

# Spontaneous emission of an excited atom in a random absorbing medium

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The problem of spontaneous emission of an excited atom inside a random medium formed by randomly distributed dipoles is theoretically discussed by explicitly calculating the decay rate of the atom. After averaging over all the density fluctuations of the medium, we obtain an analytical expression of the decay rate in the low-density limit, which shows that inside a random medium the relaxation of an excited state is enhanced due to the density-fluctuation correlations of the random medium.

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## I. INTRODUCTION

The lifetime of an excited atom is a standard problem in quantum optics; it is well known that in the dipole transition approximation an excited state of a single atom in free space can be shown to relax spontaneously to the ground state at a rate proportional to the modular square of the dipole matrix element that links these states. Such a result can be obtained by treating spontaneous emission as a process of either the interaction between an excited atom and the fluctuations of the ground state of the quantized electromagnetic field or the reaction of the atom to its own radiation field. These two mechanisms are equally important in many situations in determining the lifetime of an excited atom [1]. In the present paper we will choose to work in the vacuum fluctuation model.

It is also a well-established fact that, if other objects are brought to the vicinity of an excited atom, the lifetime of the atom will be modified [2]. In the view point of the vacuum fluctuation theory, the presence of these objects inevitably modifies the electromagnetic field operators and subsequently changes the structure of the fluctuating field of the vacuum. It was by following this line that Power calculated the Lamb shift in as early as 1966 [3]. In the literature these external objects are usually assumed to form a regular structure, such as a dielectric sphere [4], a perfect mirror [1,5,6], a photonic crystal [7], and so on, and the lifetime of an excited atom inside an irregular structure has largely evaded proper treatment. Since in the past two decades, we have seen interesting results being revealed from the studies of light propagation inside random media, three examples of which are backscattering enhancement [8], photon localization [9], and random lasing [10], it is natural to ask if the randomness of a medium also has any effects on light emission from an atom situated inside the medium. The present paper is devoted to answering such a question by explicitly computing the spontaneous decay rate of an excited atom inside a random medium.

The process of spontaneous emission can be formulated

by starting from Fermi's golden rule. It then follows that, in the framework of the linear-response theory [11], the transition rate between an excited state and the ground state can be related to the imaginary part of the response function of the system [5,12,13] by using the fluctuation-dissipation theorem:

$$\Gamma = \frac{2d_i^* d_j}{\hbar} \text{Im}[G_{ij}(\mathbf{r}_A, \mathbf{r}_A; \omega_A)], \quad (1)$$

where the excited atom is assumed to be fixed in space at  $\mathbf{r}_A$ ,  $\omega_A$  is the atomic transition frequency, and  $\mathbf{d}$  is the dipole matrix element linking the excited and ground states. The repeated indices in Eq. (1) have to be summed over the Cartesian coordinates. The response function  $\mathbf{G}(\mathbf{r}, \mathbf{r}_1)$  is, in fact, the familiar dyadic Green function in the classical theory of electromagnetic fields, which is in turn equal to the electric field at position  $\mathbf{r}$  due to a unit dipole at  $\mathbf{r}_1$  subject to all boundary conditions presented in a problem. Our task is, therefore, reduced to evaluating the Green function corresponding to the random medium.

The paper is organized as follows. We will first calculate the Green function inside (a particular realization of) a random medium in Sec. II, and then average it over the randomness of the medium in Sec. III to make our discussion conformable to the usual experimental realities. When the averaged Green function is substituted back into Eq. (1) in Sec. IV, an ensemble averaged spontaneous emission rate will result. The paper will be summarized in Sec. V.

## II. GREEN FUNCTION INSIDE A RANDOM MEDIUM

We assume that the random medium, in which the excited atom is situated, is formed by  $N$  dipoles randomly distributed in the whole space, and that each dipole is characterized by an isotropic polarizability  $\alpha$  and a diminishing physical size. The Green function  $\mathbf{G}(\mathbf{r}, \mathbf{r}_1)$  (in Gaussian units) needed in Eq. (1), consequently, should be built up as the sum of all fields radiated from  $\mathbf{r}_1$  and multiply scattered by the dipoles:

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$$\begin{aligned}
\mathbf{G}(\mathbf{r}, \mathbf{r}_1) &= \mathbf{G}_0(\mathbf{r}, \mathbf{r}_1; k_A) + \mu \int \mathbf{G}_0(\mathbf{r}, \mathbf{r}_2; k_A) \cdot \mathbf{G}_0(\mathbf{r}_2, \mathbf{r}_1; k_A) n(\mathbf{r}_2) d\mathbf{r}_2 \\
&\quad + \mu^2 \int \mathbf{G}_0(\mathbf{r}, \mathbf{r}_2; k_A) n(\mathbf{r}_2) d\mathbf{r}_2 \cdot \int \mathbf{G}_0(\mathbf{r}_2, \mathbf{r}_3; k_A) n(\mathbf{r}_3) d\mathbf{r}_3 \cdot \mathbf{G}_0(\mathbf{r}_3, \mathbf{r}_1; k_A) + \cdots \\
&= \overline{\mathbf{G}_0(\mathbf{r}, \mathbf{r}_1; k_A)} + \mu \int \overline{\mathbf{G}_0(\mathbf{r}, \mathbf{r}_2; k_A)} \cdot \overline{\mathbf{G}_0(\mathbf{r}_2, \mathbf{r}_1; k_A)} n(\mathbf{r}_2) d\mathbf{r}_2,
\end{aligned} \tag{2}$$

where  $k_A = \omega_A/c$ ,  $\mu = 4\pi k_A^2 \alpha$ , and  $\mathbf{G}_0(\mathbf{r}, \mathbf{r}_1; k_A)$  is the free space dyadic Green function

$$\mathbf{G}_0(\mathbf{r}, \mathbf{r}_1; k_A) = \frac{1}{4\pi} \left( \mathbf{I} + \frac{1}{k_A^2} \nabla \nabla \right) \frac{e^{ik_A |\mathbf{r} - \mathbf{r}_1|}}{|\mathbf{r} - \mathbf{r}_1|}. \tag{3}$$

We have suppressed a common multiplicity factor  $4\pi k_A^2$  on both sides of Eq. (2) for simplicity, and in Eq. (3)  $\mathbf{I}$  denotes the unit dyadic.

By writing the microscopic density function of the dipoles  $n(\mathbf{r}) = \sum_{i=1}^N \delta(\mathbf{r} - \mathbf{r}_i)$  as the sum of its spatially averaged value  $n_0 = \langle n(\mathbf{r}) \rangle$  and density fluctuations about that average  $\delta n(\mathbf{r})$ , we can use a linear-operator method to transform  $\mathbf{G}$  into a series in ascending powers of  $\delta n$  [14]:

$$\begin{aligned}
\mathbf{G}(\mathbf{r}, \mathbf{r}_1) &= \mathbf{g}(\mathbf{r}, \mathbf{r}_1) + \mu \int \mathbf{g}(\mathbf{r}, \mathbf{r}_2) \delta n(\mathbf{r}_2) \cdot \mathbf{g}(\mathbf{r}_2, \mathbf{r}_1) d\mathbf{r}_2 \\
&\quad + \mu^2 \int \mathbf{g}(\mathbf{r}, \mathbf{r}_2) \delta n(\mathbf{r}_2) d\mathbf{r}_2 \cdot \int \mathbf{g}(\mathbf{r}_2, \mathbf{r}_3) \delta n(\mathbf{r}_3) d\mathbf{r}_3 \\
&\quad \cdot \mathbf{g}(\mathbf{r}_3, \mathbf{r}_1) + \cdots,
\end{aligned} \tag{4}$$

with  $\mathbf{g}$  obeying the following integral equation:

$$\mathbf{g}(\mathbf{r}, \mathbf{r}_1) = \mathbf{G}_0(\mathbf{r}, \mathbf{r}_1; k_A) + \mu n_0 \int \mathbf{G}_0(\mathbf{r}, \mathbf{r}_2; k_A) \cdot \mathbf{g}(\mathbf{r}_2, \mathbf{r}_1) d\mathbf{r}_2. \tag{5}$$

Since it exists in an unrestricted, uniform space characterized by  $\mu n_0$ , it follows that the  $\mathbf{g}$  function assumes an outgoing-wave solution that can be readily obtained by operating an operator  $(\nabla \times \nabla \times - k_A^2)$  on both sides of Eq. (5) and solving the derived differential equation. Explicit calculation shows

$$\mathbf{g}(\mathbf{r}, \mathbf{r}_1) = \mathbf{G}_0(\mathbf{r}, \mathbf{r}_1; \sqrt{k_A^2 + \mu n_0}). \tag{6}$$

Formally, the total Green function  $\mathbf{G}$  expressed in the series in Eq. (4) can be regarded as a multiple scattering process of  $\mathbf{g}$  by the density fluctuations: The  $\mathbf{g}$  function can go directly from  $\mathbf{r}_1$  to  $\mathbf{r}$  inside the uniform medium  $\mu n_0$  as the first term on the right-hand side (RHS) of Eq. (4) shows, or it has to be scattered by the density fluctuations once (the second term), twice (the third term),  $\cdots$ , before arriving at  $\mathbf{r}$ . As in Ref. [14] we assume that the dipoles are independent with each other, that is  $\langle \delta n(\mathbf{r}) \delta n(\mathbf{r}_1) \rangle = \langle \delta n(\mathbf{r}) \rangle \langle \delta n(\mathbf{r}_1) \rangle = 0$ , when  $\mathbf{r} \neq \mathbf{r}_1$ , so that the statistics of the density fluctuations of the dipoles presented there can be applied directly to the present case.

### III. ENSEMBLE AVERAGE OF $\mathbf{G}$

A specific form for  $\delta n$ 's in Eq. (4) corresponds to a specific configuration of the dipoles. In a typical experiment, a random medium is often formed by suspending scattering particles in a liquid [10,15,16]. If the observation time is long enough, one can argue that since every configuration of the scattering particles is reached, the observed result is actually an ensemble averaged result. To make our discussion conformable to such an experimental reality, we will in the following average  $\mathbf{G}$  over all possible density fluctuations; the decay rate in Eq. (1) accordingly becomes an average rate  $\langle \Gamma \rangle$ .

Since  $\langle \delta n(\mathbf{r}) \rangle = 0$  from its definition, we know the first-order density fluctuation does not contribute to  $\langle \Gamma \rangle$ . The second- and third-order density-fluctuation correlations make no contributions either. To see this, we have to note from the following two relations that in these two correlations the density fluctuations are forced to be at same places:

$$\langle \delta n(\mathbf{r}) \delta n(\mathbf{r}_1) \rangle = n_0 \delta(\mathbf{r} - \mathbf{r}_1),$$

$$\langle \delta n(\mathbf{r}) \delta n(\mathbf{r}_1) \delta n(\mathbf{r}_2) \rangle = n_0 \delta(\mathbf{r} - \mathbf{r}_1) \delta(\mathbf{r}_1 - \mathbf{r}_2).$$

When the preceding two relations are substituted into Eq. (4), they produce, respectively, the first- and second-order self-field interactions: interactions between the radiation from a dipole and the dipole itself. Higher orders of such interactions will appear in correlations with orders higher than three; see Eq. (7) for another example. Quantum-mechanically self-field interactions are known to be responsible for atomic energy shifts and spontaneous emission; classically they amount to a correction to the polarizability of the dipoles. In our classical theory of Green function, we shall assume that such a correction has already been made in our definition of  $\alpha$ . As a result of this assumption, any further account of self-field interactions in the following discussion becomes redundant.

The first nonzero contribution to the ensemble averaged Green function  $\langle \mathbf{G} \rangle$  actually comes from the fourth-order correlation, although we know from Ref. [14] that the four density fluctuations in this correlation can only be so grouped that they are at either one common location or two different ones:

$$\begin{aligned}
 \langle \delta n(\mathbf{r}) \delta n(\mathbf{r}_1) \delta n(\mathbf{r}_2) \delta n(\mathbf{r}_3) \rangle &= n_0 \delta(\mathbf{r} - \mathbf{r}_1) \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(\mathbf{r}_2 - \mathbf{r}_3) \\
 &+ n_0^2 [\delta(\mathbf{r} - \mathbf{r}_1) \delta(\mathbf{r}_2 - \mathbf{r}_3) \\
 &+ \delta(\mathbf{r} - \mathbf{r}_2) \delta(\mathbf{r}_1 - \mathbf{r}_3) \\
 &+ \delta(\mathbf{r} - \mathbf{r}_3) \delta(\mathbf{r}_1 - \mathbf{r}_2)]. \quad (7)
 \end{aligned}$$

To be out of this apparent dilemma, it is essential to realize that the integration orders in Eq. (4) formally specify radiative paths. For example, the radiation path represented by the third term on the RHS of Eq. (4) implies that a light wave can only get  $\mathbf{r}$  from  $\mathbf{r}_1$  by following the sequence of  $\mathbf{r}_1 \rightarrow \mathbf{r}_3 \rightarrow \mathbf{r}_2 \rightarrow \mathbf{r}$ . This point has also been made implicitly before in the paragraph immediately following Eq. (6). It then becomes evident that two density fluctuations at a common location, depending on the radiation sequence, can in fact represent that radiation fields are scattered to that common place twice by way of another scatterer. Substitution of the expression (7) into Eq. (4) reveals that only one (out of four) of the  $\delta$  products on the RHS of Eq. (7) survives, and it represents a third-order scattering event between two dipoles at  $\mathbf{r}_2$  and  $\mathbf{r}_3$ :

$$\mu^4 n_0^2 \int \mathbf{g}(\mathbf{r}, \mathbf{r}_2) \cdot \mathbf{g}^3(\mathbf{r}_2, \mathbf{r}_3) \cdot \mathbf{g}(\mathbf{r}_3, \mathbf{r}_1) d\mathbf{r}_2 d\mathbf{r}_3. \quad (8)$$

Other  $\delta$  function products reproduce the self-field interactions and should be ignored accordingly. However, since it can be shown by Fourier transformation that the transverse component of Eq. (8) is rather weak compared with its longitudinal component, the term of Eq. (8) has to be dropped from  $\langle \mathbf{G} \rangle$  hereafter for their limited contribution to  $\langle \Gamma \rangle$ . This is so, because for an absorbing medium only the transverse part of  $\mathbf{G}$ , denoted as  $\mathbf{G}^T$ , is responsible for radiative decay of an excited atom while the longitudinal part of  $\mathbf{G}$  only causes nonradiative decay in the form of Joule heating [12].

Similarly, we note that if two density fluctuations are set to be at one position  $\mathbf{r}_2$  and three at another position  $\mathbf{r}_3$ , the fifth-order correlation makes a contribute to  $\langle \mathbf{G} \rangle$  as follows:

$$\mu^5 n_0^2 \int \mathbf{g}(\mathbf{r}, \mathbf{r}_3) \cdot \mathbf{g}^4(\mathbf{r}_2, \mathbf{r}_3) \cdot \mathbf{g}(\mathbf{r}_3, \mathbf{r}_1) d\mathbf{r}_2 d\mathbf{r}_3. \quad (9)$$

In this case the  $\mathbf{g}$  function is formally scattered between  $\mathbf{r}_2$  and  $\mathbf{r}_3$  four times. The difference between the expressions (8) and (9) is that in Eq. (9), after multiple scattering, the  $\mathbf{g}$  function returns to the position from which it was scattered before while in Eq. (8)  $\mathbf{g}$  settles down at another location. As long as the density fluctuations are only distributed on two locations, even-order correlations will yield similar results as that in Eq. (8) and will be ignored for their negligible role in the radiative relaxation of an excited state.

The seventh correlation can also be treated in this fashion by setting three fluctuations at one position and the remaining four at another position. This arrangement leads to formal scattering of the  $\mathbf{g}$  function six times between two density fluctuations and produces

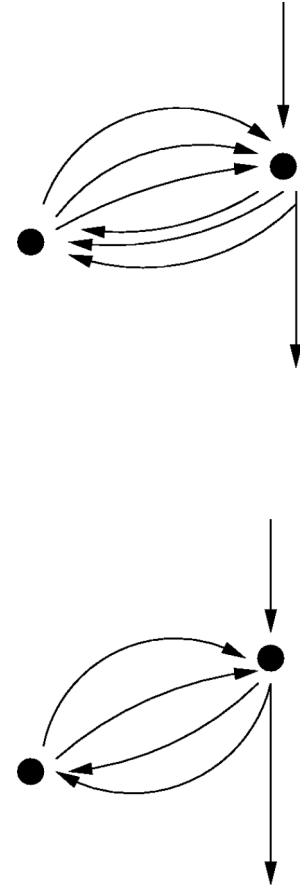


FIG. 1. Graphical representation of the first (lower) and second (upper) terms in the series in Eq. (11), with the filled circles representing dipoles and the arrows indicating scattering events.

$$\mu^7 n_0^2 \int \mathbf{g}(\mathbf{r}, \mathbf{r}_3) \cdot \mathbf{g}^6(\mathbf{r}_2, \mathbf{r}_3) \cdot \mathbf{g}(\mathbf{r}_3, \mathbf{r}_1) d\mathbf{r}_2 d\mathbf{r}_3. \quad (10)$$

A series is, therefore, formed by applying such density-fluctuation grouping to higher orders of odd correlations, in which the multiple scattering part of the  $\mathbf{g}$  function forms a geometrical subseries. This observation thus enables us to write the sum of the this series into closed form

$$\mu^5 n_0^2 \int \mathbf{g}(\mathbf{r}, \mathbf{r}_3) \cdot \frac{\mathbf{g}^4(\mathbf{r}_2, \mathbf{r}_3)}{1 - \mu^2 \mathbf{g}^2(\mathbf{r}_2, \mathbf{r}_3)} \cdot \mathbf{g}(\mathbf{r}_3, \mathbf{r}_1) d\mathbf{r}_2 d\mathbf{r}_3. \quad (11)$$

See Fig. 1 for a graphical representation of the first two terms in this series.

So far we have confined our discussion to the situation where the density-fluctuations are only on two positions. From the general discussion of density fluctuation correlations in Ref. [14], we find more combinations of density fluctuations become possible as still higher order correlations are taken into account. In the tenth-order correlation, if we divide the ten fluctuations into two groups, each of which contains five fluctuations, and then in each group we set two fluctuations at one position and three at another, we can get a contribution to  $\langle \mathbf{G} \rangle$  from this order of correlation whose ex-

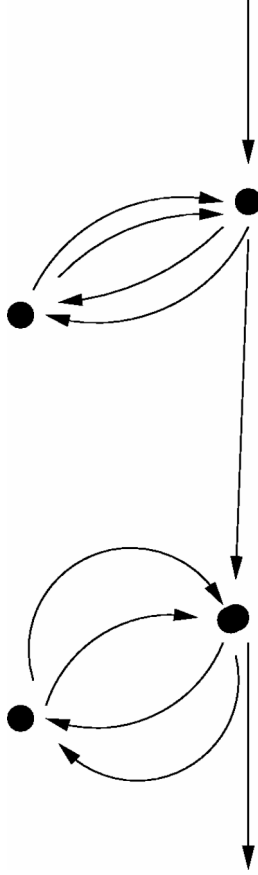


FIG. 2. Graphical representation of the contributions to  $\langle \mathbf{G} \rangle$  from the tenth-order correlation function, with filled circles representing dipoles and the arrows indicating scattering events.

pression seems to be formed by juxtaposing two expressions like that in Eq. (9)

$$\mu^{10} n_0^4 \int \mathbf{g}(\mathbf{r}, \mathbf{r}_2) \cdot \mathbf{g}^4(\mathbf{r}_2, \mathbf{r}_3) \cdot \mathbf{g}(\mathbf{r}_2, \mathbf{r}_4) \cdot \mathbf{g}^4(\mathbf{r}_4, \mathbf{r}_5) \cdot \mathbf{g}(\mathbf{r}_4, \mathbf{r}_1) \times d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 d\mathbf{r}_5. \quad (12)$$

Formally, Eq. (12) represents a scattering event where the  $\mathbf{g}$  function is first scattered four times between two density fluctuations at  $\mathbf{r}_4$  and  $\mathbf{r}_5$ , and then scattered another four times between a different pair of density fluctuations at  $\mathbf{r}_2$  and  $\mathbf{r}_3$ ; see Fig. 2 for an illustration. In view of this, the 12 density fluctuations in the 12th-order correlation can also be separated into two groups with one group containing five fluctuations and the other containing seven fluctuations. For the group that contains five fluctuations, we set two fluctuations at one position and the rest three at another position; for the group that contains seven fluctuations, we set three fluctuations at one position and the remaining four at another position. Under such an arrangement of the density fluctuations, the 12th correlation yields two contributions to  $\langle \mathbf{G} \rangle$ : in the first one (13) the first group has seven fluctuations and the second group has five; in the second one (14) the two groups are switched,

$$\mu^{12} n_0^4 \int \mathbf{g}(\mathbf{r}, \mathbf{r}_2) \cdot \mathbf{g}^6(\mathbf{r}_2, \mathbf{r}_3) \cdot \mathbf{g}(\mathbf{r}_2, \mathbf{r}_4) \cdot \mathbf{g}^4(\mathbf{r}_4, \mathbf{r}_5) \cdot \mathbf{g}(\mathbf{r}_4, \mathbf{r}_1) \times d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 d\mathbf{r}_5, \quad (13)$$

and

$$\mu^{12} n_0^4 \int \mathbf{g}(\mathbf{r}, \mathbf{r}_2) \cdot \mathbf{g}^4(\mathbf{r}_2, \mathbf{r}_3) \cdot \mathbf{g}(\mathbf{r}_2, \mathbf{r}_4) \cdot \mathbf{g}^6(\mathbf{r}_4, \mathbf{r}_5) \cdot \mathbf{g}(\mathbf{r}_4, \mathbf{r}_1) \times d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 d\mathbf{r}_5. \quad (14)$$

From all those even correlations with orders higher than 12, by following the same procedure to group the density fluctuations, we can get all the permutations that, when added, reduce to the following expression:

$$\mu^{10} n_0^4 \int \mathbf{g}(\mathbf{r}, \mathbf{r}_2) \cdot \frac{\mathbf{g}^4(\mathbf{r}_2, \mathbf{r}_3)}{1 - \mu^2 \mathbf{g}^2(\mathbf{r}_2, \mathbf{r}_3)} \cdot \mathbf{g}(\mathbf{r}_2, \mathbf{r}_4) \cdot \frac{\mathbf{g}^4(\mathbf{r}_4, \mathbf{r}_5)}{1 - \mu^2 \mathbf{g}^2(\mathbf{r}_4, \mathbf{r}_5)} d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 d\mathbf{r}_5. \quad (15)$$

It follows that, in this fashion, we can express  $\langle \mathbf{G} \rangle$  again in an integral equation

$$\langle \mathbf{G}(\mathbf{r}, \mathbf{r}_1) \rangle = \mathbf{g}(\mathbf{r}, \mathbf{r}_1) + \mu^5 n_0^2 \int \mathbf{g}(\mathbf{r}, \mathbf{r}_2) \cdot \frac{\mathbf{g}^4(\mathbf{r}_2, \mathbf{r}_3)}{1 - \mu^2 \mathbf{g}^2(\mathbf{r}_2, \mathbf{r}_3)} \cdot \langle \mathbf{G}(\mathbf{r}_2, \mathbf{r}_1) \rangle d\mathbf{r}_2 d\mathbf{r}_3, \quad (16)$$

with the expressions (11) and (15) as the second and third terms, respectively, if Eq. (16) is written out iteratively.

Other kinds of density-fluctuation grouping are of course also possible. For example, in the six-order correlation, we can set the six fluctuations into three groups (rather than two), each containing two fluctuations, and get a closed path composed of three scatterers. The contribution to  $\langle \mathbf{G} \rangle$  from such a configuration is then proportional to  $\mu^6 n_0^3$ . This term, and its higher-order counterparts, obviously gives a less important contribution to  $\langle \mathbf{G} \rangle$  when the average density  $n_0$  of the dipoles is low. In fact the theory of statistics of density fluctuations reported in Ref. [14] was based on the assumption of independent dipoles and the validity of this assumption requires that the density of dipoles be so low that the occurrence of a dipole at  $\mathbf{r}$  does not depend on whether another point  $\mathbf{r}_1$ , however close to  $\mathbf{r}$ , is occupied or not. One result of this theory is that density fluctuations are only correlated to themselves. To simplify our analysis and to consider only the most important contributions to  $\langle \mathbf{G} \rangle$  at the same time, we shall only include those terms that have entered the relation (16), and our formulation consequently works the best in the low-density limit.

When we separated the five density fluctuations in Eq. (9) into two groups, we have implicitly assumed that  $\mathbf{r}_2 \neq \mathbf{r}_3$ ; when  $\mathbf{r}_3$  is integrated over in the integral equation (16), a small volume  $V$  has to be excluded around  $\mathbf{r}_2$ . Besides, since the integral over  $\mathbf{r}_3$  represents a multiple scattering process of the  $\mathbf{g}$  function between two density fluctuations superposed on a uniform absorbing medium, it is reasonable to keep in every  $\mathbf{g}$  function involved only its transverse compo-

nent  $\mathbf{g}^T$ . From the expression of  $\mathbf{g}^T(\mathbf{R})$  (see Ref. [12]), it is clear that aside from the intermediate zone, where  $R \sim 2\pi/(\text{Re } k_1)$ , it is the term  $e^{ik_1 R}/R$  that dominates. Therefore, we can approximately have

$$\mathbf{g}^T(\mathbf{R}) = \frac{1}{4\pi R} e^{ik_1 R} (\mathbf{I} - \hat{\mathbf{R}}\hat{\mathbf{R}}), \quad (17)$$

where  $\hat{\mathbf{R}} = \mathbf{R}/R$  and  $k_1 = \sqrt{k_A^2 + \mu n_0}$ . Explicit calculation shows that any power of  $\mathbf{g}^T(\mathbf{R})$ , when integrated over the orientation of  $\mathbf{R}$ , becomes proportional to a unit dyadic. It follows then that the integral over  $\mathbf{r}_3$  in Eq. (16) can be regarded as a scalar constant  $f$ , whose expression becomes, after we set  $V \rightarrow 0$ ,

$$f = \frac{3 - 4\pi}{3(4\pi)^2} \int_0^\infty \frac{e^{i4k_1 R} dR}{(4\pi R)^2 - \mu^2 e^{2ik_1 R}}. \quad (18)$$

Again, by operating  $(\nabla \times \nabla \times - k_1^2)$  on both sides of Eq. (16), it is straightforward to get

$$\langle \mathbf{G}(\mathbf{r}, \mathbf{r}_1) \rangle = \mathbf{G}_0(\mathbf{r}, \mathbf{r}_1; \sqrt{k_A^2 + \mu n_0 + \mu^5 n_0^2 f}). \quad (19)$$

#### IV. AVERAGED DECAY RATE

After having obtained the ensemble averaged Green function, we are now in a position to calculate the average decay rate  $\langle \Gamma \rangle$ . By following the method used by Barnett *et al.* [12] it is easy to get the transverse Green function  $\langle \mathbf{G}^T(\mathbf{r}, \mathbf{r}_1) \rangle$  expressed in ascending powers of  $|\mathbf{r} - \mathbf{r}_1|$ . We then take the imaginary part of  $\langle \mathbf{G}^T(\mathbf{r}, \mathbf{r}_1) \rangle$  after setting  $\mathbf{r} \rightarrow \mathbf{r}_1 \rightarrow \mathbf{r}_A$ , and substitute it, together with the suppressed factor  $4\pi k_A^2$ , into Eq. (1). The ensemble averaged radiative decay rate of the excited atom is finally found to be

$$\langle \Gamma \rangle = \frac{4k_A^2 |\mathbf{d}|^2}{3\hbar} \text{Re} \sqrt{k_A^2 + \mu n_0 + \mu^5 n_0^2 f}, \quad (20)$$

where  $\text{Re}$  denotes the real part of a complex quantity. It is convenient to define an effective dielectric constant  $\epsilon_{eff}$  for the ensemble averaged medium through the following relation:

$$\sqrt{\epsilon_{eff}} = \text{Re} \frac{\sqrt{k_A^2 + \mu n_0 + \mu^5 n_0^2 f}}{k_A}, \quad (21)$$

which, together with the familiar decay rate in free space  $\Gamma_0 = (4|\mathbf{d}|^2 k_A^3)/(3\hbar)$  (see Refs. [2,13]), enables us to reexpress  $\langle \Gamma \rangle$  into a more compact form

$$\langle \Gamma \rangle = \Gamma_0 \text{Re} \sqrt{\epsilon_{eff}}. \quad (22)$$

This expression is analogous to that obtained by Barnett *et al.* [12]. However, since they only considered a uniform medium, the contribution from the density-fluctuation correlations  $\mu^5 n_0^2 f$  was absent in their dielectric constant. The local-field corrections have been neglected in our formulation [17], since they were pointed out to just amount to appending a multiplicity factor to  $\langle \Gamma \rangle$  [12]. In the resonant situation, where  $\omega_A$  is equal to the internal oscillation fre-

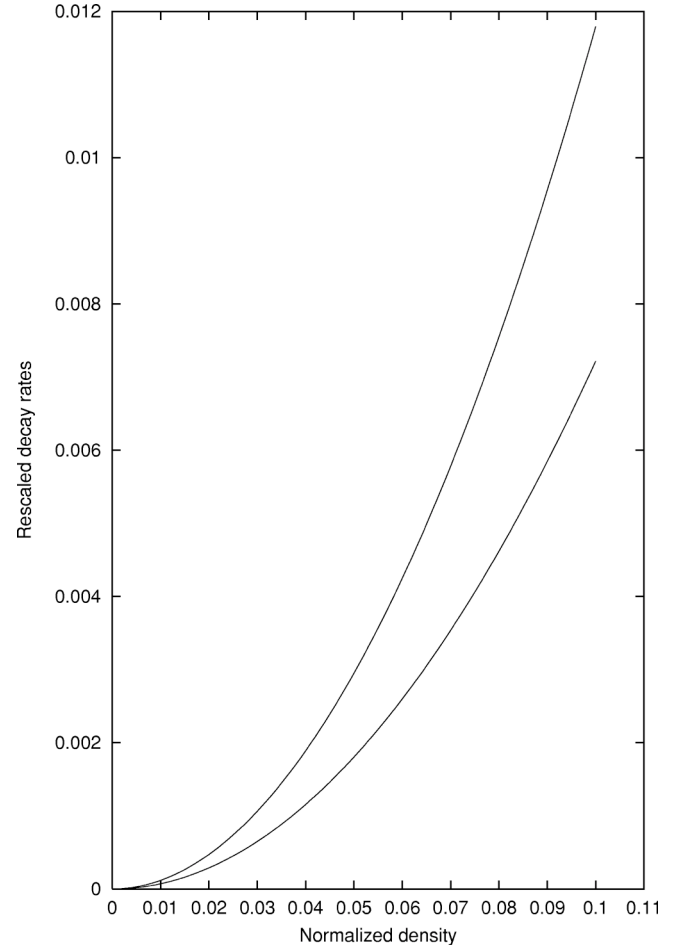


FIG. 3. Rescaled decay rate  $(\langle \Gamma \rangle / \Gamma_0 - 1) \times 10^3$  against the normalized density  $\lambda_A^3 n_0$ , with the contribution from the density-fluctuation correlations included in the upper curve but excluded in the lower one.

quency of the dipoles, the polarizability becomes  $\alpha = 3i/(2k_A^3)$ , and the decay rate  $\langle \Gamma \rangle$  is rescaled and plotted in Fig. 3 both with and without the contribution from the density-fluctuation correlations. It is clear that the presence of the density-fluctuation correlations causes the lifetime of an excited atom to decrease. This can be understood as resulting from the increment of the electric field acting on the atom due to the density-fluctuation correlations; see expression (4). From the fluctuation-dissipation theorem, one finds that the imaginary part of the Green function is proportional to the local density of electromagnetic states (LDOS) [18]; the enhanced spontaneous emission rate in our case, therefore, implies that in a random medium LDOS is also increased by the density fluctuations of the medium.

#### V. CONCLUSION

In this paper we have discussed theoretically the problem of spontaneous emission of an excited atom surrounded by randomly distributed dipoles. By separating the microscopic density of the dipoles, which is a random function, into its spatially averaged value  $n_0$  and fluctuations about that aver-

age  $\delta n$ , we are able to evaluate the Green functions with respect to both a uniform distribution of the dipoles and the density fluctuations. Since the Green function is related to the spontaneous emission rate of the excited atom through the fluctuation-dissipation theorem, our knowledge about the

Green functions has enabled us to get, in the low-density limit, an analytical expression for the ensemble averaged decay rate, from which we find that inside a random medium an excited atom has a shorter lifetime than it does inside a uniform medium.

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