Effects of spontaneously generated coherence on the conditions for exhibiting lasing without inversion in a V system

Yanfeng Bai,[1,2] Hong Guo,[1,2,*] Hui Sun,[1,2] Dingan Han,[2] Cheng Liu,[1] and Xuzong Chen[1]
[1]*QO01 Group, Key Laboratory for Quantum Information and Measurements of Ministry of Education, School of Electronics Engineering & Computer Science, Peking University, Beijing 100871, People's Republic of China*
[2]*QO01 Group, Laboratory of Light Transmission Optics, South China Normal University, People's Republic of China*
(Received 24 August 2003; published 19 April 2004)

In this paper, we investigate the effects of spontaneously generated coherence (SGC) on the conditions for exhibiting lasing without inversion (LWI) in the presence of a weak probe and a strong-coupling field. We find that owing to the effect of SGC, LWI can be realized without the incoherent pumping or the decay between two upper levels. However, in the region where LWI can be exhibited, the effect of SGC term on the exhibition of LWI is not always positive, sometimes it can even cumber the realization of LWI for some values of SGC. These values are given in this paper. In addition, we also analyze the essence of the generation of the probe field.

DOI: 10.1103/PhysRevA.69.043814 PACS number(s): 42.50.Gy, 42.50.Hz, 42.50.Ar

I. INTRODUCTION

There are various schemes which have been studied over the past few decades to realize lasing without population inversion (LWI) [1–7]. It has been proposed that a closed V-type, three-level system, which is incoherently pumped on the transition at which lasing occurs and coherently pumped on the other transition, can exhibit LWI [8]. A closed three-level atomic system was considered with the decay between two excited states, and the conditions for exhibiting LWI are changed into two inequalities under the resonant excitations [9]. Continuous-wave LWI in a closed four-level system coupled with field reservoir has been also discussed, and a practical implementation of the scheme is suggested [10]. Recently, spontaneous emission reduction and cancellation have been extensively studied. It is shown that it can be manipulated by a external pumping in a four-level system, a realizable scheme of which has been provided [11]. Based on these, there has been considerable interest in spontaneously generated coherence (SGC) arising from spontaneous emission of an atom with two close lying levels. It has been shown that SGC can change the steady-state response of the medium, and can modify significantly the absorption or spontaneous emission spectra of a near-degenerate system [12–15]. The response of SGC effects in a Λ system has been studied, where, in particular, the response of the Λ system to an external pump of arbitrary intensity is examined. It was demonstrated that the darked state could disappear in the presence of a strong SGC [16].

In this paper, we investigate the effects of SGC on the conditions for exhibiting LWI in a three-level V-type system with near-degenerate excited levels in the case of a weak probe. We show that due to the effects of SGC, the conditions are quite different, i.e., LWI can be realized without an incoherent pumping or the decay between the two excited states. Rather, the contributions to the probe gain not only originates from the population inversion and the induced coherence, but also from the SGC term. Moreover, in the region where LWI can be exhibited, the contribution from the SGC term is not always positive, sometimes the probe laser can even be attenuated due to the SGC term.

II. THE SYSTEM AND DENSITY-MATRIX EQUATIONS

We consider a V-type three-level system [see Fig. 1(a)] with the lower state $|1\rangle$ and the excited states $|2\rangle$ and $|3\rangle$. The transition $|1\rangle \leftrightarrow |2\rangle$ with frequency ω_{21} is driven by a strong coherent coupling field of frequency ω_c with a Rabi frequency $G=(\vec{\varepsilon}_c \cdot \vec{d}_{12})/(2\hbar)$. A weak, coherent probe field of frequency ω_p with Rabi frequency $g=(\vec{\varepsilon}_p \cdot \vec{d}_{13})/(2\hbar)$ is applied to the transition $|1\rangle \leftrightarrow |3\rangle$ without incoherent pump,

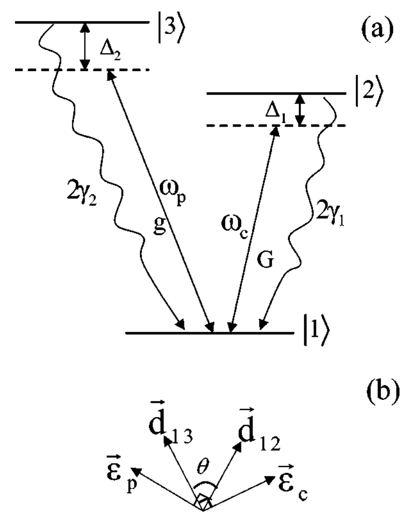

FIG. 1. (a) Schematic diagram of a three-level V-type system driven by two coherent fields with Rabi frequencies $2g$ (probe) and $2G$ (pump), respectively. (b) The field polarizations are chosen such that one field only drives one transition.

*Author to whom correspondence should be addressed. FAX: +86-10-6275-3208; email address: hongguo@pku.edu.cn

where $\vec{\varepsilon}_p$ and $\vec{\varepsilon}_c$ represent the amplitudes of two coherent fields, respectively. The spontaneous decay rates from the states $|2\rangle$ and $|3\rangle$ to the state $|1\rangle$ are $2\gamma_1$ and $2\gamma_2$, respectively. $\Delta_1=\omega_{21}-\omega_c$, and $\Delta_2=\omega_{31}-\omega_p$ are the detunings of the two corresponding fields, respectively. For simplicity, we assume g and G to be real. Since the dipole moments $\vec{d}_{12}$ and $\vec{d}_{13}$ are not orthogonal, which is necessary for the existence of the SGC effect, we have to consider an arrangement shown in Fig. 1(b) that one field acts on only one transition, where θ represents the angle between the two induced dipole moments $\vec{d}_{12}$ and $\vec{d}_{13}$. To avoid a field simultaneously affecting on two transitions, θ can be a random angle between 0 and 2π except for 0 and π.

The density-matrix equations [17] under the electric-dipole and rotating-wave approximation are

$$\dot{\rho}_{22}=-2\gamma_1\rho_{22}+iG\rho_{12}-iG^*\rho_{21}-\eta_0\cos\theta\sqrt{\gamma_1\gamma_2}(\rho_{23}+\rho_{32}), \quad (1)$$

$$\dot{\rho}_{33}=-2\gamma_2\rho_{33}+ig\rho_{13}-ig^*\rho_{31}-\eta_0\cos\theta\sqrt{\gamma_1\gamma_2}(\rho_{23}+\rho_{32}), \quad (2)$$

$$\dot{\rho}_{12}=-(\gamma_1+i\Delta_1)\rho_{12}+iG^*(\rho_{22}-\rho_{11})+ig^*\rho_{32} - \eta_0\cos\theta\sqrt{\gamma_1\gamma_2}\rho_{13}, \quad (3)$$

$$\dot{\rho}_{13}=-(\gamma_2+i\Delta_2)\rho_{13}+iG^*\rho_{23}+ig^*(\rho_{33}-\rho_{11}) - \eta_0\cos\theta\sqrt{\gamma_1\gamma_2}\rho_{12}, \quad (4)$$

$$\dot{\rho}_{23}=-(\gamma_1+\gamma_2)\rho_{23}+i(\Delta_1-\Delta_2)\rho_{23}+iG\rho_{13}-ig^*\rho_{21} - \eta_0\cos\theta\sqrt{\gamma_1\gamma_2}(\rho_{22}+\rho_{33}), \quad (5)$$

along with the equations of their complex conjugates, and the requirement of closing system, i.e., $\rho_{11}+\rho_{22}+\rho_{33}=1$. The terms including $\eta_0\sqrt{\gamma_1\gamma_2}\rho_{ij}\cos\theta$ which represents the quantum interference effects resulting from the cross coupling between spontaneous emissions $|3\rangle\leftrightarrow|1\rangle$ and $|2\rangle\leftrightarrow|1\rangle$, i.e., the SGC terms. If the two excited levels lie so closely that the SGC effects have to be taken into account, then $\eta_0=1$, otherwise $\eta_0=0$. It should be noted that only for small energy spacing [15] between the two excited levels are the SGC effects remarkable, as for large energy spacing, the oscillatory terms will average out to zero and such effects will vanish [12].

At the same time, we also note that it is difficult for the effects of the atomic coherence between the upper levels generated via spontaneous emission to be observed [18]. There is a restriction on the levels, i.e., they should have the same quantum numbers. While, with the progresses being made in modern physics, specifically semiconductor quantum dots and photonic band-gap materials, it is possible to fabricate semiconductor (quantum dots), with optical properties similar to atomic optical properties. However, unlike the atom, the quantum dot has the energy levels that can be adjusted for particular purposes [10]. So, we can fabricate a quantum dot with sufficiently close upper levels to implement our scheme. We will rediscuss this in Sec. IV.

III. THE EFFECT OF SGC ON THE CONDITIONS FOR EXHIBITING LWI

It has been known that the gain coefficient for the probe field coupled to the transition $|3\rangle\leftrightarrow|1\rangle$ is proportional to the imaginary part of the complex polarization, i.e., $\mathrm{Im}(\rho_{13})$. If $\mathrm{Im}(\rho_{13})>0$, the system exhibits gain for the probe field, that is, lasing can be established on the transition $|3\rangle\leftrightarrow|1\rangle$; whereas if $\mathrm{Im}(\rho_{13})>0$, the probe field is attenuated. In the following, $\Delta_1=\Delta_2=0$ is firstly considered, where we obtain the analytical solutions in the steady state and derive the condition under which the system exhibits LWI. Then, $\Delta_2\neq 0$ is considered. The numerical simulation will demonstrate that the contribution of the SGC terms to the probe gain is different for different values of SGC: the probe laser can sometimes be amplified, while sometimes attenuated.

From Eqs. (1)–(5), we can get the steady-state solutions under the condition of two-photon resonance ($\Delta_1=\Delta_2=0$),

$$\rho_{33}-\rho_{11}=-\frac{1}{D}\{2[G^4-G^2(-7+\eta^2)+6(-1+\eta^2)^2] + (g^2-2\sqrt{2}gG\eta)(4+G^2+2\eta^2)\}, \quad (6)$$

$$\rho_{22}-\rho_{11}=\frac{1}{D}[g^2(3+G^2-6\eta^2)-2G^2(6+G^2-3\eta^2) + g^4-2\sqrt{2}g^3G\eta+2\sqrt{2}gG(3+G^2)\eta], \quad (7)$$

$$\mathrm{Im}(\rho_{13})=\frac{1}{D}[-2g^3-2\sqrt{2}G\eta(-3+G^2+3\eta^2)+3\sqrt{2}g^2G\eta + 2g(3+2G^2)(-1+\eta^2)], \quad (8)$$

$$\mathrm{Im}(\rho_{12})=\frac{1}{D}\{-2G^3-\sqrt{2}g\eta(-6+g^2+6\eta^2)+G[12(-1+\eta^2) + g^2(-1+4\eta^2)]\}, \quad (9)$$

$$D=2[g^4+2G^4-2\sqrt{2}g^3G\eta+6(-1+\eta^2)^2+g^2(5+3G^2 + 4\eta^2)+G^2(13+5\eta^2)-2\sqrt{2}gG\eta(8+G^2+\eta^2)], \quad (10)$$

where $\eta=\eta_0\cos\theta$, and we set $2\gamma_1=\gamma_2=2\gamma$. All the parameters are reduced to dimensionless units by scaling with γ. We take the proportional relation of the two decays because it is satisfied with the inequalities proposed to exhibit LWI [9].

From Eq. (8), it is shown that $\mathrm{Im}(\rho_{13})$ is composed of two parts. The first part is a function of g, while the other is only that of G. It can be found that, when $g=0$, the first part will be zero, while the second part will not. This indicates that $\mathrm{Im}(\rho_{13})\neq 0$ even when $g=0$, which implies that the polarization does not vanish and will induce the generation of the

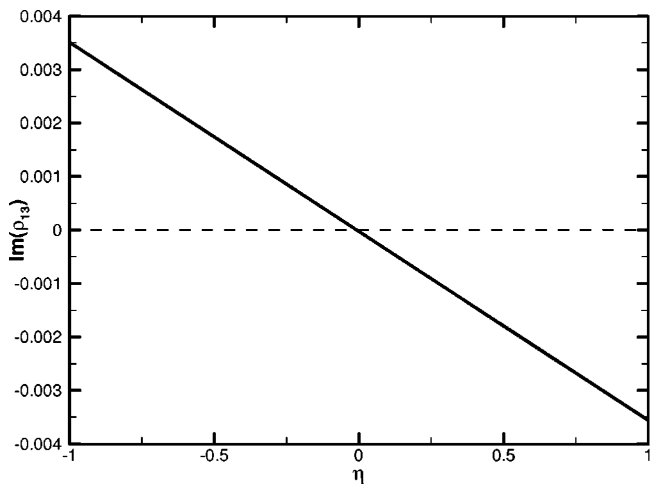

FIG. 2. Dependence of the probe gain $\mathrm{Im}(\rho_{13})$ upon η with $G=200\gamma$, $g=\gamma$.

probe field. So, the generation of the probe field is not only due to itself, but also to another driving field G. In this respect, this can also be viewed as a new coherence and interference phenomenon.

Lasing without inversion in any atomic state basis requires that $\mathrm{Im}(\rho_{13})>0$, $\rho_{33}<\rho_{11}$, and $\rho_{33}<\rho_{22}$ [19–21]. In the limit of a strong coherent field, $G\gg\gamma_1,\gamma_2$ and g, from Eqs. (6) and (7) it can be shown that $\rho_{33}<\rho_{11}$ and $\rho_{33}<\rho_{22}$ are always satisfied for all η's in its permitted range, i.e., $\eta\in(-1,1)$. From Eq. (8), we derive the gain condition $[\mathrm{Im}(\rho_{13})>0]$

$$\frac{\eta}{\eta^2-1}<\sqrt{2}\frac{g}{G}. \tag{11}$$

Here, the inequality is also called the condition for exhibiting LWI. From Eq. (11), we know that the range of η is determined by the ratio of two Rabi frequencies. LWI can be exhibited as long as the values of η are within the range, which is very different from Eq. (14) in Ref. [9] where the gain condition will not be satisfied, i.e., LWI cannot be exhibited if the decay between the two excited states is not included. It is also different from the conclusions mentioned in Ref. [8]. In the limit of a strong coherent field, $G\gg\gamma_1,\gamma_2$ and g, the V-type three-level system does not exhibit LWI if driven by a single coherent field only. We get LWI in the presence of SGC without considering the decay between two excited states or the incoherent pumping.

In Fig. 2, we plot $\mathrm{Im}(\rho_{13})$ vs η for a given ratio of the two Rabi frequencies: $G/g=200$. It is shown that the curve approximates to a straight line. Under a given ratio of two Rabi frequencies, one can get a range of η, where, $\eta\in(-1,-0.0073)$, in which $\mathrm{Im}(\rho_{13})>0$, i.e., the three-level system exhibits LWI.

When $\Delta_1\neq\Delta_2$, we investigate the physical mechanism leading to LWI in the atomic system. From Eq. (4), the steady-state solution of ρ_{13} with the SGC effect can be written as

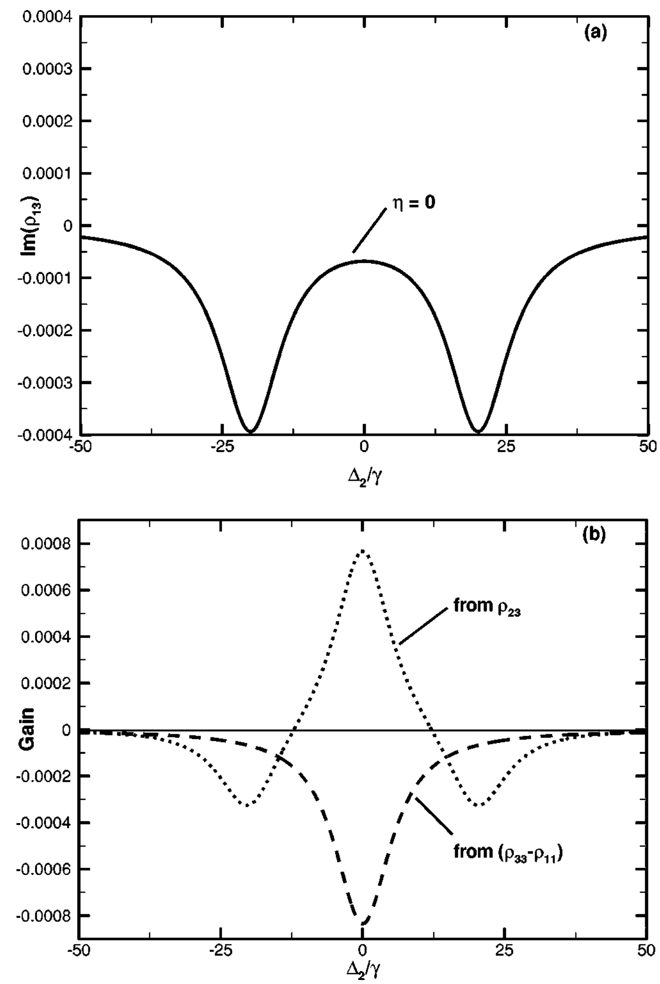

FIG. 3. (a) Plot of $\mathrm{Im}(\rho_{13})$ without the effect of SGC ($\eta=0$). (b) The gain contributions from ρ_{23} (dotted line) and $\rho_{33}-\rho_{11}$ (dashed line). Other parameters are chosen as $\gamma_1=\gamma$, $\gamma_2=2\gamma$, $G=20\gamma$, $g=0.01\gamma$, and $\Delta_1=0$.

$$\rho_{13}=\frac{iG\rho_{23}+ig(\rho_{33}-\rho_{11})-\eta\sqrt{\gamma_1\gamma_2}\rho_{12}}{\gamma_2+i\Delta_2}. \tag{12}$$

From Eq. (12), the induced polarization ρ_{13} is related to three terms, which are the induced coherence between the states $|2\rangle$ and $|3\rangle$, ρ_{23}, the population inversion between the states $|3\rangle$ and $|1\rangle$, $(\rho_{33}-\rho_{11})$, and the SGC term, $\eta\sqrt{\gamma_1\gamma_2}\rho_{12}$. It is known from Eq. (6) that $\rho_{33}-\rho_{11}<0$ for $\eta\in(-1,1)$, so the contribution to $\mathrm{Im}(\rho_{13})$ from the population inversion is always negative. Further, the contributions from the three terms to the probe gain are numerically computed and are shown in Fig. 4, while the case without SGC is also computed and is shown in Fig. 3.

In Fig. 3(a), it is shown that LWI cannot be exhibited without the effect of SGC. Also, it is apparent from Fig. 3(b) that the probe gain is contributed by ρ_{23} only. The term from $\rho_{33}-\rho_{11}$ presents an absorptive Lorentzian line profile centered at $\Delta_2=0$ [13], which coincides with the conclusions in Ref. [8].

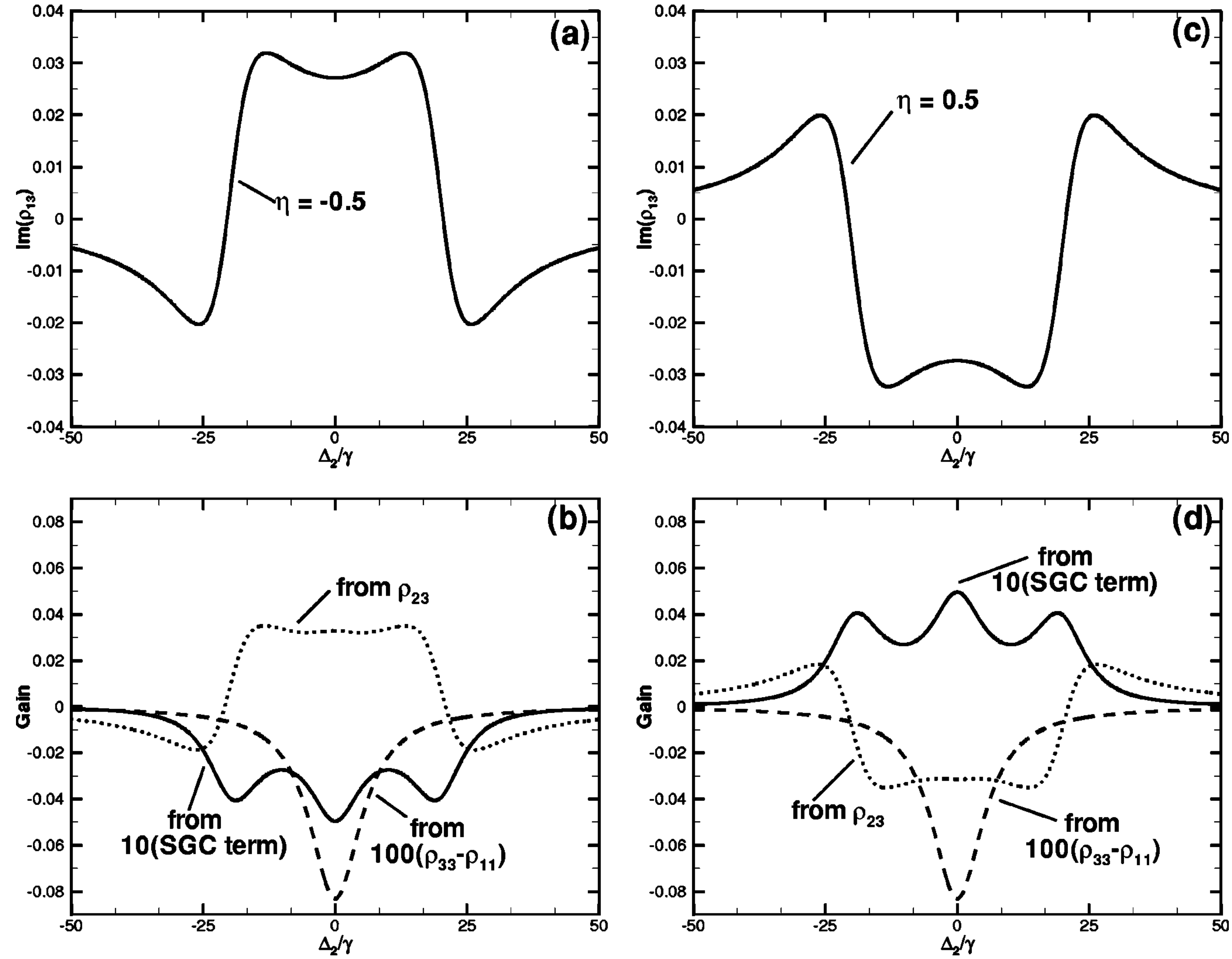

FIG. 4. Spectra for different η. When $\eta=-0.5$, the curves for (a) the gain line and (b) the gain contributions from three terms. The curves for the situations with $\eta=0.5$ are plotted in (c) and (d). The solid, dashed, and dotted curves represent the contribution from the SGC, the population inversion, and the induced coherence terms, respectively. Other parameters are the same as those in Fig. 2.

We give the gain line and the separate contributions from the three terms (including SGC term) in Fig. 4, in which the plots when $\eta=-0.5$ are shown in Figs. 4(a) and 4(b), while the case of $\eta=0.5$ are plotted in Figs. 4(c) and 4(d). From Figs. 4(a) and 4(c), it is shown that LWI occurs in different regions for different η. When $\eta=-0.5$, the area is closed to the center at $\Delta_2=0$ and is symmetrical; when $\eta=0.5$, the region is far away from the center, but it is still symmetrical. So, it is shown that due to the effects of SGC, a three-level V-type system can exhibit LWI without incoherent pumping or the decay between two excited states, which differs from the conclusions drawn previously [8,9].

Shown in Figs. 4(b) and 4(d) are the contributions to Im(ρ_{13}) from ρ_{23}, $\rho_{33}-\rho_{11}$, and the SGC term, respectively. For the sake of clarity, the contributions from the SGC and $(\rho_{33}-\rho_{11})$ terms are magnified for 10 and 100 times, respectively. It is shown that their behaviors are very different from those in Fig. 3. When $\eta=-0.5$, the gain is solely generated by ρ_{23}, the contribution from $\rho_{33}-\rho_{11}$ presents an absorptive Lorenzian line profile centered at $\Delta_2=0$, which is similar to that in Fig. 3. The part from the SGC term also presents an absorptive line profile, but it has two side bands, which correspond to the transitions from the state $|3\rangle$ to the Autler-Towner doublet states. So, though LWI is exhibited when $\eta=-0.5$, the SGC term cumbers its exhibition. At the same time, it should be noted, by comparing with Fig. 3, that the amplitude of ρ_{23} is greatly enhanced due to the effect of SGC, where, a part of the increased contribution is used to compensate the attenuation of the probe laser which is induced owing to the existence of the SGC term, the other realizes the probe gain. However, the case of $\eta=0.5$ is quite different. The SGC term begins to impulse the occurrence of LWI, and the change of η affects the line profile of ρ_{23}, which reduces the contribution from ρ_{23} on the probe gain in the region where LWI is exhibited. In addition, from Figs. 3(b), 4(b), and 4(d), we also notice that SGC has no effect on the absorptive profile of $(\rho_{33}-\rho_{11})$. From Eq. (12), it is easy to derive that the boundary of SGC is approximate to $\eta=0$. When $\eta>0$, the SGC term drives the realization of LWI; while, if $\eta<0$, its effect is negative.

IV. A POSSIBLE SCHEME FOR THE IMPLEMENTATION OF SGC

It has been shown in Sec. III that there is a principle difficulty in the realization of atomic interference via spontaneous emission for atomic transitions in free space. To realize such an interference, Kocharovskaya *et al.* proposed to take advantage of the technique of photonic band-gap materials and semiconductor quantum dots [10].

From their idea, we know that the pseudophotonic bandgap structures have a special purpose [22], they allow the propagation of electromagnetic waves with a certain polarization and strongly forbid those with orthogonal polarizations. So, the spontaneous decay with forbidden polarization is also forbidden and this allows us to remove the cancellation of different contributions originated from different polarizations of spontaneous emission occurring in free space, which is just what we need. Further, in the recent years, great effects have been focused on the fabrication of the semiconductor microstructure, which can allow for the creation of artificial quantum systems with desirable properties, based on which we can fabricate a quantum dot with sufficiently close upper levels to implement our scheme.

Kocharovskaya *et al.* have also designed a model of a quantum dot having different sizes a,b,c in three dimensions and infinite energy outside the box to realize their results, the so-called continuous wave LWI. They give the energy spectrum of the artificial "atom" and choose appropriate quantum dot for all levels. At last, they derive the equation of the small splitting between two upper levels. From their scheme, i.e., Fig. 1 in Ref. [10], we can see that it is similar to that we proposed in this paper. A difference is that an external level is used to populate other levels in their paper. So, we can implement our scheme only by removing the level from their scheme.

V. CONCLUSIONS

In this paper, we investigate the effects of SGC on the conditions for exhibiting LWI in a three-level V-type system. We find, due to the effects of SGC, that the conditions for exhibiting LWI may be expanded, i.e., LWI can be obtained without incoherent pumping [8] or the decay between two excited states [9] if the effect of SGC is considered in the limit of a weak probe field. However, this does not mean that the SGC term always impulses the exhibition of LWI. Actually, when $\eta > 0$, the effect is positive, while if $\eta < 0$, it will cumber the occurrence of LWI. In addition, it should be noted that the generation of the probe field is not only due to itself, but also to its partner field, i.e., the coupling field.

ACKNOWLEDGMENTS

One of the authors (Y.B.) is sincerely grateful to Dr. Sunish Menon for many fruitful discussions on this problem. This work was partially supported by the state Key Development Program for Basic Research of China (Grant No. 2001CB309308), the Key Project of National Natural Science Foundation of China (Grant No. 69789801), the Team Project of Guangdong Provincial Natural Science Foundation (Grant No. 20003061), the National Hi-Tech ICF program, and the Key Project of Natural Science Foundation of the Ministry of Education of China (Grant No. 00-09).

[1] S. E. Harris, Phys. Rev. Lett. **62**, 1033 (1989).
[2] V. G. Arkhipkin and Yu. I. Heller, Phys. Lett. **98A**, 12 (1983).
[3] M. O. Scully, S. Y. Zhu, and A. Gavrielides, Phys. Rev. Lett. **62**, 2813 (1989).
[4] G. S. Agarwal, S. Ravi, and J. Cooper, Phys. Rev. A **41**, 4721 (1990); **41**, 4727 (1990).
[5] O. Kocharovskaya and P. Mandel, Phys. Rev. A **42**, 523 (1990).
[6] S. Basil and P. Lambropoulos, Opt. Commun. **78**, 163 (1990); A. Lyras, X. Tang, P. Lambropoulos, and J. Zhang, Phys. Rev. A **40**, 4131 (1989).
[7] V. R. Blok and G. M. Krochik, Phys. Rev. A **41**, 1517 (1990).
[8] Y. Zhu and Min Xiao, Phys. Rev. A **49**, 2203 (1994).
[9] Shang-qing Gong, Hua-guo Teng, and Zhi-zhan Xu, Phys. Rev. A **51**, 3382 (1995).
[10] Olga Kocharovskaya, Andrey B. Matsko, and Yuri Rostovtsev, Phys. Rev. A **65**, 013803 (2001).
[11] Kishore T. Kapale, Marlan O. Scully, Shi-Yao Zhu, and M. Suhail Zubairy, Phys. Rev. A **67**, 023804 (2003).
[12] E. Paspalakis, S. Q. Gong, and P. L. Knight, J. Mod. Opt. **45**, 2433 (1998).
[13] Jin-Hui Wu and Jin-Yue Gao, Phys. Rev. A **65**, 063807 (2002).
[14] Wei-Hua Xu, Jin-Hui Wu, and Jin-Yue Gao, Opt. Commun. **215**, 345 (2003).
[15] Sunish Menon and G. S. Agarwal, Phys. Rev. A **57**, 4014 (1998).
[16] J. Javanainen, Europhys. Lett. **17**, 407 (1992).
[17] S. Menon and G. S. Agarwal, Phys. Rev. A **61**, 013807 (1999).
[18] A. Imamoglu, Phys. Rev. A **40**, 2835 (1989); P. Zhou and S. Swain, *ibid.* **56**, 3011 (1997).
[19] Yifu Zhu, Phys. Rev. A **53**, 2742 (1996).
[20] A. Imamoglu, J. E. Field, and S. E. Harris, Phys. Rev. Lett. **66**, 1154 (1991).
[21] Shi Yao Zhu and Ernst E. Fill, Phys. Rev. A **42**, 5684 (1990).
[22] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1989).