Quantum entanglement of four distant atoms trapped in different optical cavities

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We propose a unified scheme to generate W states, Greenberger-Horne-Zeilinger (GHZ) states, and cluster states of four distant atoms, which are trapped separately in leaky cavities. The proposed schemes require linear optical elements and photon detectors with single-photon sensitivity. The quantum noise influences the fidelity of the generated states. Further, based on the four-photon coincidence detection, we propose another scheme to generate the W states and GHZ states of four distant atoms. The scheme is insensitive to the quantum noise, but cannot be used to generate the cluster states.

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I. INTRODUCTION

Quantum entanglement is one of the most striking features of quantum mechanics [1,2]. The recent surge of interest and progress in quantum information theory allows one to take a more positive view of entanglement and regard it as an essential resource for many ingenious applications such as quantum cryptography [3], quantum dense coding [4], and quantum teleportation [5]. Most of the research is based on two-qubit entanglement. Since the Greenberger-Horne-Zeilinger (GHZ) state was introduced to test quantum nonlocality without inequality [6], there has been much interest in the classification of multiparticle entangled states. It has become clear that for the system shared by three parties there are two inequivalent classes of entangled states, i.e., the GHZ state and the W state [7]. For entangled states of four or more parties, Briegel and Raussendorf introduced a new class of entangled state, i.e., the cluster state [8]. The GHZ and W classes have found applications in testing quantum nonlocality without inequality [6,9] and realizing quantum information processing [10]. By employing the cluster class, the concept of the one-way quantum computer was introduced, which describes a realization of quantum computation which goes beyond the usual network picture [11]. These works motivated an intensive research in the generation and the manipulation of entangled states of many qubits. However, how to design and realize quantum entanglement is extremely challenging due to the coupling with an environmental degree of freedom. Various quantum systems have been suggested as possible candidates for engineering quantum entanglement. Among them the cavity QED system is an almost ideal system to generate entangled states and to perform small scale quantum information processing [12]. In the context of cavity QED, a number of theoretical methods were proposed to generate GHZ and W states [12,13]. In particular, the generation of the GHZ state of three particles has been demonstrated experimentally in high-Q cavities [14]. Schemes of this type are based on the controlling of an effective interaction between atoms, which are intended to be entangled. Since most of these schemes require a high-Q cavity field, decoherence caused by cavity decay can be neglected.

Conditioned measurements offer a promising way to generate entangled states. In Ref. [15], authors use photon detection on leaky cavities to implement the teleportation of atomic states and generate the Einstein-Podolsky-Rosen (EPR) state between two distant atoms. In Ref. [16], Fidio and Vogel proposed a scheme for preparing the W state of three trapped atoms in leaky cavities. In Ref. [17], we proposed a scheme for generating GHZ states of many distant atoms trapped in different cavities. Recently several efficient schemes are proposed to engineer two-atom entanglement by using two-photon coincidence detection [18]. However, no proposal is proposed for generating cluster states of many distant atoms. The aim of the present paper is to propose schemes to generate quantum entanglement of four distant atoms.

The paper is organized as follows. In Sec. II, based on the photon detectors with single-photon sensitivity, we present a unified scheme to generate the W states, GHZ states, and cluster states of four distant atoms. In the scheme, the photons, which leak out of the four cavities, are emitted into a symmetric eight-port device [19] to become entangled and detected by four single-photon detectors. Based on the photon detection of the output modes, the atom-cavity-system state is projected into the expected entangled states. The disadvantage of proposed schemes is to require a single-photon detector to distinguish zero photons, one photon, and more than one photon. The quantum noise influences the fidelity of the generated states. In Sec. III, by using four-photon coincidence detection, we propose another scheme to generate the W states and the GHZ states of four distant atoms. The scheme is insensitive to the quantum noise, but cannot be used to generate the cluster states of the four distant atoms. Finally the conclusions are given in Sec. IV.

II. GENERATION OF THE *W* STATES, GHZ STATES, AND CLUSTER STATES BASED ON THE SINGLE-PHOTON DETECTORS

The experimental setup is shown in Fig. 1(a), which consists of four three-level atoms confined separately in four optical cavities, and a symmetric eight-port device and four

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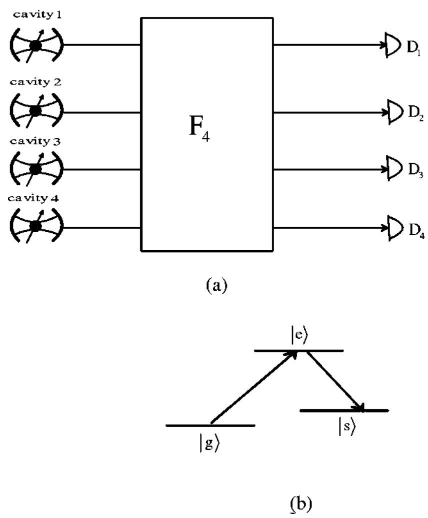

FIG. 1. (a) The schematic setup for generating W states, GHZ states, and cluster states of four distant atoms based on the single photon detection. F_4 is a symmetric eight-port device [19] and D_i are photon detectors. (b) The relevant level structure with ground states $|g\rangle$, $|s\rangle$, and excited state $|e\rangle$.

single-photon detectors. An extended introduction to the symmetric multiport device is given in Ref. [19]. The action of the symmetric eight-port device can be described by the unitary operator U. The matrix element of U is given by

$$U_{kl} = \frac{1}{\sqrt{N}}\gamma_4^{(k-1)(l-1)}, \tag{1}$$

where $\gamma_4=\exp(i\pi/2)$ and indices k, l denote the input and exit port. The matrix element U_{kl} gives the probability amplitude for single photon entering via input k and leaving the device by output l $(k,l=1,\ldots,4)$. Reference [19] has shown how to construct a symmetric eight-port device from mirrors, beam splitters, and phase shifters. Each of the three-level atoms (index j) has a ground state $|g_j\rangle$, a metastable state $|s_j\rangle$, and an excited state $|e_j\rangle$ [see Fig. 1(b)]. The lifetime of the atomic levels $|s_j\rangle$ and $|g_j\rangle$ are assumed to be comparatively long so that spontaneous decay of these states can be neglected. The $|s_j\rangle \Leftrightarrow |e_j\rangle$ transition is driven by the classical field and the $|g_j\rangle \Leftrightarrow |e_j\rangle$ transition is driven by the quantized cavity field. Both the classical laser field and the cavity field are assumed to be detuned from their respective transition by the same amount. In the case of large detuning the excited state $|e_j\rangle$ can be eliminated adiabatically to obtain the effective interaction Hamiltonian (in the interaction picture),

$$H_j = i\Omega(a_j\sigma_{+j} - a_j^\dagger\sigma_{-j}), \tag{2}$$

with the operators $\sigma_{+j}=|s_j\rangle\langle g_j|$ and $\sigma_{-j}=|g_j\rangle\langle s_j|$. $a_j^\dagger$ and a_j are the creation and annihilation operators of the jth cavity field. Here we assumed that the effective coupling constants of all the atoms coupled with their cavities is the same, denoted by Ω. In order to investigate the quantum dynamics of the system, it is convenient to follow a quantum trajectory description [20]. The evolution of the system's wave function is governed by a non-Hermitian Hamiltonian

$$H_j' = H_j - i\kappa_j a_j^\dagger a_j, \tag{3}$$

as long as no photon decays from the cavity. Here $2\kappa_j$ is the decay rate of the jth cavity field, which is assumed to be for all four cavities the same: $\kappa_j=\kappa$. If a single-photon detector D_j $(j=1,2,3,4)$ detects a photon, the coherent evolution according to H_j' is interrupted by a quantum jump. This corresponds to a quantum jump, which can be formulated with the operators b_j on the joint state vectors of four atom-cavity systems

$$b_1 = \frac{1}{2}(a_1 + a_2 + a_3 + a_4), \quad b_2 = \frac{1}{2}(a_1 + ia_2 - a_3 - ia_4),$$

$$b_3 = \frac{1}{2}(a_1 - a_2 + a_3 - a_4), \quad b_4 = \frac{1}{2}(a_1 - ia_2 - a_3 + ia_4). \tag{4}$$

In the preparation stage the initial state of each atom-cavity system is $|s_j\rangle|0\rangle_j$ $(j=1,2,3,4)$, i.e., each atom is initially in the metastable state and the cavity is in the vacuum state. Now we switch on the Hamiltonian (2) in each atom-cavity system for a time τ. If no photon is emitted from the cavity, the jth atom-cavity system is governed by the interaction H_j'. In this case the atom-cavity state evolves to the entangled state

$$|\Psi\rangle_j = \alpha|s_j\rangle|0\rangle_j + \beta|g_j\rangle|1\rangle_j, \tag{5}$$

with

$$\alpha = \frac{\cos(\Omega_\kappa\tau) - \dfrac{\sin(\Omega_\kappa\tau)\kappa}{2\Omega_\kappa}}{\sqrt{\left[\cos(\Omega_\kappa\tau) - \dfrac{\sin(\Omega_\kappa\tau)\kappa}{2\Omega_\kappa}\right]^2 + \dfrac{\sin^2(\Omega_\kappa\tau)\Omega^2}{\Omega_\kappa^2}}},$$

$$\beta = \frac{\sin(\Omega_\kappa\tau)\Omega}{\Omega_\kappa\sqrt{\left[\cos(\Omega_\kappa\tau) - \dfrac{\sin(\Omega_\kappa\tau)\kappa}{2\Omega_\kappa}\right]^2 + \dfrac{\sin^2(\Omega_\kappa\tau)\Omega^2}{\Omega_\kappa^2}}},$$

$$\Omega_\kappa = \sqrt{\Omega^2 - \kappa^2/4}. \tag{6}$$

The probability that no photon is emitted during this evolution becomes

$$P_{single} = e^{-\kappa\tau}\left\{\left[\cos(\Omega_\kappa\tau) - \frac{\sin(\Omega_\kappa\tau)\kappa}{2\Omega_\kappa}\right]^2 + \frac{\sin^2(\Omega_\kappa\tau)\Omega^2}{\Omega_\kappa^2}\right\}. \tag{7}$$

We assume that the interaction Hamiltonian (2) is applied to each atom-cavity system simultaneously, so that the preparation of the atom-cavity states $|\Psi\rangle_j$ ends at the same time. This concludes the preparation stage of the protocol. The probability that this stage is equal to the probability that no photon decays during the preparation stage. This quantity is given by $P_{suc}=P_{single}^4$.

Now we consider the detection stage and demonstrate the generation of W states, GHZ states, and cluster states. In this stage, we assume that we have prepared the four atom-cavity systems in the form

$$|\Phi(0)\rangle = |\Psi\rangle_1|\Psi\rangle_2|\Psi\rangle_3|\Psi\rangle_4, \tag{8}$$

where the state $|\Psi\rangle_j$ is given by Eq. (5). When the state (8) has been prepared, we turn off the laser pulse and wait for the one or two of four detectors D_1, D_2, D_3, and D_4 to click. We assume that photons are detected at the time t. This assumption is posed to calculate the system's time evolution during this time interval in a consistent way with the "no-photon emission Hamiltonian" (3). Notice that, in the detection stage, we turn off the laser pulses, the atom-cavity interaction term H_j of Eq. (3) is set to zero. In this case, the state of the jth atom-cavity system at the time t evolves into

$$|\Psi(t)\rangle_j = \exp(-iH_j't)|\Psi\rangle_j = \frac{1}{\sqrt{\alpha^2+\beta^2 e^{-2\kappa t}}}(\alpha|s_j\rangle|0\rangle_j + \beta e^{-\kappa t}|g_j\rangle|1\rangle_j). \tag{9}$$

The probability that no photon decay takes place during this evolution is given by $(\alpha^2+\beta^2 e^{-2\kappa t})$. At the time t, the joint state of the total system becomes

$$|\Phi(t)\rangle = |\Psi(t)\rangle_1|\Psi(t)\rangle_2|\Psi(t)\rangle_3|\Psi(t)\rangle_4. \tag{10}$$

The detection of one photon with the detector D_l ($l=1,2,3,4$) corresponds to a quantum jump, which can be formulated with the operator b_l on the joint state $|\Phi(t)\rangle$. In order to generate the W states, we consider the events that one of the four detectors registers one photon and the other three detectors do not register any counts. For simplicity, we assume that the detector D_1 detects one photon, and the detectors D_2, D_3, and D_4 do not detect any photon. In this case, the combined state of the four atom-cavity systems is projected into

$$\begin{aligned}|\Psi_1\rangle &= b_1|\Phi(t)\rangle \\ &= \frac{1}{2}(|g_1\rangle|0\rangle_1|\Psi(t)\rangle_2|\Psi(t)\rangle_3|\Psi(t)\rangle_4 + |\Psi(t)\rangle_1|g_2\rangle \\ &\quad\times|0\rangle_2|\Psi(t)\rangle_3|\Psi(t)\rangle_4 + |\Psi(t)\rangle_1|\Psi(t)\rangle_2|g_3\rangle|0\rangle_3|\Psi(t)\rangle_4 \\ &\quad+|\Psi(t)\rangle_1|\Psi(t)\rangle_2|\Psi(t)\rangle_3|g_4\rangle|0\rangle_4),\end{aligned} \tag{11}$$

with the success probability $\beta^2(1-e^{-2\kappa t})$. By tracing over the cavity field, we obtain the state of four atoms

$$\begin{aligned}\rho_W = \frac{1}{(\alpha^2+\beta^2 e^{-2\kappa t})^3}\Big\{&\alpha^6|W_4\rangle\langle W_4| + \beta^6 e^{-6\kappa t}|g_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle\langle g_1|\langle g_2|\langle g_3|\langle g_4| + \frac{\alpha^4\beta^2 e^{-2\kappa t}}{4}[(|g_1\rangle|s_2\rangle|s_3\rangle|g_4\rangle + |s_1\rangle|g_2\rangle|s_3\rangle|g_4\rangle \\ &+|s_1\rangle|s_2\rangle|g_3\rangle|g_4\rangle)(\langle g_1|\langle s_2|\langle s_3|\langle g_4| + \langle s_1|\langle g_2|\langle s_3|\langle g_4| + \langle s_1|\langle s_2|\langle g_3|\langle g_4|) + (|g_1\rangle|s_2\rangle|g_3\rangle|s_4\rangle + |s_1\rangle|g_2\rangle|g_3\rangle|s_4\rangle + |s_1\rangle|s_2\rangle|g_3\rangle|g_4\rangle) \\ &\times(\langle g_1|\langle s_2|\langle g_3|\langle s_4| + \langle s_1|\langle g_2|\langle g_3|\langle s_4| + \langle s_1|\langle s_2|\langle g_3|\langle g_4|) + (|g_1\rangle|g_2\rangle|s_3\rangle|s_4\rangle + |s_1\rangle|g_2\rangle|g_3\rangle|s_4\rangle + |s_1\rangle|g_2\rangle|s_3\rangle|g_4\rangle)(\langle g_1|\langle g_2|\langle s_3|\langle s_4| \\ &+\langle s_1|\langle g_2|\langle g_3|\langle s_4| + \langle s_1|\langle g_2|\langle s_3|\langle g_4|) + (|g_1\rangle|g_2\rangle|s_3\rangle|s_4\rangle + |g_1\rangle|s_2\rangle|g_3\rangle|s_4\rangle + |g_1\rangle|s_2\rangle|s_3\rangle|g_4\rangle)(\langle g_1|\langle g_2|\langle s_3|\langle s_4| + \langle g_1|\langle s_2|\langle g_3|\langle s_4| \\ &+\langle g_1|\langle s_2|\langle s_3|\langle g_4|)] + \frac{\alpha^2\beta^4 e^{-4\kappa t}}{4}[(|g_1\rangle|g_2\rangle|g_3\rangle|s_4\rangle + |s_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle)(\langle g_1|\langle g_2|\langle g_3|\langle s_4| + \langle s_1|\langle g_2|\langle g_3|\langle g_4|) + (|g_1\rangle|g_2\rangle|s_3\rangle|g_4\rangle \\ &+|s_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle)(\langle g_1|\langle g_2|\langle s_3|\langle g_4| + \langle s_1|\langle g_2|\langle g_3|\langle g_4|) + (|g_1\rangle|s_2\rangle|g_3\rangle|g_4\rangle + |s_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle)(\langle g_1|\langle s_2|\langle g_3|\langle g_4| + \langle s_1|\langle g_2|\langle g_3|\langle g_4|) \\ &+(|g_1\rangle|g_2\rangle|g_3\rangle|s_4\rangle + |g_1\rangle|s_2\rangle|g_3\rangle|g_4\rangle)(\langle g_1|\langle g_2|\langle g_3|\langle s_4| + \langle g_1|\langle s_2|\langle g_3|\langle g_4|) + (|g_1\rangle|g_2\rangle|s_3\rangle|g_4\rangle + |g_1\rangle|s_2\rangle|g_3\rangle|g_4\rangle)(\langle g_1|\langle g_2|\langle s_3|\langle g_4| \\ &+\langle g_1|\langle s_2|\langle g_3|\langle g_4|) + (|g_1\rangle|g_2\rangle|g_3\rangle|s_4\rangle + |g_1\rangle|g_2\rangle|s_3\rangle|g_4\rangle)(\langle g_1|\langle g_2|\langle g_3|\langle s_4| + \langle g_1|\langle g_2|\langle s_3|\langle g_4|)]\Big\},\end{aligned} \tag{12}$$

where $|W_4\rangle$ is the W-type entangled states of the four atoms

$$|W_4\rangle = \frac{1}{2}(|g_1\rangle|s_2\rangle|s_3\rangle|s_4\rangle + |s_1\rangle|g_2\rangle|s_3\rangle|s_4\rangle + |s_1\rangle|s_2\rangle|g_3\rangle|s_4\rangle + |s_1\rangle|s_2\rangle|s_3\rangle|g_4\rangle). \tag{13}$$

If the time $t \gg \kappa^{-1}$, $e^{-\kappa t} \approx 0$ and we can only retain the first term $|W_4\rangle\langle W_4|$ of Eq. (12) and neglect all the other terms. This shows the obvious physical result that the W state is created in the event that only one photon is registered during the time $t \gg \kappa^{-1}$. During this period, the event that two, three, or four photons are registered has to be dropped.

In order to quantify how close the state (12) comes to the W state, we calculate the fidelity

$$F = \langle W_4|\rho_W|W_4\rangle = \frac{\alpha^6}{(\alpha^2+\beta^2 e^{-2\kappa t})^3}. \tag{14}$$

The fidelity increases with increasing detection time and increasing ratio $|\alpha/\beta|$.

Now we show how to generate GHZ states and cluster states of four distant atoms. For this purpose, we consider the events that both of the four detectors detect one photon, respectively, and the other two do not detect any photon. If detectors D_2 and D_4 detect one photon and D_1 and D_3 do not detect any photon, the state of the total system becomes projected into

$$\begin{aligned}|\Psi_{24}\rangle &= b_2 b_4|\Phi(t)\rangle \\ &= \frac{1}{\sqrt{2}}(|g_1\rangle|0\rangle_1|\Psi(t)\rangle_2|g_3\rangle|0\rangle_3|\Psi(t)\rangle_4 \\ &\quad + |\Psi(t)\rangle_1|g_2\rangle|0\rangle_2|\Psi(t)\rangle_3|g_4\rangle|0\rangle_4), \end{aligned} \tag{15}$$

with success probability $\beta^4(1-e^{-2\kappa t})^2$. If detectors D_1 and D_3 detect one photon and D_2 and D_4 do not detect any photon, the state of the total system becomes projected into

$$\begin{aligned}|\Psi_{13}\rangle &= b_1 b_3|\Phi(t)\rangle \\ &= \frac{1}{\sqrt{2}}(|g_1\rangle|0\rangle_1|\Psi(t)\rangle_2|g_3\rangle|0\rangle_3|\Psi(t)\rangle_4 \\ &\quad - |\Psi(t)\rangle_1|g_2\rangle|0\rangle_2|\Psi(t)\rangle_3|g_4\rangle|0\rangle_4), \end{aligned} \tag{16}$$

which can be transformed into Eq. (15) by local operations. Thus we only consider the state (15). By tracing over the cavity field, we obtain the state of four atoms

$$\begin{aligned}\rho_{\mathrm{GHZ}} = \frac{1}{(\alpha^2+\beta^2 e^{-2\kappa t})^2}\Bigg[&\alpha^4|\mathrm{GHZ}_4\rangle\langle\mathrm{GHZ}_4| + \beta^4 e^{-4\kappa t}|g_1\rangle|g_2\rangle \\ &\times|g_3\rangle|g_4\rangle\langle g_1|\langle g_2|\langle g_3|\langle g_4| + \frac{\alpha^2\beta^2 e^{-2\kappa t}}{2} \\ &\times(|s_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle\langle s_1|\langle g_2|\langle g_3|\langle g_4| + |g_1\rangle|s_2\rangle|g_3\rangle|g_4\rangle \\ &\times\langle g_1|\langle s_2|\langle g_3|\langle g_4| + |g_1\rangle|g_2\rangle|s_3\rangle|g_4\rangle\langle g_1|\langle g_2|\langle s_3|\langle g_4| \\ &+ |g_1\rangle|g_2\rangle|g_3\rangle|s_4\rangle\langle g_1|\langle g_2|\langle g_3|\langle s_4|)\Bigg], \end{aligned} \tag{17}$$

with

$$|\mathrm{GHZ}_4\rangle = \frac{1}{\sqrt{2}}(|g_1\rangle|s_2\rangle|g_3\rangle|s_4\rangle + |s_1\rangle|g_2\rangle|s_3\rangle|g_4\rangle). \tag{18}$$

We calculate the fidelity

$$F = \langle\mathrm{GHZ}_4|\rho|\mathrm{GHZ}_4\rangle = \frac{\alpha^4}{(\alpha^2+\beta^2 e^{-2\kappa t})^2}, \tag{19}$$

which again increases with increasing detection time and increasing ratio $|\alpha/\beta|$.

If detectors D_1 and D_2 detect one photon and D_3 and D_4 do not detect any photon, or vice versa, the state of the total system becomes projected into

$$\begin{aligned}|\Psi_{12}\rangle = \frac{1}{2}(&e^{i\pi/4}|g_1\rangle|0\rangle_1|g_2\rangle|0\rangle_2|\Psi(t)\rangle_3|\Psi(t)\rangle_4 \\ &- e^{i\pi/4}|\Psi(t)\rangle_1|\Psi(t)\rangle_2|g_3\rangle|0\rangle_3|g_4\rangle|0\rangle_4 + e^{-i\pi/4}|g_1\rangle \\ &\times|0\rangle_1|\Psi(t)\rangle_2|\Psi(t)\rangle_3|g_4\rangle|0\rangle_4) - e^{-i\pi/4}|\Psi(t)\rangle_1|g_2\rangle \\ &\times|0\rangle_2|g_3\rangle|0\rangle_3|\Psi(t)\rangle_4, \end{aligned} \tag{20}$$

with the probability $\beta^4(1-e^{-2\kappa t})^2$. If detectors D_1 and D_4 detect one photon and D_2 and D_3 do not detect any photon, or vice versa, the state of the total system becomes projected into

$$\begin{aligned}|\Psi_{14}\rangle = \frac{1}{2}(&||g_1\rangle|0\rangle_1|g_2\rangle|0\rangle_2|\Psi(t)\rangle_3|\Psi(t)\rangle_4 - |\Psi(t)\rangle_1|\Psi(t)\rangle_2|g_3\rangle \\ &\times|0\rangle_3|g_4\rangle|0\rangle_4 + i|g_1\rangle|0\rangle_1|\Psi(t)\rangle_2|\Psi(t)\rangle_3|g_4\rangle|0\rangle_4) \\ &- i|\Psi(t)\rangle_1|g_2\rangle|0\rangle_2|g_3\rangle|0\rangle_3|\Psi(t)\rangle_4), \end{aligned} \tag{21}$$

which can be transformed into Eq. (20) by local operations. The success probability of the outcome is $\beta^4(1-e^{-2\kappa t})^2$. By tracing over the cavity field of the state (20), we obtain the state of four atoms

$$\begin{aligned}\rho_C = \frac{1}{(\alpha^2+\beta^2 e^{-2\kappa t})^2}\Bigg[&\alpha^4|C_4\rangle\langle C_4| + \beta^4 e^{-4\kappa t}|g_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle\langle g_1| \\ &\times\langle g_2|\langle g_3|\langle g_4| + \frac{\alpha^2\beta^2 e^{-2\kappa t}}{2}(|s_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle\langle s_1|\langle g_2|\langle g_3|\langle g_4| \\ &+ |g_1\rangle|s_2\rangle|g_3\rangle|g_4\rangle\langle g_1|\langle s_2|\langle g_3|\langle g_4| + |g_1\rangle|g_2\rangle|s_3\rangle|g_4\rangle\langle g_1|\langle g_2| \\ &\times\langle s_3|\langle g_4| + |g_1\rangle|g_2\rangle|g_3\rangle|s_4\rangle\langle g_1|\langle g_2|\langle g_3|\langle s_4|)\Bigg], \end{aligned} \tag{22}$$

with

$$\begin{aligned}|C_4\rangle = \frac{1}{2}(&|g_1\rangle|g_2\rangle|s_3\rangle|s_4\rangle - |s_1\rangle|s_2\rangle|g_3\rangle|g_4\rangle + i|g_1\rangle|s_2\rangle|s_3\rangle|g_4\rangle \\ &- i|s_1\rangle|g_2\rangle|g_3\rangle|s_4\rangle). \end{aligned} \tag{23}$$

Using the local operation, we can transform the state (23) into the normal cluster state

$$\begin{aligned}&\frac{1}{2}(|g_1\rangle|g_2\rangle|s_3\rangle|s_4\rangle - |s_1\rangle|s_2\rangle|g_3\rangle|g_4\rangle \\ &+ |g_1\rangle|s_2\rangle|s_3\rangle|g_4\rangle - |s_1\rangle|g_2\rangle|g_3\rangle|s_4\rangle). \end{aligned} \tag{24}$$

We calculate the fidelity $F=\langle C_4|\rho|C_4\rangle$, which is equal to the Eq. (19).

We now give a brief discussion on the influence of photon losses on the scheme. In the generation process, the dominant noise is the photon loss, which includes the contribution from channel attenuation and the inefficiency of the single-photon detectors. All these kinds of noise can be considered by an overall photon loss probability η [15]. For simplicity, we only consider the influence of the noise on the preparation of the cluster state. In this case, the effective state of the four atoms is actually described by

$$\begin{aligned}\rho_{loss} = \frac{1}{N}\Bigg\{ &(\alpha^2+\beta^2 e^{-2\kappa t})^2\rho_C + \frac{1}{2}\eta\alpha^2\beta^2(1-e^{-2\kappa t}) \\ &\times(4\beta^2 e^{-2\kappa t}|g_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle\langle g_1|\langle g_2|\langle g_3|\langle g_4| \\ &+ |s_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle\langle s_1|\langle g_2|\langle g_3|\langle g_4| + |g_1\rangle|s_2\rangle|g_3\rangle|g_4\rangle \\ &\times\langle g_1|\langle s_2|\langle g_3|\langle g_4| + |g_1\rangle|g_2\rangle|s_3\rangle|g_4\rangle\langle g_1|\langle g_2|\langle s_3|\langle g_4| \\ &+ |g_1\rangle|g_2\rangle|g_3\rangle|s_4\rangle\langle g_1|\langle g_2|\langle g_3|\langle s_4|) + \frac{3}{2}\eta^2\beta^4 \end{aligned}$$

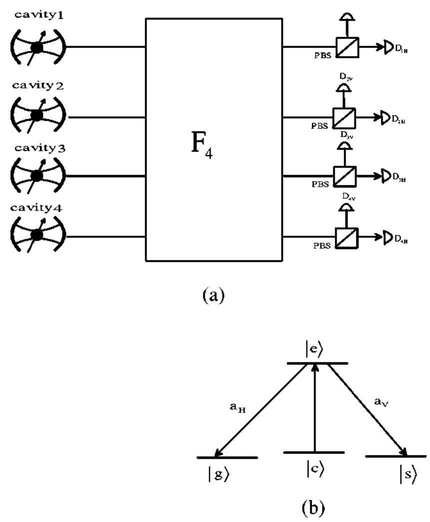

FIG. 2. (a) The schematic setup for generating W states and GHZ states of four distant atoms based on the four-photon coincidence detection. (b) The relevant level structure with ground states $|g\rangle$, $|s\rangle$, $|c\rangle$, and excited state $|e\rangle$.

$$\times(1-e^{-2\kappa t})^2|g_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle\langle g_1|\langle g_2|\langle g_3|\langle g_4|\Bigg\}, \quad (25)$$

with

$$N=(\alpha^2+\beta^2e^{-2\kappa t})^2+2(1-\eta)\alpha^2\beta^2(1-e^{-2\kappa t})+\eta\beta^4(1-e^{-2\kappa t})\times\left[2e^{-2\kappa t}+\frac{3}{2}\eta(1-e^{-2\kappa t})\right]. \quad (26)$$

The first term ρ_c of Eq. (25) is the expected state, which comes from the event that the two photons have been emitted from the cavities and two photons are detected at the detectors D_1 and D_2. The second term (or the third term) of Eq. (25) is noise contributions caused by the photon loss. It comes from the event that the three photons (or four photons) have been emitted from the cavities, but only two photons have been detected at the detectors D_1 and D_2.

In order to quantify how close the state (25) is to the cluster state we calculate the fidelity $F=\langle C_4|\rho_{loss}|C_4\rangle=\alpha^4/N$. The recalculated fidelity does not increase with increasing detection time. But this fidelity still increases with increasing ratio $|\alpha/\beta|$.

III. GENERATION OF THE *W* STATES AND GHZ STATES BASED ON THE FOUR-PHOTON COINCIDENCE DETECTION

In this section, we will present another unified scheme to generate the W states and GHZ states of four distant atoms. The scheme is based on the four-photon coincidence detection, and is insensitive to the quantum noise. But the scheme is not used to generate the cluster states of the four distant atoms.

The experimental setup is shown in the Fig. 2(a), and the atoms have the level structure shown in Fig. 2(b). Each of the four-level atoms (index j) has the Zeeman sublevels $|c_j\rangle$, $|g_j\rangle$, $|s_j\rangle$ and an excited state $|e_j\rangle$ $(j=1,2,3,4)$. The lifetime of the atomic levels $|c_j\rangle$, $|g_j\rangle$, and $|s_j\rangle$ are assumed to be comparatively long so that spontaneous decay of these states can be neglected. The transitions $|e_j\rangle\Leftrightarrow|g_j\rangle$ and $|e_j\rangle\Leftrightarrow|s_j\rangle$ are coupled to two degenerate cavity modes a_{jH} and a_{jV} with different polarization H and V. The transition $|e_j\rangle\Leftrightarrow|c_j\rangle$ is driven by the classical field. We assume that the classical laser field and the cavity field are detuned from their respective transition by the same amount. In this case of large detuning the excited state $|e_j\rangle$ can be eliminated adiabatically to obtain the effective interaction Hamiltonian (in the interaction picture):

$$H_j=\Omega(a_{jH}|c_j\rangle\langle g_j|+a_{jH}^\dagger|g_j\rangle\langle c_j|+a_{jV}|c_j\rangle\langle s_j|+a_{jV}^\dagger|s_j\rangle\langle c_j|), \quad (27)$$

where a_{jH} and a_{jV} are the annihilation operators of the H and V polarization modes of the jth cavity. Here we assumed that the effective coupling constants of the atoms coupled with their cavity modes are same, which are described by Ω. The evolution of the system's wave function is governed by a non-Hermitian Hamiltonian

$$H_j'=H_j-i\kappa(a_{jH}^\dagger a_{jH}+a_{jV}^\dagger a_{jV}), \quad (28)$$

as long as no photon decays from the cavity. Here we assume that four cavities have the same loss rate κ for all the modes. If a single-photon detector D_{jH} or D_{jV} $(j=1,2,3,4)$ detects a photon, the coherent evolution according to H_j' is interrupted by a quantum jump. This corresponds to a quantum jump, which can be formulated with the operators b_{jH} or b_{jV} on the joint state vectors of four atom-cavity systems

$$b_{1H}=\frac{1}{2}(a_{1H}+a_{2H}+a_{3H}+a_{4H}),$$

$$b_{2H}=\frac{1}{2}(a_{1H}+ia_{2H}-a_{3H}-ia_{4H}),$$

$$b_{3H}=\frac{1}{2}(a_{1H}-a_{2H}+a_{3H}-a_{4H}),$$

$$b_{4H}=\frac{1}{2}(a_{1H}-ia_{2H}-a_{3H}+ia_{4H}),$$

$$b_{1V}=\frac{1}{2}(a_{1V}+a_{2V}+a_{3V}+a_{4V}),$$

$$b_{2V}=\frac{1}{2}(a_{1V}+ia_{2V}-a_{3V}-ia_{4V}),$$

$$b_{3V}=\frac{1}{2}(a_{1V}-a_{2V}+a_{3V}-a_{4V}),$$

$$b_{4V} = \frac{1}{2}(a_{1V} - ia_{2V} - a_{3V} + ia_{4V}). \tag{29}$$

In the preparation stage the initial state of each atom-cavity system is $|c_j\rangle|0\rangle_{jH}|0\rangle_{jV}$ $(j=1,2,3,4)$, i.e., each atom is initially in the Zeeman level $|c_j\rangle$ and the cavity modes are prepared in the vacuum states. Now we switch on the Hamiltonian (27) in each atom-cavity system for a time τ. If no photon is emitted from the cavity, the jth atom-cavity system is governed by the interaction H_j'. In this case the atom-cavity state evolves to the entangled state

$$|\Psi\rangle_j = \alpha|c_j\rangle|0\rangle_{jH}|0\rangle_{jV} - i\beta(|g_j\rangle|H\rangle_j + |s_j\rangle|V\rangle_j), \tag{30}$$

with

$$\alpha = \frac{\cos(\Omega_\kappa\tau) - \dfrac{\sin(\Omega_\kappa\tau)\kappa}{2\Omega_\kappa}}{\sqrt{\left[\cos(\Omega_\kappa\tau) - \dfrac{\sin(\Omega_\kappa\tau)\kappa}{2\Omega_\kappa}\right]^2 + \dfrac{2\sin^2(\Omega_\kappa\tau)\Omega^2}{\Omega_\kappa^2}}},$$

$$\beta = \frac{\sin(\Omega_\kappa\tau)\Omega}{\Omega_\kappa\sqrt{\left[\cos(\Omega_\kappa\tau) - \dfrac{\sin(\Omega_\kappa\tau)\kappa}{2\Omega_\kappa}\right]^2 + \dfrac{2\sin^2(\Omega_\kappa\tau)\Omega^2}{\Omega_\kappa^2}}},$$

$$\Omega_\kappa = \sqrt{2\Omega^2 - \kappa^2/4}. \tag{31}$$

The probability that no photon is emitted during this evolution becomes

$$P_{single} = e^{-\kappa\tau}\left\{\left[\cos(\Omega_\kappa\tau) - \frac{\sin(\Omega_\kappa\tau)\kappa}{2\Omega_\kappa}\right]^2 + \frac{2\sin^2(\Omega_\kappa\tau)\Omega^2}{\Omega_\kappa^2}\right\}. \tag{32}$$

We assume that the interaction Hamiltonian (27) is applied to each atom-cavity system simultaneously, so that the preparation of the atom-cavity states $|\Psi\rangle_j$ ends at the same time. This implements the preparation stage of the protocol.

Now we consider the detection stage. In this stage, the joint state of three atom-cavity systems becomes prepared in the form

$$|\Phi(0)\rangle = |\Psi\rangle_1|\Psi\rangle_2|\Psi\rangle_3|\Psi\rangle_4, \tag{33}$$

where the state $|\Psi\rangle_j$ is given by Eq. (30). We assume that photons are detected at the time t and the joint state of the total system evolves into

$$|\Phi(t)\rangle = |\Psi(t)\rangle_1|\Psi(t)\rangle_2|\Psi(t)\rangle_3|\Psi(t)\rangle_4, \tag{34}$$

with

$$|\Psi(t)\rangle_j = \frac{1}{\alpha^2 + 2\beta^2 e^{-2\kappa t}}(\alpha|c_j\rangle|0\rangle_j + \beta e^{-\kappa t}[|g_j\rangle|H\rangle_j + |s_j\rangle|V\rangle_j)]. \tag{35}$$

If the detectors D_{1H}, D_{2H}, D_{3H}, and D_{4V} detect one photon, respectively, the state of the total system becomes projected into the W state:

$$|\text{Dick}\rangle_1 = \frac{1}{2}(|g_1\rangle|g_2\rangle|g_3\rangle|s_4\rangle + |g_1\rangle|g_2\rangle|s_3\rangle|g_4\rangle + |g_1\rangle|s_2\rangle|g_3\rangle|g_4\rangle + |s_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle). \tag{36}$$

The success probability of scheme is $P_{coin}=2\beta^8(1-e^{-2\kappa t})^4/9$.

If the detectors D_{1H}, D_{3H}, D_{2V}, and D_{4V} detect one photon, respectively, or D_{1V}, D_{3V}, D_{2H} and D_{4H} detect one photon, respectively, the state of the total system becomes projected into the GHZ state

$$|\text{Dick}\rangle_1 = \frac{1}{\sqrt{2}}(|g_1\rangle|s_2\rangle|g_3\rangle|s_4\rangle - |s_1\rangle|g_2\rangle|s_3\rangle|g_4\rangle). \tag{37}$$

The success probability of scheme is $P_{coin}=\beta^8(1-e^{-2\kappa t})^4/4$.

We now give a brief discussion on the influence of photon losses on the scheme. Since the scheme is based on the four-photon coincidence detection, this requires that each of the four detectors has to detect the photon. If one photon is loss, a click from each of the detectors is never recorded and the scheme fails. Therefore the photon loss has no influence on the fidelity of the generated states, but decreases the success probability P_{coin} by a factor $(1-\eta)^4$.

IV. CONCLUSION

In summary, we have proposed schemes for creating quantum entanglement of four distant atoms, which are trapped separately in different optical cavities. Based on the photon detectors with single-photon sensitivity, a unified scheme was presented to generate the W states, the GHZ states, and cluster states of four distant atoms. The disadvantage of the scheme is to require the single-photon detector to distinguish zero photons, one photon, and more than one photon. The detection inefficiency influences the fidelity of the generated states. Based on the four-photon coincidence detection, the scheme was presented to generate the W states and GHZ states of four distant atoms. They are insensitive to the detection inefficiency, which has no influence on the fidelity of the generated states, but decreases the success probability. But the scheme cannot be used to generate the cluster states.

Finally, we should mention that, in the proposed schemes, we consider the case of the large detuning on the excited state transition and neglect the effect of the spontaneous emission from the excited states of atoms. Taking into account that the adiabatic transition time increases with the detuning, it is not obvious that the large detuning eliminates the noise contribution caused by the spontaneous emission. The influence of the spontaneous emission on the schemes needs to be further studied.

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