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Observation and characterization of an optical spring

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Recent theoretical developments have highlighted the potential importance of "optical springs" in interferometers for gravitational wave detection as a means for beating the standard quantum limit. We have observed an optical spring effect experimentally in a detuned Fabry-Perot resonator in which one mirror is mounted on a flexure so that it has a significant response to radiation pressure. The main effect of the optical spring, an observed shift in the mechanical resonance frequency of the moveable mirror, agrees well with a simple model.

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It has long been recognized that interferometers for gravitational wave detection would eventually have to face the sensitivity limits imposed by the quantum nature of light [1]. The uncertainty in the position measurement of the optics due to photon counting statistics and radiation pressure noise, also due to the discrete nature of light, combine to give the standard quantum limit (SQL) as it applies to free masses. Much effort, both theoretical [2,3] and experimental [4], has gone into the use of squeezed light to circumvent the nominal quantum limits. Recent analyses have shown that radiation pressure can be used in a different way; certain optical configurations show a radiation pressure force which varies with the position of the mirror on which it acts, effectively an optical spring [5–7]. These optical springs can alter the response of an interferometer, so that the free mass SQL no longer applies, but rather the SQL fora simple harmonic oscillator, which is lower in the frequency band around the resonance. However, there is little experimental characterization of optical springs.

A simple system which exhibits an optical spring effect is an optical cavity tuned away from exact resonance (see Fig. 1). The radiation force on the mirror is always positive, and must be balanced by another force—typically from a mechanical spring or a servosystem. However, if the cavity is detuned from resonance, there will be a component of the radiation force which (for small displacements) depends linearly on the position of the mirror. If the cavity is longer than the resonant length the "optical spring constant" will be positive, increasing the total spring constant. If it is shorter the "optical spring constant" will be negative, reducing the total spring constant. If the optical spring effect is large enough to reduce the total spring constant below zero then the system will be unstable.

Mathematically, the optical spring constant for such a cavity in steady state is

$$k_{\text{opt}} = -\frac{dF_{rp}(x)}{dx} = -\frac{d}{dx}\left[\frac{(2R_2 + A_2)T_1 P_{\text{in}}}{|1-\sqrt{R_1 R_2}e^{-i(4\pi x/\lambda)}|^2 c}\right], \quad (1)$$

where x is the mirror displacement from cavity resonance, P_{in} is the input power, R_1 and R_2 are the mirror reflectivities of the input and output couplers, respectively, T_1 is the transmittance of the input coupler, A_2 is the fraction of power absorbed by the mirror, c is the speed of light, and λ is the optical wavelength. The total spring constant is given by the sum of the mechanical and optical spring constants, i.e., $k_{\text{tot}} = k_{\text{mech}} + k_{\text{opt}}$. The mechanical spring constant depends only on geometry and materials properties, not on the cavity resonance condition or optical parameters.

A full time-dependent analysis including transients must consider the velocity-dependent (damping) terms. The storage time of the cavity causes the force to lag the motion of the mirror, and in the case of a positive spring causes negative damping (i.e., excitation) of the mechanical spring. This optical damping is only significant if the lag angle of the

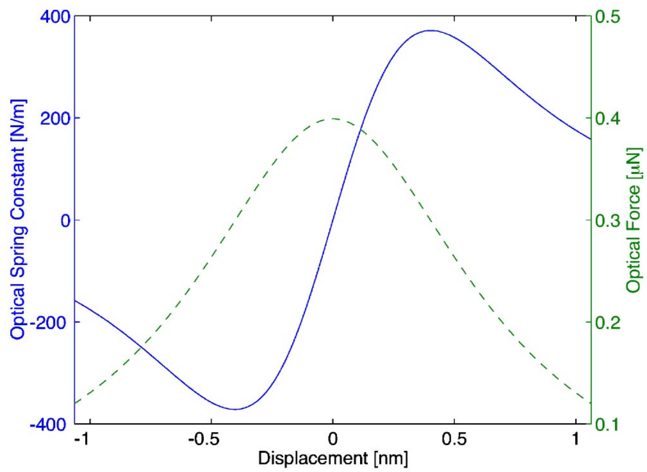

FIG. 1. The dashed curve shows the radiation pressure on the cavity mirror as a function of the detuning from resonance. The solid curve shows the derivative of the optical force, i.e., the optical spring constant. Positive displacement corresponds to increasing the cavity length. The parameters for calculating this curve are the ones from the experiment described here. The circulating power on resonance is 60 W, corresponding to the maximum input power used–400 mW.

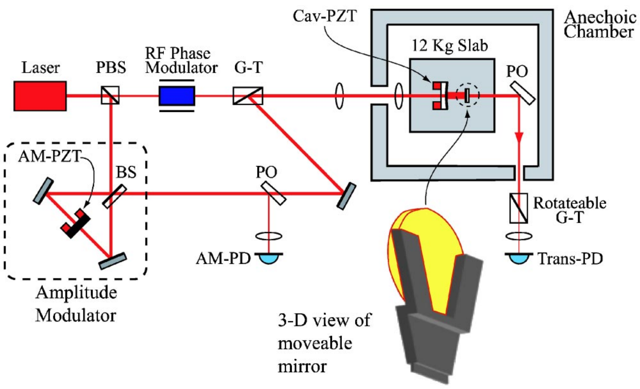

FIG. 2. A schematic diagram of the experiment. PBS, polarizing beam splitter; BS, beam splitter; G-T, Glan-Taylor polarizer; PO, pick off mirror; PZT, peizoelectric actuator; PD, photodetector.

optical field is comparable to $1/Q$ (Q is the quality factor of the mechanical spring). Optical damping can be neglected provided the following condition is met:

$$\tau_{\mathrm{cav}} < T^2/\tau_{\mathrm{mech}}, \qquad (2)$$

where τ_{cav} is the cavity storage time, T is the period of the mechanical oscillator, and τ_{mech} is the relaxation time of the mechanical spring without the presence of the optical spring. In this work the storage time of the cavity was short enough to neglect optical damping. Equation (1) is therefore an adequate description of the optical spring due to radiation pressure in this regime.

Radiation pressure induced nonlinear effects including optical bistability have been observed in a number of systems including macroscopic Fabry-Perot cavities [8] and in microelectromechanical systems Fabry-Perot etalons [9]. However, a detailed characterization of an optical spring including other nonlinear processes which may also be present, such as the photothermal effect [10–12] has, to our knowledge, not yet been reported.

In this paper we report on an experiment which shows the presence of an optical spring in a detuned Fabry-Perot cavity. Our system had a mechanical spring to counter the force of the optical spring and a servosystem to hold the cavity on resonance. The presence of the servosystem means that we had control of the cavity detuning and could lock to arbitrary positions around cavity resonance, thus providing quantitative data for comparisons with theory.

The optical layout of the experiment is shown in Fig. 2. The central element for observing the optical spring effect was a small mirror mounted on a niobium cantilever flexure. The mirror-flexure system formed a simple angular spring which was sensitive to radiation pressure. The mirror center was 12.7 mm above the flexure. Thus for small displacements ($\lesssim 1\,\mu$m), angular misalignments of the mirror can be ignored, and we can treat the mirror-flexure as a linear simple harmonic oscillator. The oscillator had a mass of approximately 1.2 g, a measured resonance frequency of 303 Hz, a Q of order 3000 (limited by gas damping), and an inferred mechanical spring constant of 2800 Nm^{-1}.

To sense the position of, and apply radiation pressure forces to the flexure mirror, it was used as the rear mirror in a Fabry-Perot cavity. The input coupler was mounted on a piezo-electric transducer which was used to control the length of the cavity. The cavity finesse was 380, which with the maximum available input power of 400 mW gave a circulating power of approximately 60 W. This corresponds to a peak radiation pressure force of 4×10^{-7} N, smaller than $k_{\mathrm{mech}}\lambda/2$ and thus much too small to observe multiple lock points. The cavity was short, 15 mm, to minimize the effect of the angular spring misalignment and to reduce the cavity storage time—approximately 6 ns. With such a short storage time the radiation inside the cavity can be modeled as reacting immediately to any change in cavity length so that there is no velocity-dependent force on the flexure-mirror oscillator.

The laser light was split by a polarizing beamsplitter into a high power, vertically polarized pump beam used to generate the optical spring, and a low power, horizontally polarized probe beam used to measure the cavity length. The pump beam passed through a common mirror Michelson interferometer which was used to provide amplitude modulation. A small portion of the beam was detected at the Michelson output to control the interferometer and to sense the power incident on the test cavity.

The probe beam passed through an electro-optic modulator which imposed 19 MHz phase modulation sidebands that were used to readout and control the length of the test cavity. The cross-polarized pump and probe were then recombined on a Glan-Taylor prism before being mode matched into the test cavity. The use of a Glan-Taylor polarizer ensured high polarization purity and good orthogonality between pump and probe. After the cavity, the pump beam was rejected by a final Glan-Taylor polarizer. This polarizer attenuated the pump beam by a factor of approximately 10^4. An rf photodiode was used to detect the 19 MHz signal on the transmitted probe beam, which was then demodulated and used as an error signal for the cavity length.

The optical system was first characterized by measuring the response of the cavity to modulation of the input power. Both amplitude and phase were measured using a network analyzer driving the amplitude modulator and reading out the cavity error signal, shown in Fig. 3. We determined that the response consists of three basic effects. At low frequencies the response is dominated by thermal expansion of the mirror substrates due to heating of the mirror coatings (the photothermal effect). The photothermal effect rolls off at higher frequencies with a $1/f$ dependence [11]. From 100 to 500 Hz the measured response fits the expected radiation pressure model. Higher frequencies are dominated by a frequency independent gain modulation effect (a spurious effect which is reduced but not entirely eliminated by the pump-probe arrangement in Fig. 2). Note that an increase in optical power causes a thermal expansion of the mirror substrates and thus the effective cavity length is shortened. Radiation pressure however responds to an increase in optical power by driving the mirrors apart and is therefore 180° out of phase with the photothermal effect at dc; however, in our system the photothermal effect dominates for frequencies below 100 Hz.

We first characterized the optical spring by scanning the laser frequency across the cavity resonance while monitoring

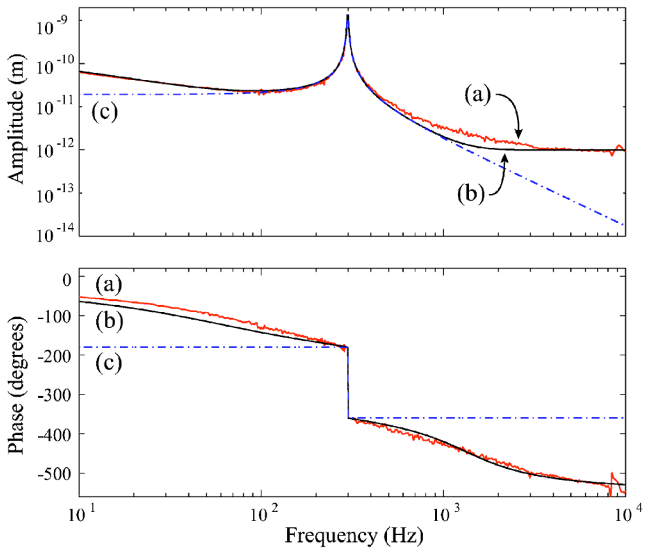

FIG. 3. Transfer function from input power to mirror displacement. Trace (a) is the measured transfer function. Trace (b) is the modeled transfer function and has three components: radiation pressure, the photothermal effect and a gain modulation coupling. Trace (c) is the theoretical response due to radiation pressure alone. Both the photothermal and radiation pressure models have no fitted parameters.

the transmitted power (the top trace in Fig. 4). Each resonance is asymmetric due to changes in the cavity length caused by the effect of circulating power on the mirror. The asymmetry occurs at low frequencies where the photothermal effect dominates; expansion of the mirror substrates due to heating from the circulating optical power either "pushes"

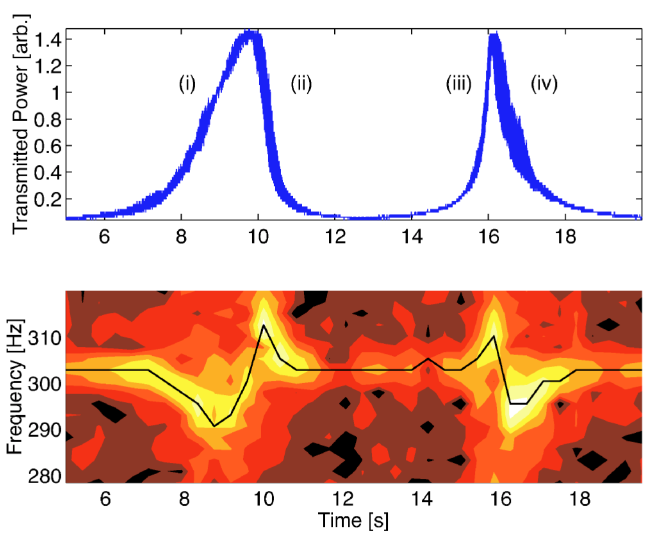

FIG. 4. The top trace shows the transmitted power as a function of time as the laser frequency is scanned through resonance. The sample rate was 500 Hz. The data in this trace has been decimated by a factor of 10. The bottom trace shows the mechanical resonant frequency shifting due to the optical spring effect either side of resonance. The thick black line in the bottom trace is the frequency with the largest amplitude corresponding to the observed mechanical resonant frequency.

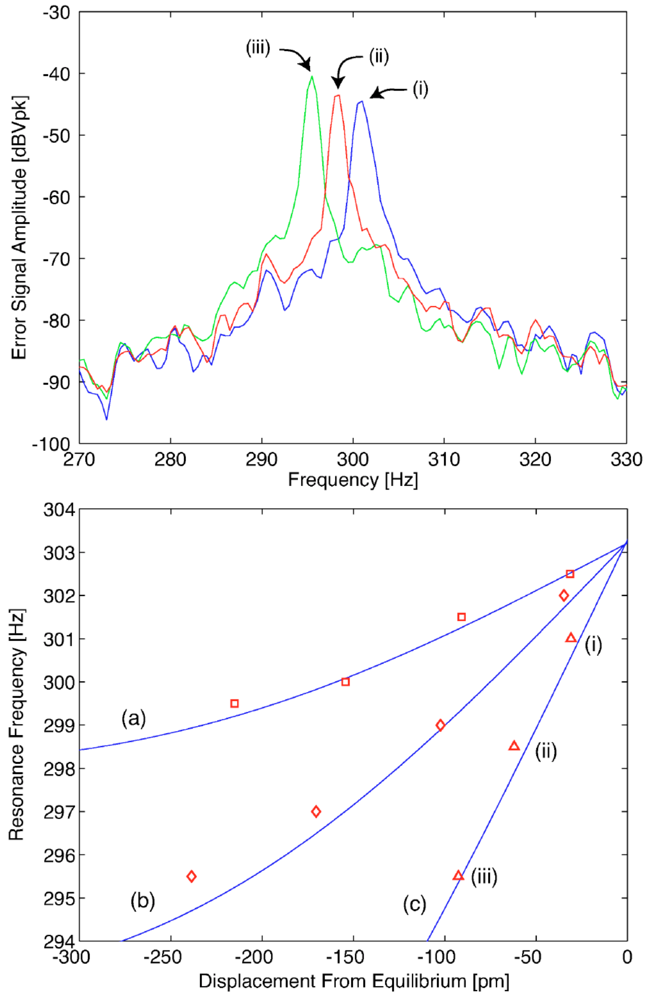

FIG. 5. The top plot shows the error signal spectrum for three different cavity detunings. The input power was 400 mW. The bottom plot shows the observed shift in resonance frequency versus offset for three different input powers: (a) 100 mW, (b) 200 mW, and (c) 400 mW. The data points corresponding to the observed frequency shifts shown in the top plot are marked (i), (ii), and (iii).

the cavity off resonance or "pulls" it onto resonance, depending on the direction of scanning. If the laser frequency is red detuned (cavity too short) and scanned towards resonance the power build up pushes the mirrors away from resonance, resulting in a broadened transition, shown in region (i). Once the laser reaches resonance and continues scanning toward the blue, the decrease in power further detunes the cavity resulting in an accelerated transition, shown in region (ii). At approximately 12.5 s the scan direction is reversed. Region (iii) shows an accelerated transition to resonance aided by the photothermal effect, while region (iv) shows a prolonged transition off resonance.

The "noise" visible on this trace is due to both acoustic and radiation pressure excitation of the mechanical resonance. Evidence of the optical spring can be observed as a shift in the mechanical resonant frequency either side of resonance which can be seen in a spectrogram of this data. The bottom plot in Fig. 4 shows a spectrogram calculated with a 2048 point FFT using a Hanning window. The frequency spacing of the FFT is 2.4 Hz and the time spacing is 0.41 s. The thick black line shows the frequency with the largest amplitude and corresponds to the observed mechanical resonant frequency. The frequency shift due to the radiation pressure optical spring can be seen on both sides of the cavity resonance. The reduction in resonant frequency occurs where the radiation pressure optical spring is negative, similarly the increase in observed resonant frequency occurs where the optical spring is positive. Both far off resonance and exactly on resonance, where the derivative of optical power with respect to mirror position is zero, no shift in the resonant frequency is observed.

Comparing Fig. 4(a) and 4(b) it is clear that the low frequency dynamics visible in Fig. 4(a) are out of phase with the frequency shifts shown in Fig. 4(b) and are not due to radiation pressure. In region (i), for example, the photothermal effect increases stability resulting in an extended transition to resonance while the resulting radiation pressure optical spring is negative resulting in a reduction in the mechanical resonance frequency. This is consistent with the transfer function measured in Fig. 3. At frequencies near the mechanical resonance, however, radiation pressure dominates, hence we assert that the frequency shifting is caused by radiation pressure.

In order to further quantify the optical spring the cavity was locked with low bandwidth feedback via the PZT-mounted input mirror. The optical spring effect relies on changes in the intensity in the cavity as a function of position of the moveable mirror. A high gain servo, desirable to suppress environmental noise acting on the moveable mirror, would effectively suppress the displacement from resonance, canceling any optical spring effect for frequencies within the control bandwidth. Hence, in order to observe the optical spring effect, we chose to lock with low bandwidth, typically less than 1 Hz so that around the mechanical resonance frequencies the mirror was effectively uncontrolled.

We observe the presence of the optical spring by measuring its effect on the resonance frequency of the mirror flexure system. To determine the mechanical resonance frequency of the flexure, free from optical effects, it was measured with the input power reduced to a minimum, yielding $f_{res} = 303$ Hz. At this low power, the mechanical resonance frequency did not change as a function of offset from cavity resonance.

To measure the frequency shift in the presence of the optical spring, the cavity was locked at full power and a spectrum of the servo error signal was recorded. With no offset added to the locking servo, the resonant peak had a frequency of 303 Hz, essentially the same as the one measured at low power. The top plot in Fig. 5 shows the traces of three different frequency shifts corresponding to curve (c) in the lower plot which shows additional data for lower power data points. Experimentally, positive frequency shifts due to radiation pressure were not observable due to instability. The photothermal effect degrades stability on this side of resonance and therefore is the likely cause of the instability. Conversely, on the side of resonance corresponding to a negative optical spring the photothermal effect enhances stability.

Equation (1) was used to calculate the effective optical spring constant using the experimentally measured parameters. The model then predicts values for the resonant frequencies as a function of the offset from resonance. There is good agreement between the measured and modeled frequency shifts. This quantitatively supports the supposition that the frequency shift was caused by radiation pressure.

In conclusion, we have accurately characterized an optical spring which is comprised of both a photothermal effect at low frequencies and dominated by radiation pressure at higher frequencies $(100\ \text{Hz}-500\ \text{Hz})$. Furthermore, due to the good agreement between the experiment results and theoretical predictions we conclude that this optical spring is the direct result of radiation pressure near the mechanical resonance (approximately 300 Hz).

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