly carried out in the nonrelativistic regime [5]. In presenting this work, we want to show that the modifications of the relativistic differential cross section corresponding to the elastic collision $e^-+H(1\ s_{1/2})\rightarrow e^-+H(1\ s_{1/2})$ due to the dressing of the Dirac-Volkov electron, in the presence of an ultraintense laser field can provide many interesting insights concerning the importance and the signatures of the relativistic effects.

In this work, we do not consider very high laser intensities that allow pair creation [6] and focus instead on the domain of intensities that justifies a strong classical electromagnetic potential [7]. The Dirac-Volkov electrons are thus dressed by a strong classical electromagnetic field with circular polarization. The organization of this paper is as follows.

In Sec. II we will present the formalism and establish the expression of the relativistic unpolarized differential scattering cross section in the absence of the laser field. This will serve as a guide and test the consistency of the calculation in presence of the laser field. In Sec. III, we give the expression of the unpolarized differential cross section in presence of a laser field with circular polarization and we compare it with the unpolarized relativistic differential cross section without laser field. We show that a simple formal analogy links these two differential cross sections. In Sec. IV, we give a brief discussion of the results. In Sec. V, we give a brief conclusion. Throughout this work, we use atomic units (a.u.) and DCS stands for differential cross section.

II. DIFFERENTIAL CROSS SECTION WITHOUT LASER FIELD

In order to recover the relativistic differential cross section without the laser field, we begin by considering the process $e^-+H(1\ s_{1/2})\rightarrow e^-+H(1\ s_{1/2})$ in the absence of radiation. The (direct) transition amplitude corresponding to this process is

$$S_{fi}=-\frac{i}{c}\int d^4x_1\overline{\psi_{pf}}(\mathbf{r}_1,t)\gamma^0\psi_{p_i}(\mathbf{r}_1,t)\langle\phi_f(\mathbf{r}_2,t)|V_d|\phi_i(\mathbf{r}_2,t)\rangle, \tag{1}$$

where $\psi_p(\mathbf{r},t)=u(p,s)e^{-ipx}/\sqrt{2EV}$ is the electron wave functions described by a free Dirac spinor normalized to the volume V and $\phi_{i,f}(\mathbf{r}_2)$ are the relativistic wave functions of the hydrogen atom where the index i stands for the initial state and the index f stands for the final state. As we study the elastic excitation by electronic impact, we have $f=i=(1s_{1/2})$. The velocity of light is $c=137.036$ in atomic units, the explicit expression of the wave functions $\phi(\mathbf{r})$ for the fundamental state (spin up) can be found in Ref. [8] and reads in atomic units as

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$$\phi(\mathbf{r}) = \frac{1}{\sqrt{4\pi}}\begin{pmatrix} ig(r) \\ 0 \\ f(r)\cos(\theta) \\ f(r)\sin(\theta)e^{i\phi} \end{pmatrix}, \tag{2}$$

with $g(r)$ given by

$$g(r) = (2Z)^{\gamma+1/2}\sqrt{\frac{1+\gamma}{2\Gamma(1+2\gamma)}}e^{-Zr}r^{\gamma-1}, \tag{3}$$

whereas $f(r)$ is given by

$$f(r) = -(2Z)^{\gamma+1/2}\sqrt{\frac{1+\gamma}{2\Gamma(1+2\gamma)}}e^{-Zr}r^{\gamma-1}\left(\frac{1-\gamma}{Z\alpha}\right). \tag{4}$$

To simplify the notation we shall use throughout this work the following abbreviations:

$$g(r) = N_g e^{-Zr}r^{\gamma-1},$$

$$f(r) = -g(r)\left(\frac{1-\gamma}{Z\alpha}\right) = N_f e^{-Zr}r^{\gamma-1}. \tag{5}$$

In Eq. (1), V_d is the direct interaction potential

$$V_d = \frac{1}{r_{12}} - \frac{Z}{r_1}, \tag{6}$$

where $\mathbf{r}_1$ are the electron coordinates, $\mathbf{r}_2$ are the atomic electron coordinates, and $r_{12}=|\mathbf{r}_1-\mathbf{r}_2|$. The parameter γ appearing in all these equations is

$$\gamma = \sqrt{1-Z^2\alpha^2}. \tag{7}$$

It is straightforward to get for the transition amplitude

$$S_{fi} = -i\frac{\overline{u}(p_f,s_f)\gamma^0 u(p_i,s_i)}{V\sqrt{2E_i 2E_f}}2\pi\delta(E_f-E_i)H(\Delta), \tag{8}$$

where γ^0 is given in the standard representation of the Dirac matrices by $\gamma^0=\mathrm{diag}(1,1,-1,-1)$. The argument of the function H is $\Delta=|\mathbf{p}_i-\mathbf{p}_f|$, the norm of the momentum transfer. The DCS is given by

$$\frac{d\sigma}{d\Omega_f} = \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}\frac{1}{(4\pi c^2)^2}\left(\frac{1}{2}\sum_{s_i,s_f}|\overline{u}(p_f,s_f)\gamma^0 u(p_i,s_i)|^2\right) \times |H(\Delta)|^2|_{E_f=E_i}. \tag{9}$$

In Eq. (9), we have summed over the final polarization s_f and averaged over the initial polarization s_i. For elastic collisions $|\mathbf{p}_f|=|\mathbf{p}_i|=|\mathbf{p}|$ so that $E_i=E_f=E$ and

$$\frac{1}{2}\sum_{s_i,s_f}|\overline{u}(p_f,s_f)\gamma^0 u(p_i,s_i)|^2 = 4E^2[1-\beta^2\sin^2(\theta/2)], \tag{10}$$

with $\beta=|\mathbf{p}|c/E$. The angle θ is the scattering angle between the vectors $\mathbf{p}_i$ and $\mathbf{p}_f$. We then have for the unpolarized DCS

$$\frac{d\sigma}{d\Omega_f} = \frac{4E^2}{(4\pi c^2)^2}[1-\beta^2\sin^2(\theta/2)]|H(\Delta)|^2\Big|_{E_f=E_i}. \tag{11}$$

We now turn to the function $H(\Delta)$ of the momentum transfer which is simply proportional to the Fourier transform of the average (static) potential felt by the incident electron in the field of the hydrogen atom [4]. Performing the various integrals, we get for this Fourier transform

$$H(\Delta) = -4\pi(N_g^2+N_f^2)\Gamma(2\gamma+1)\left(\frac{1}{(2Z)^{2\gamma+1}\Delta^2} - \frac{\sin(2\gamma\phi)}{2\gamma\lambda^{2\gamma}\Delta^3}\right), \tag{12}$$

where the quantities λ and ϕ are

$$\lambda = \sqrt{(2Z)^2+\Delta^2} \quad \text{and} \quad \phi = \arctan\left(\frac{\Delta}{2Z}\right). \tag{13}$$

Even if it may not seem so, the function $H(\Delta)$ is well behaved for the case of forward scattering $\theta=0°$ [recall that $\Delta=2|\mathbf{p}_i|\sin(\theta/2)$] and has the property that the limit for $\Delta\to 0$ of the quantity

$$\left(\frac{1}{(2Z)^{2\gamma+1}\Delta^2} - \frac{\sin(2\gamma\phi)}{2\gamma\lambda^{2\gamma}\Delta^3}\right) \tag{14}$$

is given by

$$\frac{(2\gamma+1)(2\gamma+2)}{6(2Z)^{2\gamma+3}}. \tag{15}$$

We must of course recover the result in the nonrelativistic limit ($\beta\to 0$ and $\gamma_{rel}\to 1$). In that case, the unpolarized differential cross section is simply given by

$$\frac{d\sigma}{d\Omega_f} = 4\frac{(\Delta^2+8)^2}{(\Delta^2+4)^4}. \tag{16}$$

Taking $\beta\to 0$ and $\gamma\to 1$ (for $Z=1$), one easily recovers from Eq. (11) the above-mentioned nonrelativistic limit.

III. THE DIFFERENTIAL CROSS SECTION IN THE PRESENCE OF A LASER FIELD

We turn to the calculation of the DCS for elastic scattering without exchange in the first Born approximation and in the presence of a laser field. The (direct) transition amplitude in this case is given by

$$S_{fi} = -\frac{i}{c}\int d^4x_1\overline{\psi}_{q_f}(\mathbf{r}_1,t)\gamma^0\psi_{q_i}(\mathbf{r}_1,t)\langle\phi_f(\mathbf{r}_2,t)|V_d|\phi_i(\mathbf{r}_2,t)\rangle, \tag{17}$$

where $\phi_{i,f}(\mathbf{r}_2)$ are the relativistic wave functions of the hydrogen atom and the functions $\psi_q(\mathbf{r}_1,t)$ are the Dirac-Volkov solutions normalized to the volume V

$$\psi_q(\mathbf{r},t) = \frac{1}{\sqrt{2QV}}R(q)u(p,s)e^{-iS(x)}, \tag{18}$$

with

$$R(q) = \left(1 + \frac{1}{2(kp)c}\not{k}[\not{a}_1\cos(\varphi) + \not{a}_2\sin(\varphi)]\right), \tag{19}$$

and

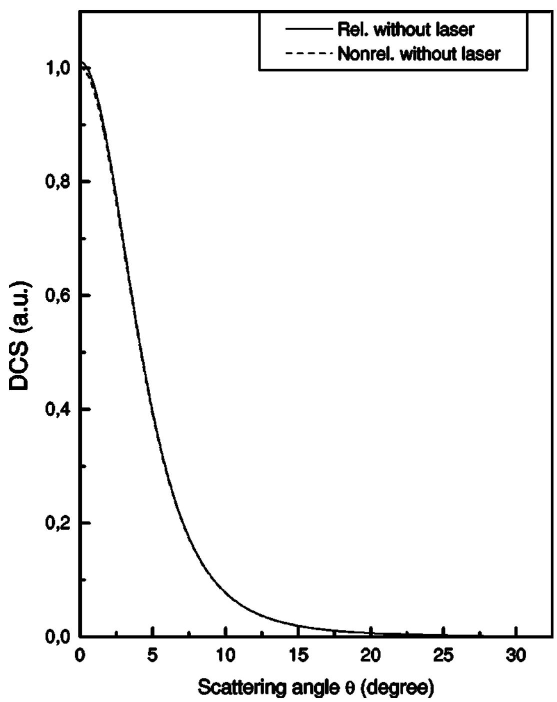

FIG. 1. Comparison between the nonrelativistic DCS and the relativistic DCS as functions of the scattering angle (θ) varying from 0° to 30°, without taking into account the spherical coordinates of (p_i, p_f); the curves are perfectly confounded.

$$S(x) = qx + \frac{(a_1 p)}{c(kp)}\sin(\varphi) - \frac{(a_2 p)}{c(kp)}\cos(\varphi), \tag{20}$$

in the case of a circularly polarized electromagnetic potential such that $A^\mu = a_1^\mu \cos(\varphi) + a_2^\mu \sin(\varphi)$ with $k_\mu A^\mu = 0$ (the Lorentz condition) and $A^2 = a_1^2 = a_2^2 = a^2$, $a_1 a_2 = 0$, and $ka_1 = ka_2 = 0$. The four-vector $q^\mu = (Q/c, \mathbf{q})$ is the four-momentum of the electron inside the laser field with wave four-vector k^μ.

We have

$$q^\mu = p^\mu - \frac{a^2}{2(kp)c^2}k^\mu. \tag{21}$$

In Eq. (21) a^2 denotes the time-averaged square of the four-vector potential of the laser field. The square of the four-vector q^μ is

$$q_\mu q^\mu = m_*^2 c^2. \tag{22}$$

The parameter m_* plays the role of an effective mass of the electron inside the electromagnetic field

$$m_*^2 = 1 - \frac{a^2}{c^4}. \tag{23}$$

The factor $R(q)$ acting on the bispinor u contains information about the spin-dressing field interaction. Thus, the Dirac-Volkov wave function represents a free-electron wave (containing a field-dependent phase) modulated by a wave generated by the interaction of the spin with the classical single mode field with four-vector potential A^μ.

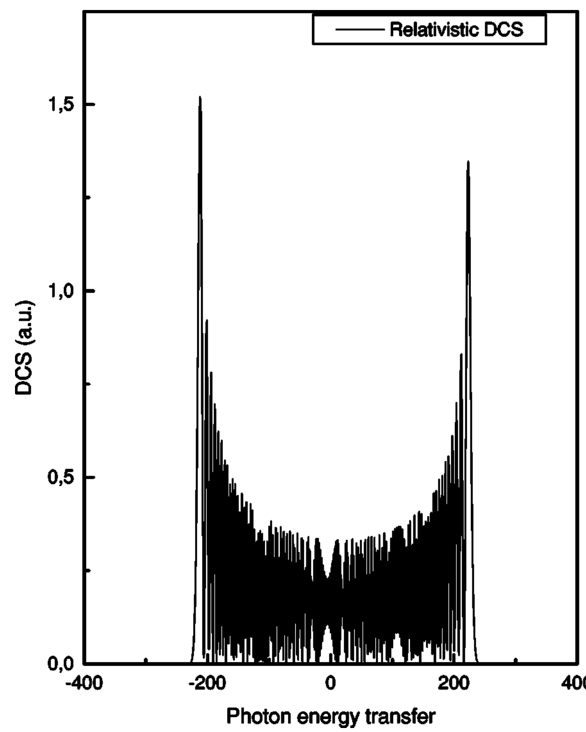

FIG. 2. The envelope of the relativistic DCS scaled in 10^{-5} as a function of the photon energy transfer in the nonrelativistic regime.

In Eqs. (19) and (20), $\varphi = kx = k_\mu x^\mu = k_0 x^0 - \mathbf{k}\cdot\mathbf{x}$ and we use throughout this work the notations and conventions of Bjorken and Drell [8]. Proceeding along the lines of standard calculations in QED [8], one has for the unpolarized DCS

$$\frac{d\sigma}{d\Omega_f} = \sum_{s=-\infty}^{\infty} \frac{d\sigma^{(s)}}{d\Omega_f}. \tag{24}$$

The sum over s in Eq. (24) stems from the well-known relation of ordinary Bessel functions $\exp[-iz\sin(\varphi)] = \sum_{s=-\infty}^{+\infty} J_s(z)\exp(-is\varphi)$ and this physically corresponds to the number of photons exchanged. The quantity $d\sigma^{(s)}/d\Omega_f$ is the DCS corresponding to the exchange of exactly s photons and reads

$$\frac{d\sigma^{(s)}}{d\Omega_f} = \frac{1}{(4\pi c)^2}\frac{|\mathbf{q}_f|}{|\mathbf{q}_i|}\left(\frac{1}{2}\sum_{s_i s_f}|M_{fi}^{(s)}|^2\right)|H(\Delta_s)|^2\Big|_{Q_f = Q_i + sw}. \tag{25}$$

In Eq. (25) $\Delta_s = |\mathbf{q}_i + s\mathbf{k} - \mathbf{q}_f|$ is the momentum transfer with the net exchange of s photons. The quantity $(\sum_{s_i s_f}|M_{fi}^{(s)}|^2)/2$ is the electronic contribution to the unpolarized differential cross section $d\sigma^{(s)}/d\Omega_f$ and has already been determined in a previous work [9]. It contains combinations of ordinary Bessel functions. The function $H(\Delta_s)$ is now given by

$$H(\Delta_s) = -4\pi(N_g^2 + N_f^2)\Gamma(2\gamma+1)\left(\frac{1}{(2Z)^{2\gamma+1}(\Delta_s)^2} - \frac{\sin(2\gamma\phi_s)}{2\gamma\lambda_s^{(2\gamma)}(\Delta_s)^3}\right), \tag{26}$$

with $\lambda_s = \sqrt{(2Z)^2 + (\Delta_s)^2}$ and $\phi_s = \arctan(\Delta_s/2Z)$. Once again,

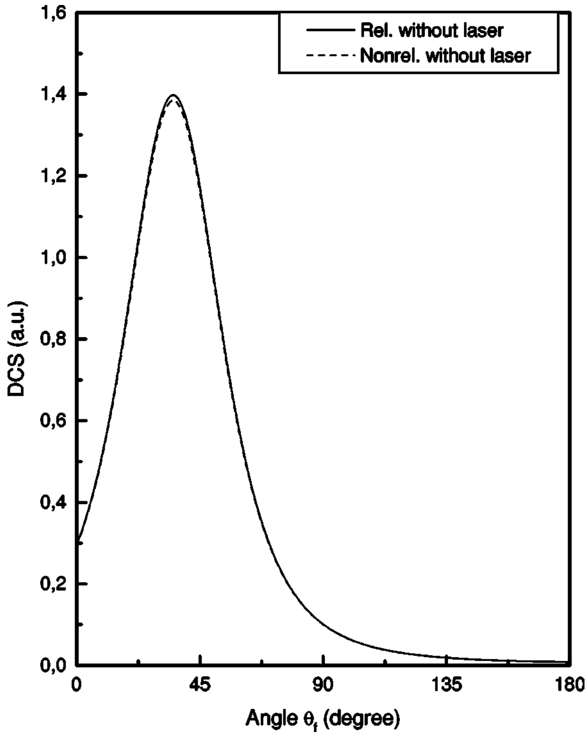

FIG. 3. Comparison in the nonrelativistic regime, between the nonrelativistic DCS and the relativistic DCS scaled in 10^{-3} as a function of (θ_f) the angle between $\mathbf{p}_f$ and the Oz axis. The relativistic parameter is $\gamma_{rel}=1.0053$. The geometry chosen is $\theta_i=\phi_i=45°$ with θ_f varying from 0° to 180° and $\phi_f=90°$.

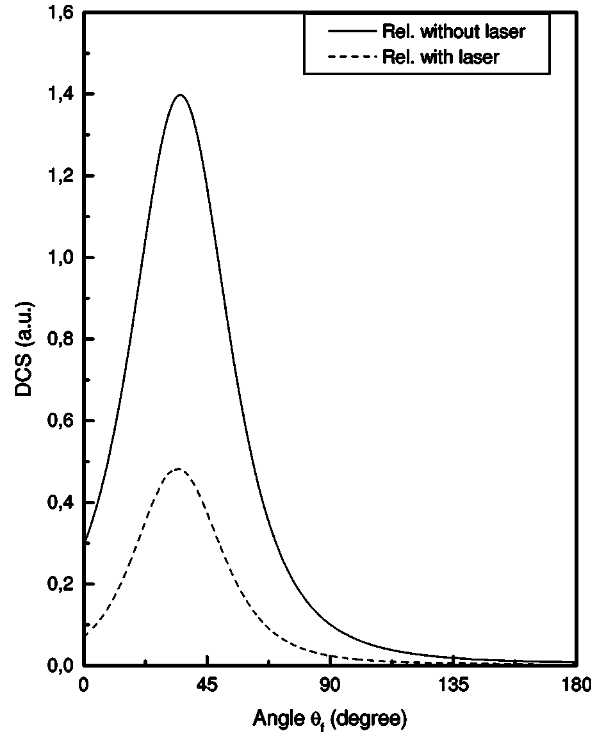

FIG. 4. Comparison in the nonrelativistic regime ($\gamma_{rel}=1.0053$, $\varepsilon=0.05$ a.u., $w=0.043$ a.u.) between the relativistic DCS with laser and the relativistic DCS without laser scaled in 10^{-3} for the exchange of ± 100 photons.

the function $H(\Delta_s)$ is well behaved for the forward scattering which corresponds to $s=0$ and $\theta=0°$. When no radiation field is present, all Bessel functions vanish except for $s=0$, we have $J_s(z=0)=\delta_{s0}$ and in this case, the result reduces to the unpolarized DCS given in Eq. (11).

IV. RESULTS AND DISCUSSIONS

The kinematics of the process is that given in Ref. [9] and we maintain the same choice for the laser angular frequency, that is, $w=0.043$ which corresponds to a near-infrared Neodymium laser.

A. The nonrelativistic regime

In the limit of low electron kinetic energy and moderate field strength, typically an electron kinetic energy $W=100$ a.u. and a field strength $\varepsilon=0.05$ a.u., the effects of the additional spin terms and the dependence of q^μ on the spatial orientation of the electron momentum due to (kp) are small. In Fig. 1, we compare the nonrelativistic DCS given by Eq. (16) and the relativistic DCS given by Eq. (11) as functions of the scattering angle $\theta(\mathbf{p}_i,\mathbf{p}_f)$ in the absence of the laser field. As expected, in the nonrelativistic limit, there is only a small difference between these two DCSs and we see that this difference becomes more pronounced for the case of forward scattering. For large-angle scattering, there is almost no difference between the nonrelativistic calculations and the relativistic calculations. In Fig. 2, we represent the envelope of the unpolarized relativistic DCS as function of the photon energy transfer for the following geometry ($\theta_i=\phi_i=45°$ and $\theta_f=45°,\phi_f=90°$). There is an asymmetry between the absorption and emission part of the spectrum due to the denominators containing powers of Δ_s appearing in Eq. (26) and a rapid falloff of the contributions of the various partial unpolarized DCSs when the arguments of the ordinary Bessel functions are close to their indices. In Fig. 3, instead of plotting $(d\sigma/d\Omega_f)_{NR}$ and $(d\sigma/d\Omega_f)_{REL}$ as functions of the scattering angle, we use the angular coordinates (θ_i,ϕ_i) of $\mathbf{p}_i$ and (θ_f,ϕ_f) of $\mathbf{p}_f$ to plot the angular dependence of the two DCSs as functions of θ_f, the angle between $\mathbf{p}_f$ and the Oz axis. This will serve as a consistency check of our next calculations in presence of an electromagnetic potential circularly polarized and whose wave vector points in the Oz direction. We have chosen a geometry where $\theta_i=\phi_i=45°$ and the angle θ_f varies from 0° to 180° with $\phi_f=90°$. The relativistic parameter $\gamma_{rel}=1/\sqrt{1-\beta^2}$ is equal to 1.0053 which corresponds to an electron kinetic energy equal to 100 a.u. $\simeq 2.721$ keV. The first observation to be made is that in the nonrelativistic regime, the nonrelativistic DCS is very close to the relativistic one, which was to be expected. Also, there is a peak in the vicinity of $\theta_f=35°$.

It is also important to compare the relativistic DCS $(d\sigma^{(s)}/d\Omega)$ corresponding to the net exchange of s photons where only the electronic dressing term is taken into account, with the corresponding nonrelativistic DCS. Working with the nonrelativistic Volkov states, one easily gets for the nonrelativistic case

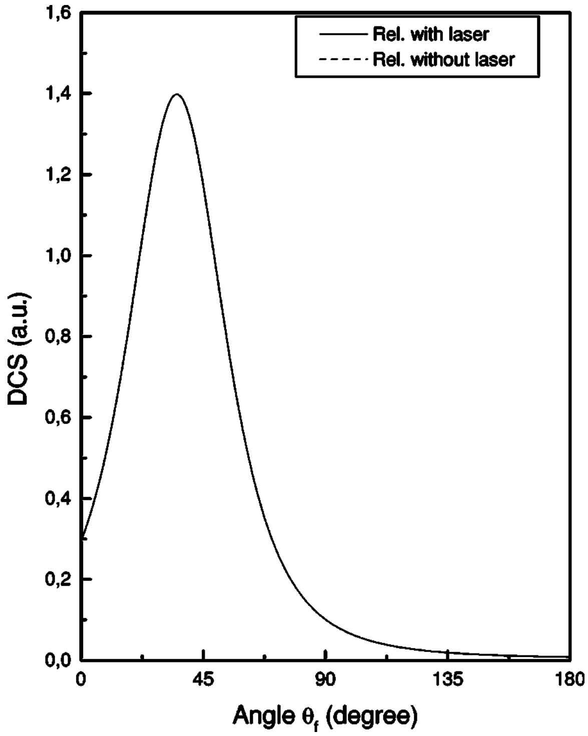

FIG. 5. Comparison between the relativistic DCS with laser and relativistic without laser scaled in 10^{-3}. The parameters are $\gamma=1.0053$, $\varepsilon=0.05$ a.u., and $w=0.043$ a.u. The geometry chosen is $\theta_i=\phi_i=45°$, where θ_f varies from 0° to 180° with $\phi_f=90°$ for an exchange of ±300 photons. The curves are indistinguishable.

$$\frac{d\sigma^{B_1,s}}{d\Omega_f}=\frac{|\mathbf{p}_f(s)|}{|\mathbf{p}_i|}J_s^2\left(\frac{|a|}{cw}\sqrt{(\Delta\cdot\hat{\boldsymbol{x}})^2+(\Delta\cdot\hat{\boldsymbol{y}})^2}\right)\frac{d\sigma^{B_1,F,F}(\Delta_s)}{d\Omega_f}, \tag{27}$$

where the first-Born DCS is given by Eq. (16) and corresponds to the field free case evaluated for Δ_s. In the argument of the ordinary Bessel function corresponding to the circular polarization of the laser field, $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are the unit vectors along the direction of the x axis and the direction of the $\boldsymbol{y}$ axis respectively. In Fig. 4, we compare the relativistic summed DCS with and without laser field for the net exchange of ±100 photons. As one can see, the laser field gives rise to important modifications of the DCS. For this collision geometry, several hundred photons can be exchanged even in the case of a moderate laser intensity of 8.75×10^{13} W/cm^2. In Fig. 5, the net exchange of ±300 photons shows that the DCS with laser field approaches very closely the DCS without laser field and we have almost two indistinguishable curves. This result is in accordance with the approximate sum rule [5]. In the nonrelativistic regime, this sum rule is obtained for a relatively small numbers of photons exchanged.

B. The relativistic regime

In the limit of high electron kinetic energy and strong-field strength, typically an electron kinetic energy $W=c^2$ a.u. and a field strength $\varepsilon=1.00$ a.u., the effects of the

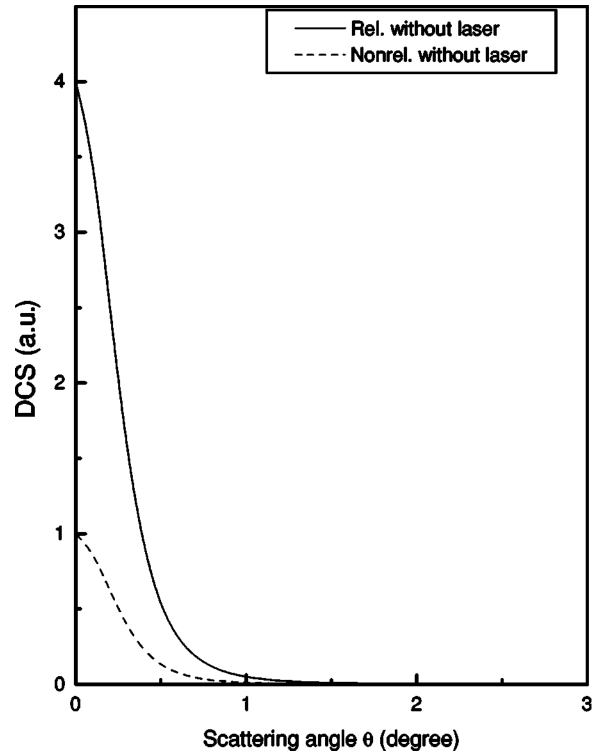

FIG. 6. Comparison in the relativistic regime $\gamma_{rel}=2$ between the nonrelativistic DCS and the relativistic DCS as functions of the scattering angle (θ) varying from 0° to 3°.

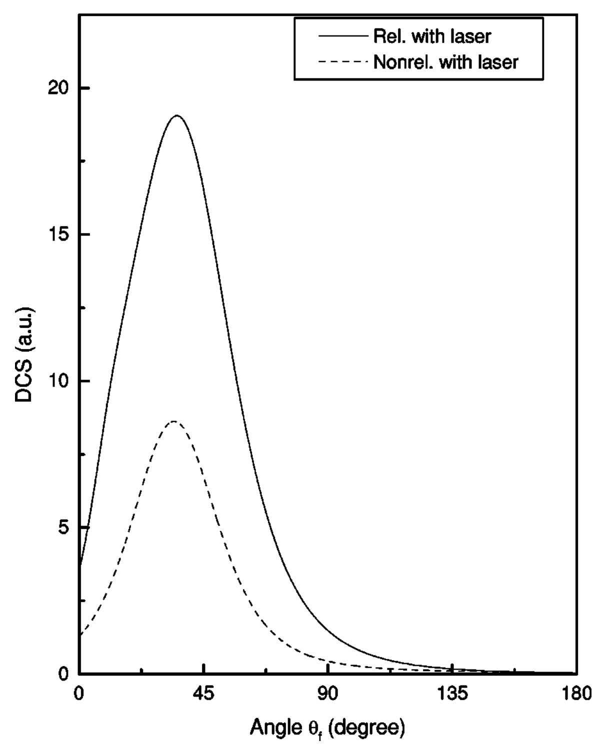

FIG. 7. Comparison in the relativistic regime ($\gamma_{rel}=2$, $\varepsilon=1$ a.u., $w=0.043$ a.u.) between the relativistic DCS with laser and the nonrelativistic DCS with laser scaled in 10^{-10}, for an exchange of ±5000 photons. The geometry chosen is $\theta_i=\phi_i=45°$, where θ_f varies from 0° to 180° with $\phi_f=90°$.

additional spin terms and the dependence of q^{μ} on the spatial orientation of the electron momentum due to (kp) begin to be noticeable.

In Fig. 6, the nonrelativistic DCS is compared to the relativistic DCS as functions of the scattering angle θ (not to be confused with the angle θ_f). The nonrelativistic formalism is no longer applicable since there is now a net difference between the DCS given by Eq. (11) and the DCS given by Eq. (16) particularly for small angles. Indeed, for the case of forward scattering $(d\sigma/d\Omega)_{NR}=1$ while $(d\sigma/d\Omega)_{R}\simeq 4$ and the difference between the DCS remains noticeable up to $\theta=1.5°$. For large angles, both approaches give nearly the same results. In Fig. 7, we compare the summed DCS relativistic and nonrelativistic where there is an exchange of ± 5000 photons. The values of the nonrelativistic DCS are more than halved with regard to the relativistic DCS.

V. CONCLUSION

In this work, we have studied the effect of the relativistic electronic dressing in laser-assisted electron-hydrogen elastic collisions. To our knowledge, this is the first time that such a calculation has been carried out at this level. Even if the formalism may seem heavy and complicated, we have (using the case with no laser field as a guide) checked every step of our calculations. What emerges is that the relativistic electronic dressing reduces considerably the magnitude of the DCS. We must sum the DCS given by Eq. (24) over a very large number of photons in order to get the same order of magnitude as that of the DCS given by Eq. (11). Of course, a more sophisticated approach is needed in order to have a complete treatment of this relativistic process. The relativistic generalization of the method due to by Byron and Joachain [10] which takes into account the atomic dressing will be presented in a separate paper. These authors and others [11] have shown that in the nonrelativistic regime, the dressing of atomic states can give rise to very important modifications of the DCS. The nonrelativistic treatment of laser-assisted electron-atom collisions taking into account both the electronic dressing and the atomic dressing has been studied by many authors [12]. All agree that at least in the nonrelativistic regime, the effects of atomic dressing can modify the behavior of the DCS. Work is in progress to include the relativistic atomic dressing in this collision process.

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