

Concatenating dynamical decoupling with decoherence-free subspaces for quantum computationYong Zhang,^{*} Zheng-Wei Zhou,[†] Bo Yu, and Guang-Can Guo*Key Laboratory of Quantum Information, University of Science and Technology of China, Chinese Academy of Sciences, Hefei, Anhui 230026, China*

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A scheme to implement a quantum computer subjected to decoherence and governed by an untunable qubit-qubit interaction is presented. By concatenating dynamical decoupling through bang-bang (BB) pulse with decoherence-free subspaces (DFSs) encoding, we protect the quantum computer from environment-induced decoherence that results in quantum information dissipating into the environment. For the inherent qubit-qubit interaction that is untunable in the quantum system, BB control plus DFSs encoding will eliminate its undesired effect which spoils quantum information in qubits. We show how this quantum system can be used to implement universal quantum computation.

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I. INTRODUCTION

Quantum computation (QC) has become a very active field ever since the discovery that quantum computers can be much more powerful than their classical counterparts [1–3]. Quantum computers act as sophisticated quantum information processors, in which calculations are made by the controlled time evolution of a set of coupled two-level quantum systems. Coherence in the evolution is essential for taking advantage of quantum parallelism, which plays an essential role in all quantum algorithms. However, real physical systems will inevitably interact with their surrounding environment. No matter how weak the coupling that prevents an open system from being isolated, the evolution of the system is eventually plagued by nonunitary features such as decoherence and dissipation [4]. Quantum decoherence, in particular, is a purely quantum-mechanical effect whereby the system loses its ability to exhibit coherent behavior by getting entangled with the ambient degrees of freedom. Decoherence stands as a serious obstacle common to all applications, including QC, which rely on the capability of maintaining and exploiting quantum coherence.

Recently, considerable effort has been devoted to designing strategies able to counteract decoherence. Roughly speaking, three classes of procedures are available to overcome the decoherence problem. Two kinds of encoding methods of these strategies in the field of quantum information are quantum error-correction codes (QECCs) [5] and decoherence-free subspaces (DFSs, also called error-avoiding codes) [6–13], both based on encoding the state into carefully selected subspaces of the Hilbert space of the system. The main difference between the two encoding strategies is that QECCs is an active strategy, in which the encoding is performed in such a way that the various errors are mapped onto orthogonal subspaces so that they can be diagnosed and reversed, and DFSs instead provide a passive strategy relying on the occurrence of specific symmetries in

the interaction with the environment, which guarantees the existence of state space regions inaccessible to noise. The third strategy can be termed dynamical decoupling or quantum “bang-bang” (BB) control [14–18] after its classical analog by using strong, fast pulses on quantum systems. The basic idea is that open-system properties, specifically decoherence, may be modified if a time-varying control field acts on the dynamics of the system over time scales that are comparable to the memory time of the environment. Dynamical decoupling has an advantage over QECCs and DFSs, because it uses external pulses (BB pulse) rather than requiring several physical qubits to encode one logical qubit.

Despite their promise to counteract decoherence in the process of QC, QECCs and DFSs, in which ancillary physical qubits are required for protecting quantum information, have their disadvantage for the construction of a large scale quantum computer, because the available physical resource is very exiguous in the present quantum engineering. Dynamical decoupling does not require an ancillary physical qubit to protect quantum information, but entirely decoupling system from the environment requires more complicated pulse operations. Moreover, the inherent qubit-qubit interaction, which is vital to the implementation of two-qubit gate, is assumed to be tunable in all the approaches given above, but this will augment further the complexity of quantum computer in microstructure. Our effort is devoted to solving the problems mentioned above. In this work we present an architecture of quantum computer with fixed coupling between qubits. In our scheme, by concatenating dynamical decoupling and DFSs encoding we can simultaneously overcome the effects from decoherence and qubit-qubit interaction and realize the scalable fault-tolerant QC.

The structure of the paper is as follows. In Sec. II, we review dynamical decoupling by BB operations, and we show how to counteract decoherence via encoding into DFSs and decoupling by BB operations. In Sec. III we deal with the inherent qubit-qubit interaction between physical qubits by BB operations. We show in Sec. IV how the universal QC can be accomplished. Section V is for discussion and concluding remarks.

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II. DECOHERENCE AND BANG-BANG OPERATION

We consider a two-level quantum system S coupled to an arbitrary bath B , which together form a closed system defined on the Hilbert space $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_B$, \mathcal{H}_S and \mathcal{H}_B denoting S and B Hilbert spaces, respectively. The dynamics of the quantum system S coupled to a bath B evolves unitarily under the Hamiltonians

$$H = H_S \otimes I_B + I_S \otimes H_B + H_{SB}, \quad (1)$$

where H_S , H_B , and H_{SB} are the system, bath, and interaction Hamiltonians, respectively. The interaction Hamiltonian between the system and bath can be written as

$$H_{SB} = \sigma_x \otimes b_x + \sigma_y \otimes b_y + \sigma_z \otimes b_z. \quad (2)$$

Here the σ_α 's ($\alpha=x,y,z$) are the spin- $\frac{1}{2}$ Pauli operators on physical qubit and the b_α 's are operators on the degrees of freedom of environment. Due to the interaction Hamiltonian, the quantum system will entangle with the environment so that the quantum information encoded into quantum states irreversibly dissipates into the environment; this is the so-called decoherence. The objective of dynamical decoupling with BB operations used in our scheme is to modify this unwanted evolution.

The process of dynamical decoupling by BB operations, which counteracts decoherence by applying sequences of strong and fast pulses, serves for protecting the evolution of S against the effect of the interaction H_{SB} . In the standard view of the dynamical decoupling, a set of realizable BB operations can be chosen such that they form a discrete (finite order) subgroup of the full unitary group of operation on the Hilbert space of the system. Denote the subgroup \mathcal{G} and its elements g_k , $k=0,1,\dots,|\mathcal{G}|-1$, where $|\mathcal{G}|$ is the order of the group. The cycle time is $T_c = |\mathcal{G}|\Delta t$, where $|\mathcal{G}|$ is now also the number of pulse operations, and Δt is the time that the system evolves freely between operations under $U_0 = \exp(-iHt)$. The evolution of the system now is given by

$$U(T_c) = \prod_{k=0}^{|\mathcal{G}|-1} g_k^\dagger U_0(\Delta t) g_k \equiv e^{iH_{eff}T_c}. \quad (3)$$

H_{eff} denotes the resulting effective Hamiltonian. Obviously, to satisfy the above equation, it is required that the pulses in the sequence are very fast and strong compared with the evolution of Hamiltonian H , which is the origin of the name ‘‘bang-bang’’ operation. In this BB limit, the system will evolve under the effective Hamiltonian

$$H \rightarrow H_{eff} = \frac{1}{|\mathcal{G}|} \sum_{k=0}^{|\mathcal{G}|-1} g_k^\dagger H g_k \equiv \prod_{\mathcal{G}}(H). \quad (4)$$

The map $\prod_{\mathcal{G}}$ commutes with all g_k so that the action of the controller over times longer than the averaging period T_c only preserves the set of operators which are invariant under \mathcal{G} , thereby enforcing a \mathcal{G} symmetrization of the evolution of S [19]. Recently, a general result has been established by Facchi *et al.* [20], which states that dynamical decoupling can be accomplished by a sequence of arbitrary (fast and strong) pulses and symmetry or group structure is not neces-

sary, and the above procedure of decoupling by ‘‘symmetrization’’ arises as a special case. The main drawback of BB pulse decoupling procedures is that the timing constraints are particularly stringent. In fact, perfect decoupling from the environment is obtained only in the infinitely fast control limit [15,17,21], but it has been established that these decoupling schemes can be effective in a realistic situation with control pulses with finite strength and time duration [17,22].

Now let us first present our approach to counteract decoherence. For modifying the coupling induced by the Hamiltonian in Eq. (2), we consider a single BB operation $U_{z1} = \exp(-i\sigma_z\pi/2) = -i\sigma_z$, and when no pulses are applied the unit operator I denotes the operation on qubits. Using the commutation relation for Pauli operators, we have

$$U_{z1}^\dagger \sigma_x U_{z1} = \sigma_z \sigma_x \sigma_z = -\sigma_x, \quad (5)$$

$$U_{z1}^\dagger \sigma_y U_{z1} = \sigma_z \sigma_y \sigma_z = -\sigma_y, \quad (6)$$

$$U_{z1}^\dagger \sigma_z U_{z1} = \sigma_z \sigma_z \sigma_z = \sigma_z. \quad (7)$$

Thus after cycles of BB operations, we can obtain the effective interaction Hamiltonian

$$H_{SB} \rightarrow \prod(H_{SB}) = \sigma_z \otimes b_z, \quad (8)$$

which still introduces phase decoherence. In order to counteract phase decoherence, we can encode quantum information into DFSs. We use a well-known code [6,7,23] in which two physical qubits encode a logical qubit,

$$|0\rangle_L = |0_1 1_2\rangle \text{ and } |1\rangle_L = |1_1 0_2\rangle. \quad (9)$$

Here $i=1,2$ indexes physical qubits. For the system consisting of two physical qubits, the BB operation on the two physical qubits, correspondingly, can be defined as collective rotation: $U_z = U_{z1} \otimes U_{z2} = \exp(-i\sigma_1^z \pi/2) \otimes \exp(-i\sigma_2^z \pi/2) = -\sigma_1^z \otimes \sigma_2^z$, and then $\prod(H_{SB}) = (\sigma_1^z + \sigma_2^z) \otimes b_z$. Clearly, such encoding on a pair of physical qubits ensures that the encoded states are decoherence-free for phase error only if the disturbances from the environment around the system are identical. In other words, the two qubits must be arranged so close to each other that they undergo collective phase decoherence. Here the DFSs encoding together with BB operations serve for combating decoherence.

In Refs. [24,25], Byrd and Lidar have proposed a comprehensive encoding and decoupling solution to problems of decoherence. Decoherence is first reduced by encoding a logical qubit into two qubits, then completely eliminated by an efficient set of decoupling pulse sequences, in which cycles of pairs of BB pulses generated from the same exchange Hamiltonian are used to eliminate errors other than dephasing. The quantum code in our scheme is analogous to the one they have proposed for reducing phase decoherence. Then we apply directly a kind of simple BB pulse on a physical qubit to selectively decouple the system from the environment, which reduces the complexity of pulse operation. In our scheme untunable qubit-qubit interaction can be controlled by BB operations as discussed in the following section.

III. INTERACTION AND BANG-BANG OPERATION

To realize QC, any universal quantum gates (quantum operations) must include single-qubit gates and two-qubit gates. A traditional way for the implementation of single-qubit and two-qubit gates requires a control on two qubits level that is an ability to “switch on” and to “switch off” interaction between qubits. But an “always on” coupling can cause certain problems for quantum information preservation and QC. For example, if the interaction between two physical qubits in the code (9) is Heisenberg exchange interaction [26], the computational basis will always be flipped under the exchange Hamiltonian, which spoils quantum information in qubits. In general, quantum computers exploit control techniques [27,28] to tune the interaction between two physical qubits to avoid the undesired effect of the coupling, and tunability of the interaction constant is at the heart of many solid-state proposals, but this may prove extremely difficult to achieve experimentally. Recently, some schemes of QC governed by always on interaction have been presented [29–31]. In our scheme, we discuss the case that the interaction is always on and untunable, and we exploit BB operations to selectively decouple two physical qubits.

Now we consider the general exchange interaction between physical qubits. The exchange interaction Hamiltonian in the system has the form

$$H_I = J_x \sigma_1^x \otimes \sigma_2^x + J_y \sigma_1^y \otimes \sigma_2^y + J_z \sigma_1^z \otimes \sigma_2^z, \quad (10)$$

where J_a 's, ($a=x,y,z$) are exchange interaction constants.

We first consider the case of a single logical qubit. Under the self-exchange interaction, we find

$$H_I|0\rangle_L = (J_x + J_y)|1\rangle_L - J_z|0\rangle_L, \quad (11)$$

$$H_I|1\rangle_L = (J_x + J_y)|0\rangle_L - J_z|1\rangle_L. \quad (12)$$

Obviously, quantum information encoded will be spoiled by the self-exchange interaction. We selectively decouple the two physical qubits encoded into a logical qubit by introducing a selective decoupling BB operation $R_z = I_1 \otimes \exp(-i\sigma_2^z \pi/2) = -iI_1 \otimes \sigma_2^z$. We obtain

$$R_z^\dagger \sigma_1^x \otimes \sigma_2^x R_z = \sigma_1^x \otimes \sigma_2^x \sigma_2^z \sigma_2^z = -\sigma_1^x \otimes \sigma_2^x, \quad (13)$$

$$R_z^\dagger \sigma_1^y \otimes \sigma_2^y R_z = \sigma_1^y \otimes \sigma_2^y \sigma_2^z \sigma_2^z = -\sigma_1^y \otimes \sigma_2^y, \quad (14)$$

$$R_z^\dagger \sigma_1^z \otimes \sigma_2^z R_z = \sigma_1^z \otimes \sigma_2^z \sigma_2^z \sigma_2^z = \sigma_1^z \otimes \sigma_2^z. \quad (15)$$

So after cycles of BB operations, we obtain effective self-interaction $\Pi(H_I) = J_z \sigma_1^z \otimes \sigma_2^z$, which is equivalent to Ising interaction; the encoded states $|0\rangle_L$ and $|1\rangle_L$ in Eq. (9) are degenerate under the effective self-interaction. Therefore, if we store information in these states, no evolution whatsoever is present. In other words, for the untunable exchange interaction quantum information is stabilized by means of BB control and quantum encoding.

Until now, we have introduced two BB operations U_z and R_z . As already noted, the two BB operations are used on qubits 1 and 2 to counteract decoherence and undesired interaction. Actually, the pulse operations $R_z = I_1$

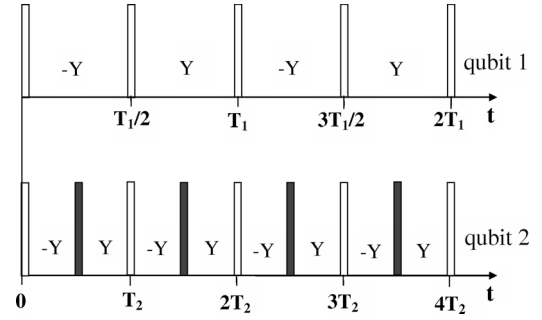


FIG. 1. The evolution of physical qubits 1 and 2 about the y axis under Hamiltonian H_{SB} and pulse operations. White and black rectangles denote strong and fast pulse operations U_z and R_z , respectively. T_1 and T_2 denote the cycle time of decoupling operations on qubits 1 and 2. $Y = \sigma_y$ is the error operator on the physical qubit.

$\otimes \exp(-i\sigma_2^z \pi/2)$ only act on physical qubit 2. For physical qubit 1, only the pulse operation σ_1^z has an effect on the decoherence. But there are two kinds of pulse operations in U_z and R_z effected on qubit 2 to selectively eliminate not only qubit-qubit interaction but also qubit-environment interaction. In other words, the number of pulse operations on qubits 1 and 2 is dissimilar. Because we apply the same pulse operations (σ^z) on every physical qubit, the time intervals Δt_1 on qubit 1 and Δt_2 on qubit 2 are different too. This implies that we have applied a kind of nonsynchronous pulse operation to overcome environment-induced decoherence and unwanted coupling between physical qubits.

Let us now show how to devise nonsynchronous pulse operations for decoupling different interactions. We can elaborately devise a set of programmed pulse operations in which the time intervals of the BB operations on two qubits are varied according to the program. In our scheme, unitary pulse operations are U_z and R_z as given above. Here we assume that the BB operation U_z begins at time $t_0=0$ and devise the time interval between two pulse operations is constant Δt . Then we devise the BB operation R_z begins at time $t_0 + \Delta t/2$ and the time interval is Δt too. So the time intervals between a pair of pulses on qubits 1 and 2 have the relation $\Delta t_1 = 2\Delta t_2$. In Fig. 1 we focus on the evolution of the y ingredient in Hamiltonian H_{SB} under the cycles of BB pulses. (The same conclusion adapts to the x ingredient in H_{SB} .) $T_1 = 2\Delta t_1$ and $T_2 = 2\Delta t_2$ denote the cycle time of decoupling operations on qubits 1 and 2, respectively. After cycles of pulse operations, the total effect of error operators (Y in the figure) on qubits 1 and 2, respectively, is zero in the cycle time NT_i ($i=1,2$), here N and N_i ($i=1,2,3$) given in the following are positive integers. This implies that decoherence on qubits 1 and 2 is held back. In addition, by similar analysis, we find that for the self-interaction between qubits 1 and 2, the total effect of the error operator $J_x \sigma_1^x \otimes \sigma_2^x + J_y \sigma_1^y \otimes \sigma_2^y$ is also eliminated in the cycle time $T = N_1 T_1 = N_2 T_2$, so in the y axis qubits 1 and 2 are decoupled. The result shows that the programmed BB pulse operations can eliminate or selectively eliminate not only qubit-environment interaction but also qubit-qubit interaction. This gives us a very heuristic solution to elimination of undesired coupling. The method of decoupling with programmed unsymmetrical

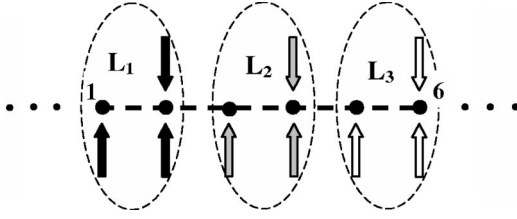


FIG. 2. Architecture of three logical qubits in the quantum computer. Each dot is a physical qubit and the dashed lines represent the interaction between qubits. Every logical qubit consists of two physical qubits. Arrows with different colors denote different pulse operations on the logical qubit.

pulse operations may be of great benefit to implementation of QC in many complicated circumstances.

In the above discussion, we present a dynamical decoupling scheme based on group averaging formulation. It is noteworthy that for the two-qubit system the operation set $\{I, U_z, R_z\}$ has no group structure, which accords with the result of Ref. [20].

We still need to show how the interaction between two logical qubits influences the encoded states of logical qubits. The exchange interaction in Eq. (10) between two logical qubits will induce unwanted flow of quantum information between two logical qubits. This will inevitably result in the failure of the preservation of quantum information and QC. In our scheme, quantum computer is constructed in a one-dimensional array of physical qubits. Now, we introduce new logical qubits L_2 and L_3 (see Fig. 2). For logical qubit L_2 , two selective decoupling BB operations are chosen as U_x and R_x , here $U_x = U_{x3} \otimes U_{x4} = \exp(-i\sigma_3^x \pi/2) \otimes \exp(-i\sigma_4^x \pi/2) = -\sigma_3^x \otimes \sigma_4^x$ and $R_x = I_3 \otimes U_{x4} = -iI_3 \otimes \sigma_4^x$. Then, we can obtain the effective interaction Hamiltonian $\Pi(H_{SB}) = (\sigma_3^x + \sigma_4^x) \otimes b_x$ and the effective self-interaction $\Pi(H_I) = J_x \sigma_3^x \otimes \sigma_4^x$. Accordingly, two encoded states of L_2 encoded in DFS can be written as

$$|0\rangle_{L_B} = \frac{1}{2}(|0_3\rangle + |1_3\rangle)(|0_4\rangle - |1_4\rangle), \quad (16)$$

$$|1\rangle_{L_B} = \frac{1}{2}(|0_3\rangle - |1_3\rangle)(|0_4\rangle + |1_4\rangle), \quad (17)$$

where the subscript B denotes the method of decoupling and encoding for logical qubit L_2 . Similarly, two selective decoupling subgroups of logical qubit L_3 are chosen as U_y and R_y . Here $U_y = U_{y5} \otimes U_{y6} = \exp(-i\sigma_5^y \pi/2) \otimes \exp(-i\sigma_6^y \pi/2) = -\sigma_5^y \otimes \sigma_6^y$ and $R_y = I_5 \otimes U_{y6} = -iI_5 \otimes \sigma_6^y$, and then, the quantum code in DFS will have the form

$$|0\rangle_{L_C} = \frac{1}{2}(|0_5\rangle + i|1_5\rangle)(|0_6\rangle - i|1_6\rangle), \quad (18)$$

$$|1\rangle_{L_C} = \frac{1}{2}(|0_5\rangle - i|1_5\rangle)(|0_6\rangle + i|1_6\rangle). \quad (19)$$

Obviously, with selective decoupling and encoding into DFSs, L_2 and L_3 can overcome decoherence and unwanted internal interaction as L_1 does.

Now, we focus on the coupling between logical qubits L_1 and L_2 that is equivalent to the coupling between physical qubit 2 and 3. The inherent interaction Hamiltonian H_{23} between qubits 2 and 3 has the form as shown in Eq. (10). For physical qubit 2, the pulse operation is σ_2^z , then the evolution of the x and y ingredients in Hamiltonian H_{23} is changed. For qubit 3, the pulse operation is σ_3^x which changes the evolution of the y and z ingredients in Hamiltonian H_{23} . Then, after cycles of pulse operations in the time $T = N_2 2\Delta t_2 = N_3 2\Delta t_3$, we obtain $\Pi(\sigma_2^x \otimes \sigma_3^x) = 0$ and $\Pi(\sigma_2^z \otimes \sigma_3^z) = 0$. So the evolution of the x and z ingredients in Hamiltonian H_{23} is eliminated. As far as the evolution of the y ingredient is concerned, since pulses have effect on qubit 2 at the interval of Δt_2 , but on qubit 3 at the interval of Δt_1 , here $\Delta t_1 = 2\Delta t_2$, the evolution about y axis on qubits 2 and 3 is unsymmetrical, then $\Pi(\sigma_2^y \otimes \sigma_3^y) = 0$, i.e., the evolution of the y ingredient in Hamiltonian is eliminated. This can also be illuminated by Fig. 1. To sum up, with cycles of pulse operations, the effect of Hamiltonian H_{23} between qubits 2 and 3 is eliminated. In other words, L_1 is entirely decoupled from L_2 . The same conclusion can be drawn for logical qubits L_2 and L_3 .

We showed above that with BB pulse operations and quantum encoding into DFS, the three logical qubits overcome not only environment-induced decoherence but also unwanted inherent interaction which is always on and untunable between physical qubits. And we devise that the three logical qubits are effected with three different BB operations so that every logical qubit is decoupled from others. Then, we can construct a scalable quantum computer with the three logical qubits as a unit of computation, i.e., the quantum computer has the periodic structure $AABBCCAABBCC \dots$, where AA , BB , and CC denote encoded logical qubits analogous to L_1 , L_2 , and L_3 , respectively.

IV. QUANTUM COMPUTATION

Our discussion so far has concentrated on the preservation of quantum information. To carry out quantum information, we must have the ability to manipulate encoded quantum information. Thus we still need to show that universal QC can actually be performed in our scheme. DiVincenzo shows that for any unitary transformation on quantum states it is sufficient to apply (a) all single-qubit rotations [SU(2)] together with (b) the two-qubit controlled-NOT (CNOT) gate on any two logical qubits [32].

In our scheme, we assume that any single-qubit operations on physical qubits are realizable at will by virtue of external pulses. We can define logical operations (denoted by a bar) which act on the encoded qubits. For example, $\bar{X}: |0_L\rangle \leftrightarrow |1_L\rangle$. For logical qubit L_1 , $\bar{X} = (J_x \sigma_1^x \otimes \sigma_2^x + J_y \sigma_1^y \otimes \sigma_2^y) / (J_x + J_y)$. Logical \bar{X} operation can be easily achieved by recoupling qubits 1 and 2 with the interaction Hamiltonian as shown in Eq. (10). We adjust the time intervals of pulses on qubits 1 and 2 both to Δt_3 , where $\Delta t_3 = \Delta t_2/2 = \Delta t_1/4$. In other words, only synchronous collective BB pulses are applied, which just eliminate the coupling from environment but have no effect on qubit-qubit coupling H_I . Then, we have

TABLE I. Comparison of properties between logical qubits L_1 , L_2 , and L_3 .

	L_1	L_2	L_3
U_α	$-\sigma_1^x \otimes \sigma_2^z$	$-\sigma_3^x \otimes \sigma_4^x$	$-\sigma_5^y \otimes \sigma_6^y$
R_α	$-iI_1 \otimes \sigma_2^z$	$-iI_3 \otimes \sigma_4^x$	$-iI_5 \otimes \sigma_6^y$
$ 0\rangle_L$	$ 0_1 1_2\rangle$	$\frac{1}{2}(0_3\rangle + 1_3\rangle)(0_4\rangle - 1_4\rangle)$	$\frac{1}{2}(0_5\rangle + i 1_5\rangle)(0_6\rangle - i 1_6\rangle)$
$ 1\rangle_L$	$ 1_1 0_2\rangle$	$\frac{1}{2}(0_3\rangle - 1_3\rangle)(0_4\rangle + 1_4\rangle)$	$\frac{1}{2}(0_5\rangle - i 1_5\rangle)(0_6\rangle + i 1_6\rangle)$
\bar{X}	$(J_x \sigma_1^x \otimes \sigma_2^x + J_y \sigma_1^y \otimes \sigma_2^y) / (J_x + J_y)$	$(J_y \sigma_3^y \otimes \sigma_4^y + J_z \sigma_3^z \otimes \sigma_4^z) / (J_y + J_z)$	$(J_x \sigma_5^x \otimes \sigma_6^x + J_z \sigma_5^z \otimes \sigma_6^z) / (J_x + J_z)$
\bar{Z}	$(\sigma_1^z - \sigma_2^z) / 2$	$(\sigma_3^x - \sigma_4^x) / 2$	$(\sigma_5^y - \sigma_6^y) / 2$

$$e^{i\theta H} |i\rangle_L = e^{i\theta(J_x \sigma_1^x \otimes \sigma_2^x + J_y \sigma_1^y \otimes \sigma_2^y + J_z \sigma_1^z \otimes \sigma_2^z)} |i\rangle_L = e^{-i\theta J_z} e^{i\theta(J_x + J_y) \bar{X}} |i\rangle_L. \quad (20)$$

By the free evolution under the inherent interaction Hamiltonian, we can easily accomplish logical \bar{X} operation. We must note that the time intervals of pulses on qubits 2 and 3 are still unequal; this implies that after cycles of pulse operations in the time $T = N_2 2\Delta t_3 = N_3 2\Delta t_1$, qubit 2 remains decoupled from qubit 3; logical \bar{X} operation on L_1 therefore has no impact on other logical qubits. We can also implement logical Z operation $\bar{Z} = (\sigma_1^z - \sigma_2^z) / 2$ by direct pulse operations on physical qubits, then \bar{X} and \bar{Z} generate all encoded-qubit SU(2) transformations.

By inspection of quantum codes of logical qubits L_1 , L_2 , and L_3 , we find that the DFSs of L_2 and L_3 can be obtained by performing a unitary transformation on that of L_1 . For example, the transformation of DFSs between L_1 and L_2 is a Hadamard transformation. Obviously, single-encoded-qubit operations, which preserve the DFSs of L_1 and L_2 , respectively, have the same unitary transformation too. Then, by performing a transformation on single-encoded-qubit gate given above, all single-encoded-qubit operations [SU(2)] on L_2 and L_3 can be easily achieved (see Table I).

Two-encoded-qubit CNOT gate seems to be more complicated, but in our scheme it is very easy to accomplish the two-qubit gate. For the convenience of discussion, let us assume that we want to do a CNOT operation from logical qubit L_1 to L_2 in Fig. 2. To obtain a two-qubit gate, we consider the imprimitive gate $W = e^{i\theta \sigma^z \otimes \sigma^z}$, which is equivalent to a controlled rotation about the z axis [33],

$$e^{i\theta \sigma^z \otimes \sigma^z} \equiv |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes e^{i2|\theta| \sigma^z}. \quad (21)$$

Conjugated by single-qubit Hadamard operation

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

on the second qubit, W can be used to implement a CNOT operation:

$$\text{CNOT} \equiv |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes e^{i(\pi/2)\sigma^z}. \quad (22)$$

To implement an encoded CNOT between L_1 and L_2 , we must recouple the two logical qubits with an interaction in the form $\bar{Z}_{L_1} \otimes \bar{Z}_{L_2}$. We perform a unitary Hadamard transfor-

ation on L_2 . In other words, we change the BB pulses characterized by $-\sigma_3^x \otimes \sigma_4^x$ and $-iI_3 \otimes \sigma_4^x$ to the same with L_1 , and the quantum code in Eqs. (16) and (17) to the same with that in Eq. (9), i.e., $|0\rangle_{L_B} \rightarrow |0\rangle_{L_A} = |0_3 1_4\rangle$ and $|1\rangle_{L_B} \rightarrow |1\rangle_{L_A} = |1_3 0_4\rangle$. It should be noted that L_2 and L_3 are still entirely decoupled after the unitary transformation. Then the effective interactions between qubits 1, 2, 3, and 4 all are in the form of Ising interaction. In this system we assume that the interaction only exists between any nearest-neighbor physical qubits. Obviously, $\bar{Z}_{L_1} \otimes \bar{Z}_{L_2} = \sigma_2^z \otimes \sigma_3^z$, two-encoded-qubit CNOT gate can be implemented by the evolution under the effective interaction $\sigma_2^z \otimes \sigma_3^z$ and single-qubit Hadamard operation conjugately effected on a physical qubit. Similarly, we can implement a CNOT operation between L_2 and L_3 .

As above, we showed that it is possible to perform all single- and two-encoded-qubit operations by means of pulse operations and evolution under inherent interaction. In our scheme, single- and two-encoded-qubit operations do not influence decoupling operations and preserve DFSs all the time, so quantum states encoded with quantum information will not undergo decoherence, then we implement universal, fault-tolerant QC.

V. DISCUSSION AND CONCLUSION

In this paper we have presented a scheme of scalable quantum computer governed by an untunable exchange Hamiltonian. We combine ideas from the theory of decoherence-free subspaces and BB control to solve the problem of strong decoherence. Cycles of simple BB pulses are used to selectively decouple the system from external environment, then by encoding two physical qubits into a DFS, we obtain full protection against strong decoherence. By concatenating BB control with the DFSs encoding, our scheme decreases the number of physical qubits required to counteract decoherence. It is highly important for the physicist to reduce the physical resource needed for implementation of scalable quantum computer, because quantum computing resources available are still a stringent requirement for practical quantum engineering. Comparing with other decoupling scheme, in our scheme only very simple BB pulses are applied which is easy to accomplish.

Furthermore, we have discussed the influence of an always on and untunable interaction between physical qubits on the logical qubits. The undesired effects of the internal interaction can be eliminated via cycles of BB operations,

which simplifies the physical structure of quantum computer that is devised in a very complicated manner for implementing the tunability of the coupling strength in many QC proposals. By different unsymmetrical decoupling operations, every logical qubit is entirely decoupled from others. With direct pulse operations on physical qubits and effective interaction, we can achieve all single- and two-encoded-qubit gates for implementing universal QC. Moreover, in our scheme all single- and two-encoded-qubit operations preserve logical

qubits in a DFS all the time, so we implement universal, fault-tolerant QC.

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