

Scattering of dark-state polaritons in optical lattices and quantum phase gate for photons

M. Mašalas^{1,2} and M. Fleischhauer¹

¹Technische Universität Kaiserslautern, 67663 Kaiserslautern, Germany

²Vilnius University Research Institute of Theoretical Physics and Astronomy, 2600 Vilnius, Lithuania

(Received 19 February 2004; published 3 June 2004)

We discuss the quasi one-dimensional (1-D) scattering of two counterpropagating, dark-state polaritons (DSPs), each containing a single excitation. DSPs are formed from photons in media with electromagnetically induced transparency and are associated with ultraslow group velocities. State-dependent elastic collisions of atoms at the same lattice site lead to a nonlinear interaction. It is shown that the scattering process in a deep optical lattice filled by cold atoms generates a large and homogeneous conditional phase shift between two individual polaritons. The latter has potential applications for a photonic phase gate. The quasi-1-D scattering problem is solved analytically and the influence of degrading processes such as dephasing due to collisions with ground-state atoms is discussed.

DOI: 10.1103/PhysRevA.69.061801

PACS number(s): 42.50.Gy, 42.65.Hw, 32.80.Qk, 03.75.Lm

A major challenge for quantum information processing using individual photons as qubits is the implementation of logic operations. Such operations require efficient nonlinear interactions for pairs of photons, which cannot be achieved in conventional optical materials. As the effect of the photon-photon coupling depends on the nonlinear susceptibilities as well as on the interaction time, it has been suggested to use ultraslow light in resonant systems with electromagnetically induced transparency (EIT), where the interaction time is long and the nonlinear susceptibilities become large due to resonance enhancement [1–3]. At the boundary of a stationary EIT medium, light pulses become spatially compressed in the propagation direction by the ratio of group velocity v_{gr} to the vacuum speed of light [3]. As a consequence, the number of photons in the pulse decreases by the same factor. Excitations are temporarily transferred to the medium by the formation of quasiparticles, so-called dark-state polaritons (DSPs), which are superpositions of electromagnetic and atomic degrees of freedom [4]. As another consequence of the pulse compression the interaction time in a head-on collision stays constant irrespective of the value of v_{gr} . Thus, in order to achieve long interaction times, copropagating pulses were considered [5]. In this case, the interaction is, however, not homogeneous and it is difficult to avoid spectral broadening of the wave packet.

Here, we suggest a completely different mechanism for an efficient nonlinear interaction between ultraslow light pulses. The slow-down corresponds to a shift of the polariton composition from pure photons to matter-waves [4]. Furthermore, the pulse compression leads to an increasing density of the matter component. Thus, collisional interactions between atomic excitations can yield an effective nonlinear coupling between two wave packets. To further enhance the strength of this interaction, we consider a lattice potential in the tight-binding limit. Starting from a fully quantized effective one-dimensional (1-D) model of light propagation in a lattice, we derive analytic solutions for the quantum scattering of two single-photon wave packets in the s -wave scattering limit. We show that the pulses attain a homogeneous conditional phase shift that may be large enough for the implementation of a quantum phase gate.

Let us consider a cold gas of bosonic, five-level atoms as shown in Fig. 1 in a deep 3-D lattice potential under tight-binding conditions. The atoms form an M -type system, with the ground state $|g\rangle$ and the excited states $|e_{\pm}\rangle$ coupled by two orthogonal polarizations of a quantized probe field \hat{E}_{\pm} propagating in the $+z$ or $-z$ direction, respectively. The excited states are, furthermore, coupled to metastable state $|q_{\pm}\rangle$ by a classical probe field of Rabi frequency Ω . All atoms are initially in the ground state $|g\rangle$. The atoms are described by five Bose fields ψ_i , where $i \in \{g, e_{\pm}, q_{\pm}\}$ denotes the internal state. The Hamiltonian of the system is given by $H = H_{at} + H_{lat} + H_{at-f} + H_{coll}$, where the atomic part reads

$$H_{at} = \sum_{i=g, e_{\pm}, q_{\pm}} \int d^3r \psi_i^{\dagger} \left(-\frac{\hbar^2}{2m} \nabla^2 + \hbar \omega_i \right) \psi_i. \quad (1)$$

H_{lat} is the lattice potential, which is assumed to be the same for all internal states. The interaction with electromagnetic field reads in rotating-wave approximation

$$H_{at-f} = - \int d^3r \sum_{i=+, -} \psi_{e_i}^{\dagger} [\mu E_i^{(+)}(\mathbf{r})] \psi_g - \int d^3r \sum_{i=+, -} \psi_{q_i}^{\dagger} \hbar \Omega(\mathbf{r}) \psi_{e_i} + \text{H.c.} \quad (2)$$

Finally, we take into account collisions between atoms in the

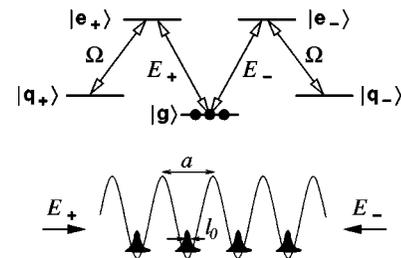


FIG. 1. *Top*: Atomic five-level system with quantized probe fields E_{\pm} of opposite circular polarization and propagation direction. Ω denotes the Rabi frequency of the classical, undepleted control field. *Bottom*: The atoms are assumed to be confined in a 3-D lattice potential with lattice constant a . In the tight-binding regime, they occupy only the lowest Wannier state of effective width l_0 .

internal states $|g\rangle$ and $|q_{\pm}\rangle$ as well as collisions between atoms in $|q_{-}\rangle$ and $|q_{+}\rangle$ in s -wave approximation:

$$H_{\text{coll}} = \frac{\hbar u_g}{2} \int d^3r \psi_g^\dagger \psi_g^\dagger \psi_g \psi_g + \sum_{i,k=+,-} \frac{\hbar u_{ik}}{2} \int d^3r \psi_{q_i}^\dagger \psi_{q_k}^\dagger \psi_{q_k} \psi_{q_i} + \sum_{i=+,-} \hbar u_{gi} \int d^3r \psi_{q_i}^\dagger \psi_g^\dagger \psi_g \psi_{q_i}. \quad (3)$$

Collisions of atoms in the excited state are irrelevant as these states will attain a vanishingly small population. Here, $E_{\pm}^{(+)}(\mathbf{r})$ are the positive frequency parts of the probe field operators corresponding to the two orthogonal polarized modes, $\Omega(\mathbf{r})$ is the Rabi frequency of the control field, and the u 's describe the collision strength, which can be expressed in terms of the corresponding s -wave scattering length $u_i = 4\pi a_i \hbar/m$. The ω_i 's are the frequencies corresponding to the electronic energy levels.

Using Eqs. (1)–(3), the Heisenberg equations for the field operators can easily be obtained:

$$i\hbar \frac{\partial \psi_g}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V + \hbar \omega_g \right) \psi_g - \mu^* E_{-}^{(-)} \psi_{e_{-}} - \mu^* E_{+}^{(-)} \psi_{e_{+}} + \hbar \left(u_g \psi_g^\dagger \psi_g + \sum_{i=+,-} u_{gi} \psi_{q_i}^\dagger \psi_{q_i} \right) \psi_g, \quad (4a)$$

$$i\hbar \frac{\partial \psi_{e_{\pm}}}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V + \hbar \omega_{e_{\pm}} \right) \psi_{e_{\pm}} - \mu E_{\pm}^{(+)} \psi_g - \hbar \Omega \psi_{q_{\pm}}, \quad (4b)$$

$$i\hbar \frac{\partial \psi_{q_{\pm}}}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V + \hbar \omega_{q_{\pm}} \right) \psi_{q_{\pm}} - \hbar \Omega^* \psi_{e_{\pm}} + \sum_{k=+,-} \hbar (u_{gk} \psi_g^\dagger \psi_g + u_{\pm k} \psi_{q_k}^\dagger \psi_{q_k}) \psi_{q_{\pm}}. \quad (4c)$$

We have not included decay from the excited states as these states will attain only a negligible population. The above operator equations are nonlinear and are thus impossible to solve exactly. Therefore, approximations are needed.

First, it is assumed that both probe fields propagate along the z axis and the control field in an orthogonal direction. Thus, $E_{\pm}^{(+)}(\mathbf{r}, t) = \mathcal{E}_{\pm}(\mathbf{r}, t) e^{i(\pm kz - \omega t)}$, $\Omega(\mathbf{r}, t) = \Omega_0(t) e^{-i\omega_c t}$. Here, both $\mathcal{E}_{\pm}(\mathbf{r}, t)$ and $\Omega_0(t)$ are slowly varying functions of \mathbf{r} and t . One also assumes that the probe field coupling $\mu \mathcal{E}_{\pm}/\hbar$ is much weaker than the control field coupling Ω_0 . In this limit, we may consider the control field undepleted and classical and can thus set Ω_0 constant. The assumption that all atoms were initially in the ground state g then also implies that only a small fraction of atoms is excited to the states q_{\pm} .

The strength of the lattice is considered to be large enough such that only the lowest energy level of each potential well is occupied and tunneling between the wells is negligible. In this limit the atomic field operators can be expanded in the basis of (real) Wannier functions [6]. Since only the lowest lattice state is occupied, only Wannier states of the lowest Bloch band $W_j(\mathbf{r}) = W_0(\mathbf{r} - \mathbf{r}_j)$ survive in this expansion. In a deep lattice Wannier functions of neighbor-

ing sites have only negligible overlap and one has $\int d^3r W_i(\mathbf{r}) W_k(\mathbf{r}) = \delta_{ik}$. This expansion along with a separation of fast oscillating terms yields $\psi_g = \sum_j W_j(\mathbf{r}) g_j(t)$, $\psi_{e_{\pm}} = \sum_j W_j(\mathbf{r}) e_{\pm j}(t) e^{i(\pm kz - \omega t)}$, and $\psi_{q_{\pm}} = \sum_j W_j(\mathbf{r}) q_{\pm j}(t) e^{\pm ikz - i(\omega - \omega_c)t}$. Here, the operators g_j , $e_{\pm j}$, and $q_{\pm j}$ are slowly varying in t and the summation runs over all lattice sites.

Substituting the Wannier expansion into Eqs. (4) yields equations for the slowly varying atomic variables. Since the Wannier functions are well localized, the slowly varying functions remain almost constant within one lattice site:

$$\dot{g}_j = i \frac{\mu}{\hbar} \mathcal{E}_{-}^{\dagger}(\mathbf{r}_j) e_{-j} + i \frac{\mu}{\hbar} \mathcal{E}_{+}^{\dagger}(\mathbf{r}_j) e_{+j} - i(u_g g_j^{\dagger} g_j + u_{g+} q_{+j}^{\dagger} q_{+j} + u_{g-} q_{-j}^{\dagger} q_{-j}) \frac{f^3}{a^3} g_j, \quad (5a)$$

$$\dot{e}_{\pm j} = -i \Delta_{e_{\pm} e_{\pm j}} + i \frac{\mu}{\hbar} \mathcal{E}_{\pm}(\mathbf{r}_j) g_j + i \Omega_0 q_{\pm j}, \quad (5b)$$

$$\dot{q}_{\pm j} = -i \Delta_{q_{\pm} q_{\pm j}} + i \Omega_0^* e_{\pm j} - i(u_{\pm\pm} q_{\pm j}^{\dagger} q_{\pm j} + u_{\pm+} q_{\mp j}^{\dagger} q_{\mp j} + u_{g\pm} g_j^{\dagger} g_j) \frac{f^3}{a^3} q_{\pm j}, \quad (5c)$$

with the detunings $\Delta_{e_{\pm}} = \omega_{e_{\pm}} - \omega + \hbar k^2/2m$, $\Delta_{q_{\pm}} = \omega_{q_{\pm}} - \omega + \omega_c + \hbar k^2/2m$. We have set $\omega_g = 0$. The factor $f = a/l_0$ describes the confinement strength in the lattice and is defined as $\int d^3r W_i(\mathbf{r})^4 = f^3/a^3$.

In order to solve the above equations of motion for the matter-field operators, a weak-probe approximation will be applied. In zeroth order of the probe field, the states $|e_{\pm}\rangle$ and $|q_{\pm}\rangle$ remain unpopulated and one finds for the ground state operators

$$\dot{g}_j = -iu_g \frac{f^3}{a^3} g_j^{\dagger} g_j. \quad (6)$$

If the lattice has a regular filling with a well defined number $N_j = N$ of atoms per site, we can make the replacement $g_j^{\dagger} g_j \rightarrow N$ in Eq. (6). A regular filling can be achieved, e.g., by employing a Mott-insulator transition in a lattice [6,7]. In this case, the self-phase modulation described by (6) can just be absorbed in the definition of the energy of the ground-state atoms. Furthermore, we can replace the ground-state operator in the equations for the operators of the other states by a constant $g_j \rightarrow g_j^{(0)}$. With this, we obtain in first order of the probe field

$$\dot{e}_{\pm j} = -i \Delta_{e_{\pm} e_{\pm j}} + i \frac{\mu g_j^{(0)}}{\hbar} \mathcal{E}_{\pm}(\mathbf{r}_j) + i \Omega_0 q_{\pm j}, \quad (7a)$$

$$\dot{q}_{\pm j} = -i(\Delta_{q_{\pm}} + u_{g\pm} f^3 n) q_{\pm j} + i \Omega_0^* e_{\pm j} - i u_{\pm\pm} \frac{f^3}{a^3} q_{\pm j}^{\dagger} q_{\pm j} q_{\pm j} - i u_{\pm+} \frac{f^3}{a^3} q_{\mp j}^{\dagger} q_{\mp j} q_{\pm j}, \quad (7b)$$

where $n = N/a^3$ is the average density of atoms. Next, we assume resonance conditions, i.e., $\Delta_{e_{\pm}} = 0$ and $\Delta_{q_{\pm}} + u_{g\pm} n f^3$

=0 and that the probe field varies sufficiently slowly. Under these conditions, we can apply an adiabatic approximation.

In zeroth order of the adiabatic approximation, the time derivative in the equations of motion for the atomic variables (5) is neglected, which yields

$$q_{\pm j}(t) = -\frac{\mu g_j^{(0)}}{\hbar \Omega_0} \mathcal{E}_{\pm j}, \quad (8a)$$

$$e_{\pm j}(t) = \frac{u_{\pm} f^3}{\Omega_0^* a^3} q_{\pm j}^\dagger q_{\pm j} q_{\pm j} + \frac{u_{\pm} f^3}{\Omega_0^* a^3} q_{\mp j}^\dagger q_{\mp j} q_{\pm j}, \quad (8b)$$

with $\mathcal{E}_{\pm j} \equiv \mathcal{E}_{\pm}(\mathbf{r}_j)$. Thus,

$$e_{\pm j}^{(0)}(t) = -\hbar u_{\pm} f^3 \frac{\mu g_j^{(0)}}{|\hbar \Omega_0|^2} \frac{|\mu|^2 n}{|\hbar \Omega_0|^2} \mathcal{E}_{\pm j}^\dagger \mathcal{E}_{\pm j} \mathcal{E}_{\pm j} - \hbar u_{\mp} f^3 \frac{\mu g_j^{(0)}}{|\hbar \Omega_0|^2} \frac{|\mu|^2 n}{|\hbar \Omega_0|^2} \mathcal{E}_{\mp j}^\dagger \mathcal{E}_{\mp j} \mathcal{E}_{\pm j}. \quad (9)$$

Proceeding in the same manner with the first order of the adiabatic approximation, we find that the operators corresponding to the excited states $|e_{\pm}\rangle$ contain a term proportional to the time derivative of the probe fields

$$e_{\pm j}^{(1)}(t) = e_{\pm j}^{(0)}(t) + \frac{i \mu g_j^{(0)}}{\hbar |\Omega_0|^2} \frac{\partial}{\partial t} \mathcal{E}_{\pm j}. \quad (10)$$

Here, higher order terms in $\mathcal{E}_{\pm j}$ containing a time derivative were neglected as they correspond to higher order corrections in both the probe field and the adiabaticity parameter.

The adiabatic solutions for the matter fields can now be used to calculate the slowly varying amplitude of the probe-field polarizations $P_{\pm}(\mathbf{r}, t) = \mu^* \sum_j |W_j(\mathbf{r})|^2 g_{\pm j}^\dagger(t) e_{\pm j}(t)$. Since the Wannier functions are strongly localized around the center of the potential wells, the microscopic polarization changes rapidly in space. On the other hand, in Maxwell's equations for the electric field, only the macroscopic polarization enters. The macroscopic polarization can be obtained by averaging over a volume small compared to the wavelength. If the lattice constant a is sufficiently smaller than the relevant wavelength of the probe field, the lattice structure disappears in the polarization. In an optical lattice, $a = \lambda_l/2$, where λ_l is the wavelength of the laser light used for the optical potential. Unless the probe field is very much detuned to the blue side of λ_l , effects from the lattice structure like Bragg scattering can be neglected.

Using the macroscopic polarization, we find the following equation of motion for the slowly varying amplitude of the field operator

$$\left(\frac{\partial}{\partial t} \pm v_{\text{gr}} \frac{\partial}{\partial z} \right) \mathcal{E}_{\pm} = -i u_{\pm} \frac{\lambda \varepsilon_0}{\hbar \pi v_{\text{gr}}} f^3 \mathcal{E}_{\pm}^\dagger \mathcal{E}_{\pm} \mathcal{E}_{\pm} - i u_{\mp} \frac{\lambda \varepsilon_0}{\hbar \pi v_{\text{gr}}} f^3 \mathcal{E}_{\mp}^\dagger \mathcal{E}_{\mp} \mathcal{E}_{\pm}, \quad (11)$$

with group velocity $v_{\text{gr}} = 2c\hbar/|\Omega_0|^2 \varepsilon_0/|\mu|^2 \omega n$ (assuming $v_{\text{gr}} \ll c$). The appearance of the group velocity in the denominator of (11) suggests at first glance a diverging nonlinear in-

teraction when the group velocity approaches zero. One should take into account, however, that due the pulse compression the total photon number $N_{\text{ph}} \sim \int d^3r \mathcal{E}^\dagger \mathcal{E}$ is only a fraction v_{gr}/c of the input value. Equation (11) becomes much more transparent if it is translated into an equation of motion of the DSPs [4]

$$\Psi_{\pm}(z, t) = \cos \theta \tilde{\mathcal{E}}_{\pm}(z, t) - \sin \theta \sqrt{A} \tilde{\psi}_{q_{\pm}}(z, t), \quad (12)$$

where $\mathcal{E} = \tilde{\mathcal{E}} \sqrt{\hbar \omega / 2 \varepsilon_0 A}$, A being the cross section of the light beam, $v_{\text{gr}} = c \cos^2 \theta$, and $\tilde{\psi}_{q_{\pm}}$ are the slowly varying amplitudes of the matter fields. It may be worthwhile noting that the DSPs in an optical lattice [Eq. (12)] have some resemblance to exciton-polaritons in a semiconductor. In the adiabatic limit considered here, the orthogonal quasiparticles to Ψ_{\pm} , the bright-state polaritons are not excited and thus $\tilde{\mathcal{E}}_{\pm} = \Psi_{\pm} \cos \theta$. With this, we find the following propagation equation inside the medium

$$\left(\frac{\partial}{\partial t} \pm v_{\text{gr}} \frac{\partial}{\partial z} \right) \Psi_{\pm} = -i \frac{2a_{\pm} \lambda v_{\text{rec}}}{A} f^3 \Psi_{\pm}^\dagger \Psi_{\pm} \Psi_{\pm} - i \frac{2a_{\mp} \lambda v_{\text{rec}}}{A} f^3 \Psi_{\mp}^\dagger \Psi_{\mp} \Psi_{\pm}, \quad (13)$$

where we have substituted the s -wave scattering length a_{ij} and the recoil velocity $v_{\text{rec}} = \hbar \omega / mc$. The polariton number densities $n_{\pm} = \Psi_{\pm}^\dagger(z) \Psi_{\pm}(z)$ undergo a sudden increase at the boundary of the medium since the electric field is continuous there. Inside the medium, they propagate form-stable with v_{gr} .

Equation (13) leads to a self- and cross-phase modulation of the DSPs. If initially only one polariton of each sort is excited, the self-phase modulation vanishes. To solve the quasi-1-D scattering problem, it is convenient to introduce the two-particle wave function

$$w(z, z', t) = \langle 0 | \Psi_{+}(z, t) \Psi_{-}(z', t) | \phi \rangle, \quad (14)$$

where $|\phi\rangle$ is the initial state vector of the system and $|0\rangle$ corresponds to the polariton vacuum. One can show that in the case considered here, namely, where only one polariton of each class is initially excited, all information is contained in w . In terms of center-of-mass and difference coordinates $R = (z+z')/2$ and $\xi = z-z'$, the equation of motion for w reads

$$\left(\frac{\partial}{\partial t} + 2v_{\text{gr}} \frac{\partial}{\partial \xi} \right) w = -2i \frac{\delta(\xi) a_{+} \lambda v_{\text{rec}}}{A} f^3 w. \quad (15)$$

This equation has a simple interpretation. The left-hand side describes the propagation of the two components in opposite directions. The right-hand side describes an interaction for $z=z'$, i.e., when the two polaritons meet. The interaction conserves the center-of-mass of the two polaritons and the solution of (15) reads

$$w(R, \xi, t) = w(R, \xi - 2v_{\text{gr}}t, 0) \exp[-i \Delta \phi \Theta(\xi)], \quad (16)$$

where $\Theta(\xi)$ is the Heaviside step function. One recognizes that the shape of the two-photon wave function remains unchanged by the collisions and there is only a homogeneous

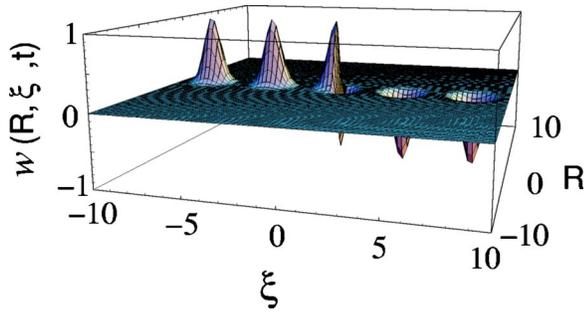


FIG. 2. Snapshots of two-particle wave-function $w(R, \xi, t)$ at equidistant times as function of center-of-mass coordinate R and difference coordinate $\xi = z - z'$. $\xi < 0$ ($\xi > 0$) corresponds to polaritons propagating toward (away from) each other. Overlapping components generate a phase flip $\Delta\phi$, which is taken to be π .

collision phase. This is illustrated for the case $\Delta\phi = \pi$ in Fig. 2.

The conditional phase shift between the polaritons that originate from two single-photon wave packets is then transferred back to two photons at the exit of the medium. In this way, a quantum phase gate between two individual photons could be implemented, if the conditional phase shift can reach the value of π . The collision phase in (16) is given by

$$\Delta\phi = \frac{a_{+} - \lambda}{A} \frac{v_{\text{rec}}}{v_{\text{gr}}} f^3. \quad (17)$$

It is interesting to note that $\Delta\phi$ does not depend on the pulse parameters. It is not necessary that the two pulses have a certain length or the same shape. Both should, however, occupy the same transverse mode. One also recognizes that the use of a lattice potential has two important effects. First of

all, phase diffusion of the individual DSPs caused by the scattering of atoms in states $|q_{\pm}\rangle$ with ground-state atoms is eliminated by the regular filling. Secondly, the local enhancement of the density leads to an enhancement factor f^3 . In a deep lattice, f can be as large as 10. To give an estimate of achievable phase shifts, let us assume $a_{+} = 10$ nm, $v_{\text{gr}} = 10v_{\text{rec}}$, $A = \lambda^2$, $\lambda = 800$ nm, and $f = 10$. This yields a phase shift on the order of unity, which is of the required order of magnitude.

The main limitation of the present scheme is set by the dephasing of the DSPs during their propagation time T in the medium. T can be chosen as small as the initial pulse length, but needs to be sufficiently large such that the compressed pulse length in the medium $L = v_{\text{gr}}T$ is still sufficiently larger than the wavelength λ . Assuming for the above given parameters $T \approx 50$ μsec , which is typical for the experiments in [8–10], and $v_{\text{rec}} \approx 5$ cm/sec, one finds $L \approx 25$ μm , which fulfills $L \gg \lambda$. Since in the light-“stopping” experiments [9,10] dephasing times of milliseconds have been observed, the dephasing of the dark polariton should not be an issue.

In summary, we have shown that scattering of ultracold atoms in a deep three-dimensional lattice together with the transfer of excitations between photons and atomic excitations through dark-state polaritons can be used for a conditional homogeneous phase shift between individual photons. An essential requirement to obtain sufficiently large phase shifts is the transverse focusing of the polaritons to a cross section comparable to λ^2 and sufficiently long dephasing times.

The authors acknowledge financial support from the European Union within the network QUACS and the Marie-Curie Trainings-site at the University of Kaiserslautern as well as the German Science Foundation.

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