Problems and aspects of energy-driven wave-function collapse models

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(Received 13 October 2003; published 21 April 2004)

Four problematic circumstances are considered, involving models which describe dynamical wave-function collapse toward energy eigenstates, for which it is shown that wave-function collapse of macroscopic objects does not work properly. In one case, a common particle position measuring situation, the apparatus evolves to a superposition of macroscopically distinguishable states (does not collapse to one of them as it should) because each such particle/apparatus/environment state has precisely the same energy spectrum. Second, assuming an experiment takes place involving collapse to one of two possible outcomes which is permanently recorded, it is shown in general that this can only happen in the unlikely case that the two apparatus states corresponding to the two outcomes have disjoint energy spectra. Next, the progressive narrowing of the energy spectrum due to the collapse mechanism is considered. This has the effect of broadening the time evolution of objects as the universe evolves. Two examples, one involving a precessing spin, the other involving creation of an excited state followed by its decay, are presented in the form of paradoxes. In both examples, the microscopic behavior predicted by standard quantum theory is significantly altered under energy-driven collapse, but this alteration is not observed by an apparatus when it is included in the quantum description. The resolution involves recognition that the state vector describing the apparatus does not collapse, but evolves to a superposition of macroscopically different states.

DOI: 10.1103/PhysRevA.69.042106 PACS number(s): 03.65.Ta

I. INTRODUCTION

Wave-function collapse models alter Schrödinger's equation by adding a term which depends upon a stochastically fluctuating quantity. The equation then drives a superposition of quantum states toward one or another state in the superposition (which state is realized depends upon the realization of the fluctuating quantity). Moreover, when all possible fluctuations and their probabilities are considered, the realized state appears as an outcome with the Born probability [1,2].

Following seminal ideas involving collapse of particles to localized positions in the model of Ghirardi, Rimini, and Weber [3], it became possible to incorporate these with the earlier ideas to construct a model, the continuous spontaneous localization (CSL) model [4,5], which describes collapse based upon a local density, e.g., particle number density, mass density [6–8], (relativistic) energy density. This model entails rapid collapse of a superposition of macroscopically distinguishable spatially localized states (such as occurs in a measurement situation) to one of them. It does this because each state differs from the others in the superposition in its spatial distribution of the density. The CSL model, different as it is from standard quantum theory, has experimentally testable consequences [7,9,10]; so far it is consistent with experiment.

A long time ago, Bedford and Wang [11] proposed that collapse based upon differences in energy (not differences in energy density as described above) could be viable. More recently, Percival [12] has constructed a stochastic energy-driven collapse model for microscopic systems but has not extended it to macroscopic systems so one cannot say how, e.g., the behavior of an apparatus is described in his model. Most recently, Hughston [13] has given an elegant argument propounding an energy-driven collapse model, which has been followed by a number of papers [14–17] exploring mathematical and physical consequences of this proposal.

Since the purpose of collapse models is to allow (the state vector of the modified) quantum theory to describe the localized world we see around us, it is necessary to have a mechanism whereby energy-driven collapse results in localized states of, e.g., a macroscopic apparatus. Hughston [13] has suggested that exchange of environment particles (air) with the apparatus by accretion or evaporation might achieve this result, and this has been explored by Adler [17].

However, after presenting the necessary energy-driven collapse formalism in Sec. II, we argue that it cannot (by this or any other mechanism) lead to spatially localized states in commonly occurring cases. Section III shows that this is so in a familiar measurement situation, the position measurement of a superposition of two mutually translated particle states, when the particle, apparatus, and environment are collectively described by the state vector. The reason is that the evolving superposed macroscopically distinguishable states have precisely the same energy spectra so one state is not singled out by energy-driven collapse, which is only sensitive to energy spectra differences. It is not sensitive to what does distinguish the different macroscopic states: their (generally degenerate) energy states have different phase factors, whose cancellation or augmentation, when projected into the position representation, gives rise to the spatial distinctiveness.

In Sec. IV, after laying out the time-translation invariant properties of the formalism not immediately evident in Sec. II, the results are used to prove that, for an experiment which produces a permanent record, the associated apparatus states

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must have nonoverlapping energy spectra (which is most unlikely).

An aspect of energy-driven collapse is that it does not allow objects to change too rapidly. The reason is that each object has a smallest time τ which characterizes the time evolution of its fastest elements. Unless the state vector describing it contains a superposition of energy eigenstates with spread $\Delta E \geqslant \hbar/\tau$, it cannot evolve over τ. However, in the energy-driven collapse model, *every* state vector has its energy bandwidth progressively narrowed as time wears on, to $\Delta E \approx (\lambda t)^{-1/2}$. Here, λ is a parameter in the model of dimension (energy)$^{-2}$ (time)$^{-1}$, which characterizes the collapse rate and t is the time that has elapsed since the energy-driven collapse began, presumably the beginning of the universe. Section V shows that any expectation value in standard quantum theory with $\tau < \mathcal{T} \equiv \hbar(\lambda t)^{1/2}$ has its time behavior "smeared" so that, under energy-driven collapse, it evolves over the time interval $\mathcal{T}$. We show that this is not only true for the ensemble of universes, but also true for a temporarily noninteracting subsystem of a single universe.

In Sec. VI we show the effect this has on a precessing spin and on an excitation/decay of a bound state. The time parameters of the models, described by standard quantum theory, are chosen $\ll \mathcal{T}$. The result is that, under energy-driven collapse, the spin does not precess and nonexponential excitation/decay takes place over $\mathcal{T}$. However, when the apparatus is included in the state vector, the altered behavior is not recorded by it. We couch this as a "paradox," a contradiction between the description of the microscopic system alone, and the description of its measurement. The resolution involves realizing that the measurement description does not produce collapse as it should.

II. ENERGY COLLAPSE DESCRIPTION

The most transparent formulation of the energy-driven collapse model is as follows [18]. The state vector $|\psi,t\rangle$ at time t is assumed to obey the nonunitary evolution

$$|\psi,t\rangle_B = e^{-iHt}e^{-(1/4\lambda t)[B(t)-2\lambda tH]^2}|\psi,0\rangle. \tag{2.1}$$

In Eq. (2.1), H is the energy operator and λ is the aforementioned constant parameter which characterizes the rate of energy collapse. $B(t)$ is the stochastic variable whose time dependence is that of Brownian motion (i.e., it is continuous but not differentiable). At any time t, it takes on the value $(B, B+dB)$ with probability

$$\mathcal{P}(B)dB = \frac{dB}{\sqrt{2\pi\lambda t}}{}_B\langle\psi,t|\psi,t\rangle_B, \tag{2.2}$$

i.e., state vectors of largest norm occur with greatest probability. Since the state vector (2.1) is not normed to 1, the expectation value of an operator A in this state is

$$\langle A\rangle_B(t) \equiv {}_B\langle\psi,t|A|\psi,t\rangle_B / {}_B\langle\psi,t|\psi,t\rangle_B, \tag{2.3}$$

and the ensemble expectation value of an operator A follows from Eqs. (2.2) and (2.3):

$$\langle A\rangle(t) = \int dB\mathcal{P}(B)\langle A\rangle_B(t) = \int \frac{dB}{\sqrt{2\pi\lambda t}}{}_B\langle\psi,t|A|\psi,t\rangle_B. \tag{2.4}$$

All the conclusions about energy-driven collapse in this paper follow from these equations.

For example, from Eq. (2.1) expressed in the energy basis, we may write

$$\langle E;j|\psi,t\rangle_B = e^{-iEt}e^{-\lambda t[E-\{B(t)/2\lambda t\}]^2}\langle E;j|\psi,0\rangle \tag{2.5}$$

(j is the energy degeneracy index). As $t\to\infty$, in Eq. (2.5), the Gaussian $\to(\pi/\lambda t)^{1/2}\delta[E-(B(t)/2\lambda t)]$, showing that the state vector collapses to an energy eigenstate [the value of the eigenstate determined by the asymptotic value of $B(t)/2\lambda t$].

III. POSITION MEASUREMENT

In most of this section we establish the structure of an initial state describing a particle in a superposition of two locations, accompanied by a position measuring apparatus immersed in a gaseous environment. The evolution of this state in standard quantum theory is given by Eq. (3.9) and, in the energy-driven collapse model, by Eq. (3.10). Equation (3.11) shows that proper collapse does not take place.

Consider first an isolated particle which, at $t=0$, is in a localized state (for example, a Gaussian wave packet with some mean position, mean momentum, and width). The state vector of that particle, translated by the distance vector $\mathbf{a}$ but identical in every other respect, differs only in that the momentum eigenstates (and therefore energy eigenstates) of which it is composed are multiplied by the phase factor $\exp(-i\mathbf{k}\cdot\mathbf{a})$. Label these two localized particle states $|\phi^1,0\rangle$, $|\phi^2,0\rangle$. Their expansion in energy eigenstates of the Hamilton H_{part} may be written as

$$|\phi^\alpha,0\rangle = \sum_{kj} a_{kj}e^{i\theta^\alpha_{kj}}|\epsilon_k;j\rangle, \tag{3.1}$$

where a_{kj} (which does *not* depend upon α) and θ^α_{kj} ($\theta^1_{kj}-\theta^2_{kj}$ $=\mathbf{k}\cdot\mathbf{a}$) are real. For simplicity of notation we have written Eq. (3.1) as a sum, although the possible energy values ϵ_k $(=k^2/2m)$ and the degeneracy index j (which is the direction $\mathbf{k}/k$) are really continuous.

We assume that an appropriate beam splitter can put the particle in an initial state which is an arbitrary superposition $|\psi,0\rangle=\Sigma^2_{\alpha=1}\beta_\alpha|\phi^\alpha,0\rangle$ with $\Sigma^2_{\alpha=1}|\beta_\alpha|^2=1$.

Consider next a position measuring apparatus together with environmental particles all around, but without the particle to be detected. The initial state here is

$$|A,0\rangle = \sum_{mn} c_{mn}|\epsilon^A_m;n\rangle, \tag{3.2}$$

where $|\epsilon^A_m;n\rangle$ are energy eigenstates of the apparatus/environment Hamiltonian $H_{a/e}$ with energy ϵ^A_m, n is the degeneracy label, and c_{mn} is generally a complex number.

Now, when the particle to be detected is brought together with the apparatus/environment, the energy eigenstates of the complete Hamiltonian, $H=H_{\rm part}+H_{\rm a/e}+H_{\rm int}\equiv H_0+H_{\rm int}$, must be considered. Lest the environment knock the particle away from the detector, take $H_{\rm int}$ not to include an environment-particle interaction (alternatively, one could choose the initial state so that the particle reaches the apparatus before it is hit by an environment particle, but that complicates proofs). Thus, $H_{\rm int}=\Sigma_{iapp}\,V_i(\mathbf{X}_{iapp}-\mathbf{X})$, where $\mathbf{X}$ is the particle's position operator, V_i is the interaction potential between the particle and each relevant apparatus particle (i.e., particles in the detector part of the apparatus, such as the gas in a geiger tube) whose position operators are $\mathbf{X}_{iapp}$. Thus, in particular, if $|\mathbf{x}\rangle$ is an eigenstate of $\mathbf{X}$, then

$$H_{\rm int}|\mathbf{x}\rangle=0 \text{ when } \mathbf{x} \text{ lies appreciably outside the particle detector}. \tag{3.3}$$

It is necessary to write the initial state of particle together with the apparatus/environment in terms of the energy eigenstates of H. Given an energy eigenstate of the noninteracting particle $|\epsilon_k;j\rangle$ (i.e., a plane-wave state) and an energy eigenstate of the apparatus/environment $|\epsilon_m^A;n\rangle$, there corresponds a unique energy eigenstate $|E_{km};j;n\rangle$ of H satisfying the incoming Lippmann-Schwinger equation with energy $E_{km}=\epsilon_k+\epsilon_m^A$, which may be written as

$$|E_{km};j;n\rangle=|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle+\frac{1}{E_{km}+i\epsilon-H_0}H_{\rm int}|E_{km};j;n\rangle \tag{3.4a}$$

$$=|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle+\frac{1}{E_{km}+i\epsilon-H}H_{\rm int}|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle \tag{3.4b}$$

$$\equiv|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle+|\psi_{\rm int};k;m;j;n\rangle. \tag{3.4c}$$

In Eq. (3.4c), $|\psi_{\rm int};k;m;j;n\rangle$ describes the various outcomes of detecting the particle by the apparatus along with scattering, accretion, excitation, etc., involving the apparatus and environment. One may think of this as a scattering situation, with the apparatus a bound state of its constituent particles, and even the environment particles can be imagined as confined in a large box surrounding the apparatus so that they too are part of the bound state. The analysis may then be considered under the rubric of multichannel scattering [19]: the in-states $|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle$ are then a complete set in their subspace of the Hilbert space.

The labels k,m,j,n of $|E_{km};j;n\rangle$ do not describe eigenvalues of operators which commute with H; they describe eigenvalues of operators which commute with $H_{\rm part}$ and $H_{\rm a/e}$. Nevertheless, it is well known from scattering theory that, as a consequence of Eq. (3.4),

$$\langle E_{k'm'};j';n'|E_{km};j;n\rangle=\langle\epsilon_{k'};j'|\langle\epsilon_{m'}^A;n'||\epsilon_k;j\rangle|\epsilon_m^A;n\rangle$$
$$=\delta_{k'k}\delta_{j'j}\delta_{m'm}\delta_{n'n}. \tag{3.5}$$

Equation (3.5) can be derived algebraically by taking the scalar product of Eq. (3.4a) with (the primed version of) itself and utilizing the Low equation [19] [which also follows from Eq. (3.4a)] to eliminate all terms involving $|\psi_{\rm int};k;m;j;n\rangle$. It is most easily derived (utilizing $\exp it[H_0-E_{km}]|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle=|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle$) by writing Eq. (3.4b) as

$$|E_{km};j;n\rangle=\left[1-i\int_0^\infty dte^{-it[H-E_{km}-i\epsilon]}H_{\rm int}e^{it[H_0-E_{km}]}\right]\times|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle \tag{3.6a}$$

$$=\left[1-i\int_0^{1/\epsilon}dte^{-itH}H_{\rm int}e^{itH_0}\right]|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle$$
$$=\left[1+\int_0^{1/\epsilon}d(e^{-itH}e^{itH_0})\right]|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle \tag{3.6b}$$

$$=\lim_{t\to\infty}e^{-itH}e^{itH_0}|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle$$
$$\equiv\Omega^+|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle. \tag{3.6c}$$

Here, Ω^+ is the Møller matrix. It is well known to be isometric, $\Omega^{+\dagger}\Omega^+=1$ on the space of in-states, the proof of which we now sketch for completeness. As shown above, so it is also readily shown from Eq. (3.4a) that

$$|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle=\lim_{t\to\infty}e^{-itH_0}e^{itH}|E_{km};j;n\rangle=\Omega^{+\dagger}|E_{km};j;n\rangle, \tag{3.7}$$

so $\Omega^{+\dagger}\Omega^+|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle=|\epsilon_k;j\rangle|\epsilon_m^A;n\rangle$ follows from Eqs. (3.6) and (3.7). Equation (3.5) is an immediate consequence of the scalar product of Eq. (3.6c) with (the primed version of) itself, and use of $\Omega^{+\dagger}\Omega^+=1$.

The αth initial state can be written as a sum over eigenstates of H as follows. From Eqs. (3.1) and (3.2) and again using the form (3.6a), one sees that

$$\sum_{kjmn}a_{kj}e^{i\theta_{kj}^\alpha}c_{mn}|E_{km};j;n\rangle$$
$$=|\phi^\alpha,0\rangle|A,0\rangle-i\int_0^\infty dte^{-it[H-i\epsilon]}H_{\rm int}\sum_{kj}a_{kj}e^{i\theta_{kj}^\alpha}e^{it\epsilon_k}|\epsilon_k;j\rangle\times\sum_{mn}c_{mn}e^{it\epsilon_m^A}|\epsilon_m^A;n\rangle \tag{3.8a}$$

$$=|\phi^\alpha,0\rangle|A,0\rangle-i\int_0^\infty dte^{-it[H-i\epsilon]}H_{\rm int}|\phi^\alpha,-t\rangle|A,-t\rangle \tag{3.8b}$$

$$=|\phi^\alpha,0\rangle|A,0\rangle. \tag{3.8c}$$

The second term in Eq. (3.8b) vanishes because $|\phi^\alpha,0\rangle$ is a wave packet outside the apparatus with mean momentum heading toward the apparatus. Therefore, $|\phi^\alpha,-t\rangle$ $(t>0)$ is the wave packet even further away from the apparatus, so Eq. (3.8c) follows from Eq. (3.3). (This result is well known for potential scattering [21].) Equations (3.5) and (3.8c) are what we need as we now consider the time evolution.

In standard quantum theory, using Eq. (3.8c), the time evolution of the αth initial state is

$$|\phi^{\alpha},0\rangle|A,0\rangle \to |\Psi^{\alpha},t\rangle = \sum_{kjmn} a_{kj}e^{i[\theta^{\alpha}_{kj}-E_{km}t]}c_{mn}|E_{km};j;n\rangle, \tag{3.9}$$

where $|\Psi^1,t\rangle$, $|\Psi^2,t\rangle$ eventually describe the apparatus as having detected the particle's location (and also describe the battering by the environment particles of the apparatus and each other) and having indicated this by a macroscopically distinguishable spatially localized feature.

According to the energy-driven collapse model, Eq. (2.1), and once again employing Eq. (3.8c), we see that the initial superposed state $|\Psi,0\rangle = \Sigma_{\alpha=1}^{2}\beta_{\alpha}|\phi^{\alpha},0\rangle|A,0\rangle$ evolves into the (unnormalized) state

$$|\Psi,t\rangle_B = \sum_{\alpha=1}^{2}\beta_{\alpha}\sum_{kjmn} e^{i[\theta^{\alpha}_{kj}-E_{km}t]}e^{-(1/4\lambda t)[B(t)-2\lambda tE_{km}]^2}a_{kj}c_{mn}|E_{km};j;n\rangle \tag{3.10a}$$

$$\equiv \sum_{\alpha=1}^{2}\beta_{\alpha}|\Psi^{\alpha},t\rangle_B \tag{3.10b}$$

for some $B(t)$. [For simplicity we have taken $t=0$ as the time the collapse process begins: replacement of t by $t+T$ in the Gaussian in Eq. (3.10a) does not affect the result (3.11).] Assuming that the energy-driven collapse does not destroy the functioning of the apparatus, the orthogonal states $|\Psi^1,t\rangle_B$, $|\Psi^2,t\rangle_B$, like the comparable state vectors (3.9) of standard quantum theory, each describe the apparatus as possessing a spatially localized macroscopically distinguishable feature.

However, the squared amplitudes of the states $\beta_{\alpha}|\Psi^{\alpha},t\rangle_B$ in the superposition (3.10) are (apart from an overall normalization factor which does not affect their ratio)

$$|\beta_{\alpha}|^2_B\langle\Psi^{\alpha},t|\Psi^{\alpha},t\rangle_B = |\beta_{\alpha}|^2\sum_{kjmn} a^2_{kj}|c_{mn}|^2 e^{-(1/2\lambda t)[B(t)-2\lambda tE_{km}]^2} \tag{3.11}$$

on account of the orthonormality (3.5). But, the α dependence is absent from the sum in Eq. (3.11). Thus, as in standard quantum theory, there is no collapse to one of these two apparatus states since these superposed states at time t are in the same proportion, $|\beta_2|^2/|\beta_1|^2$, as they were initially.

IV. ENERGY-DRIVEN COLLAPSE AND PERMANENT RECORDS

According to Eq. (2.1), at time t, each value of $B(t)$ corresponds to a different possible universe in the ensemble of universes, all of which evolved from a single state vector $|\psi,0\rangle$ starting at time $t=0$. If collapse takes place as purported, each $B(t)$ of non-negligible probability should characterize a recognizable state of the universe, in the sense that each macroscopic object in a universe is not in a superposed state. That a single number $-\infty < B(t) < \infty$ can characterize the myriad possible recognizable universes which evolve from an initial state $|\psi,0\rangle$ seems unlikely, and raises the suspicion that collapse cannot take place as purported.

In Sec. IV A, time-translation properties are displayed, showing how the state vector and probability characterized by $B(t)$ can be expressed in terms of $B(t_0)$ at time t_0, and not just in terms of $B(0)=0$ at time 0, as given in Eqs. (2.1) and (2.2).

In Sec. IV B, any experiment is considered whose outcome, one of two possible values, is macroscopically permanently recorded at time t_0 (i.e., the record is unaltered for $t>t_0$). It is shown that this can only be possible if, at time t_0, there is no overlap in the energy spectra corresponding to the two different states describing the two different macroscopic outcomes. Since there is no reason why a measurement should lead to such a bifurcation of the energy spectrum, it must be concluded that there cannot be collapse to a state which describes a unique macroscopic permanent record.

A. Time-translation invariant description

Corresponding to Eq. (2.1), it is useful to define the (unnormalized) state vector at time t,

$$|\psi,t\rangle_{B(t),B(t_0)} \equiv e^{-iH(t-t_0)}e^{-[1/4\lambda(t-t_0)][B(t)-B(t_0)-2\lambda(t-t_0)H]^2}|\psi,t_0\rangle_{B(t_0)}, \tag{4.1}$$

which evolves from the (unnormalized) state vector $|\psi,t_0\rangle_{B(t_0)}$ at time t_0 [we take $B(0)=0$ and $|\psi,t_0\rangle_{B(t_0),B(0)} = |\psi,t_0\rangle_{B(t_0),0} \equiv |\psi,t_0\rangle_{B(t_0)}$ is given by Eq. (2.1)].

Corresponding to Eq. (2.2), given $B(t_0)$ (and therefore also the associated state vector $|\psi,t_0\rangle_{B(t_0)}$), the conditional probability that, at time t, B lies in the interval $[B(t),B(t)+dB(t)]$ is

$$\mathcal{P}(B(t),t|B(t_0),t_0)dB(t) = \frac{dB(t)}{\sqrt{2\pi\lambda(t-t_0)}}\frac{{}_{B(t),B(t_0)}\langle\psi,t|\psi,t\rangle_{B(t),B(t_0)}}{{}_{B(t_0)}\langle\psi,t_0|\psi,t_0\rangle_{B(t_0)}}. \tag{4.2}$$

Equations (4.1) and (4.2) agree with Eqs. (2.1) and (2.2) when $t_0=0$. It is necessary to show that Eqs. (4.1) and (4.2) are fully consistent with Eqs. (2.1) and (2.2), in the sense that the evolution and probability taken in two steps, from time 0 to t_0 and then from t_0 to t, are equivalent to one step from 0 to t.

Consider first the normalized state vector, which we shall denote by a prime. From Eq. (4.1),

$$|\psi,t\rangle'_{B(t),B(t_0)} \equiv \frac{|\psi,t\rangle_{B(t),B(t_0)}}{{}_{B(t),B(t_0)}\langle\psi,t|\psi,t\rangle^{1/2}_{B(t),B(t_0)}} = \frac{e^{-iH(t-t_0)}e^{[\{B(t)-B(t_0)\}H-\lambda(t-t_0)H^2]}|\psi,t_0\rangle_{B(t_0)}}{{}_{B(t_0)}\langle\psi,t_0|e^{2[\{B(t)-B(t_0)\}H-\lambda(t-t_0)H^2]}|\psi,t_0\rangle^{1/2}_{B(t_0)}}. \tag{4.3}$$

(In Eq. (4.3), the term $\exp-[4\lambda(t-t_0)]^{-1}[B(t)-B(t_0)]^2$, arising from the squared bracket of the exponent in Eq. (4.1), factors out of numerator and denominator and thus cancels, while the remaining terms from the squared bracket have a common factor $4\lambda(t-t_0)$, which cancels with the premultiplying inverse of this factor).

The exponent in the numerator of Eq. (4.3) has a purely linear dependence upon $B(t)-B(t_0)$ and upon $(t-t_0)$. Because of this, writing out $|\psi,t_0\rangle_{B(t_0)}$ in Eq. (4.3) in terms of $|\psi,0\rangle$ using Eq. (2.1), canceling the exponential factors exp $-[4\lambda t_0]^{-1}[B(t_0)]^2$, which then appear in numerator and denominator, and combining the remaining exponents,

$$\{-iH(t-t_0)+[B(t)-B(t_0)]H-\lambda(t-t_0)H^2\} + \{-iHt_0+B(t_0)H-\lambda t_0H^2\} = -iHt+B(t)H-\lambda tH^2,$$

results in

$$|\psi,t\rangle'_{B(t),B(t_0)} = \frac{e^{-iHt}e^{B(t)H-\lambda tH^2}|\psi,0\rangle}{\langle\psi,0|e^{2[B(t)H-\lambda tH^2]}|\psi,0\rangle} = \frac{e^{-iHt}e^{-(1/4\lambda t)[B(t)-2\lambda tH]^2}|\psi,0\rangle}{\langle\psi,0|e^{-(1/2\lambda t)[B(t)-2\lambda tH]^2}|\psi,0\rangle}. \tag{4.4}$$

We see in Eq. (4.4) that the dependence upon $B(t_0)$ and t_0 has disappeared and that the normalized state vector (4.3), constructed from Eq. (4.1), is the same normalized state vector that is constructed from Eq. (2.1).

Consider next the probability that B takes on the value $B(t)$ at time t, given that B has the value 0 at time 0. From Eqs. (4.2) and (4.1) and the usual rule for compounding probabilities,

$$\begin{aligned}
&\mathcal{P}(B(t),t|0,0)dB(t)\\
&= dB(t)\int \mathcal{P}(B(t),t|B(t_0),t_0)dB(t_0)\mathcal{P}(B(t_0),t_0|0,0)\\
&= \frac{dB(t)}{\sqrt{2\pi\lambda(t-t_0)}}\int \frac{{}_{B(t),B(t_0)}\langle\psi,t|\psi,t\rangle_{B(t),B(t_0)}}{{}_{B(t_0)}\langle\psi,t_0|\psi,t_0\rangle_{B(t_0)}}\frac{dB(t_0)}{\sqrt{2\pi\lambda t_0}}{}_{B(t_0)}\langle\psi,t_0|\psi,t_0\rangle_{B(t_0)}\\
&= \frac{dB(t)}{\sqrt{2\pi\lambda(t-t_0)}}\int \frac{dB(t_0)}{\sqrt{2\pi\lambda t_0}}{}_{B(t_0)}\langle\psi,t_0|e^{-[1/2\lambda(t-t_0)][B(t)-B(t_0)-2\lambda(t-t_0)H]^2}|\psi,t_0\rangle_{B(t_0)}\\
&= \frac{dB(t)}{\sqrt{2\pi\lambda(t-t_0)}}\int \frac{dB(t_0)}{\sqrt{2\pi\lambda t_0}}\langle\psi,0|e^{-[1/2\lambda(t-t_0)][B(t)-B(t_0)-2\lambda(t-t_0)H]^2}e^{-[1/2\lambda t_0][B(t_0)-2\lambda t_0H]^2}|\psi,0\rangle\\
&= \frac{dB(t)}{\sqrt{2\pi\lambda t}}\langle\psi,0|e^{-(1/2\lambda t)[B(t)-2\lambda tH]^2}|\psi,0\rangle,
\end{aligned} \tag{4.5}$$

which is the same as Eq. (2.2).

This time-translation invariance, especially evident in the stochastic differential Schrödinger equation (which we have not bothered to reproduce) whose solution is Eq. (4.1), seems to have misled workers into overlooking the important effects of the cumulative narrowing of the energy spectrum as the universe evolves (see Secs. V and VI). Because of this, one cannot take an initial state vector $|\psi,t_0\rangle_{B(t_0)}$ to be just any state vector (as one is free to do in standard quantum theory or CSL) if $t_0=T$ is, e.g., the age of the universe. The evolution of the state vector from time 0 when the universe began to time T restricts the spectrum of $|\psi,T\rangle_{B(T)}$, which must be respected if the theory is to be consistently applied. For example, it may not be consistent to consider as an initial condition that an electron is in an excited atomic state at time T. Although the electron is in an energy eigenstate (and therefore the energy spread is zero) of the atomic Hamiltonian, it is not in an energy eigenstate of the complete Hamiltonian (including the radiation field) and, for sufficiently large coupling (sufficiently short lifetime), the actual energy spread of this initial state may be larger than is allowed in the universe at time T.

B. Permanent records

Now, consider an experiment performed at a time earlier than t_0 which, for definiteness, has two equally likely outcomes, say 1 and −1. Suppose further that the apparatus records the result at time t_0, which is permanent thereafter in all universes. A word on terminology: we are using "universe" for what is described by the state vector $|\psi,t\rangle_{B(t)}$ corresponding to a particular $B(t)$, and "universes" to characterize the set of these. As far as the argument here is concerned, one may regard a universe as consisting of just the matter required for the experiment, or it may be so large as to represent the real universe (in this case, in order that all universes have the experiment going on as described, one might imagine that, somehow, at the beginning of the universe an isolated box containing the apparatus came into existence, with a clock set to start the experiment at a prescribed time).

If collapse takes place as is supposed, one may partition the values of $B(t)$ for $t \geqslant t_0$ into sets $\Sigma_1(t)$ [$\Sigma_1(t)$ consists of the set $\{B_1(t)\}$ for which $|\psi,t\rangle_{B_1(t)}$ describes the experimental outcome 1], $\Sigma_{-1}(t)$ (similarly defined), and $\Sigma_0(t)$ [covering the remaining values of $B(t)$ which do not lie in $\Sigma_1(t)$ or $\Sigma_{-1}(t)$ and which therefore must have negligible probability of occurring]. Then, for example, if $B_1(t_0)$ characterizes a universe (result 1 at time t_0), it is (almost) certain to evolve to a universe at time t characterized by some $B_1(t)$ and it has ≈ 0 probability to evolve to a universe characterized by a $B_{-1}(t)$ or $B_0(t)$. Thus, from the expression (4.2) for the conditional probability, we have

$$\int_{\Sigma_1(t)} \frac{dB(t)}{\sqrt{2\pi\lambda(t-t_0)}} \frac{{}_{B(t),B_1(t_0)}\langle\psi,t|\psi,t\rangle_{B(t),B_1(t_0)}}{{}_{B_1(t_0)}\langle\psi,t_0|\psi,t_0\rangle_{B_1(t_0)}} \approx 1,$$

$$\int_{\Sigma_1(t)} \frac{dB(t)}{\sqrt{2\pi\lambda(t-t_0)}} \frac{{}_{B(t),B_{-1}(t_0)}\langle\psi,t|\psi,t\rangle_{B(t),B_{-1}(t_0)}}{{}_{B_{-1}(t_0)}\langle\psi,t_0|\psi,t_0\rangle_{B_{-1}(t_0)}} \approx 0, \tag{4.6a}$$

$$\int_{\Sigma_{-1}(t)} \frac{dB(t)}{\sqrt{2\pi\lambda(t-t_0)}} \frac{{}_{B(t),B_{-1}(t_0)}\langle\psi,t|\psi,t\rangle_{B(t),B_{-1}(t_0)}}{{}_{B_{-1}(t_0)}\langle\psi,t_0|\psi,t_0\rangle_{B_{-1}(t_0)}} \approx 1,$$

$$\int_{\Sigma_{-1}(t)} \frac{dB(t)}{\sqrt{2\pi\lambda(t-t_0)}} \frac{{}_{B(t),B_1(t_0)}\langle\psi,t|\psi,t\rangle_{B(t),B_1(t_0)}}{{}_{B_1(t_0)}\langle\psi,t_0|\psi,t_0\rangle_{B_1(t_0)}} \approx 0, \tag{4.6b}$$

$$\int_{\Sigma_0(t)} \frac{dB(t)}{\sqrt{2\pi\lambda(t-t_0)}} \frac{{}_{B(t),B_1(t_0)}\langle\psi,t|\psi,t\rangle_{B(t),B_1(t_0)}}{{}_{B_1(t_0)}\langle\psi,t_0|\psi,t_0\rangle_{B_1(t_0)}} \approx 0,$$

$$\int_{\Sigma_0(t)} \frac{dB(t)}{\sqrt{2\pi\lambda(t-t_0)}} \frac{{}_{B(t),B_{-1}(t_0)}\langle\psi,t|\psi,t\rangle_{B(t),B_{-1}(t_0)}}{{}_{B_{-1}(t_0)}\langle\psi,t_0|\psi,t_0\rangle_{B_{-1}(t_0)}} \approx 0. \tag{4.6c}$$

The "$\approx$" is in these equations because a probability can differ from 0 or 1 by a negligible but nonzero amount (e.g., there is a tiny probability of a "flip" that an outcome 1 at time t_0 can evolve to -1 at a later time t) and the theory still has acceptable behavior.

The state vectors in Eqs. (4.6) may be written in the form of Eq. (4.1) and expressed in the energy basis:

$$\frac{{}_{B(t),B(t_0)}\langle\psi,t|\psi,t\rangle_{B(t),B(t_0)}}{{}_{B(t_0)}\langle\psi,t_0|\psi,t_0\rangle_{B(t_0)}} = \frac{\sum_j \int dE|\langle E;j|\psi,t_0\rangle_{B(t_0)}|^2 e^{-[1/2\lambda(t-t_0)][B(t)-B(t_0)-2\lambda(t-t_0)E]^2}}{\sum_j \int dE|\langle E;j|\psi,t_0\rangle_{B(t_0)}|^2} \tag{4.7a}$$

$$\equiv \int dE e^{-[1/2\lambda(t-t_0)][B(t)-B(t_0)-2\lambda(t-t_0)E]^2} \rho_{B(t_0)}(E) \tag{4.7b}$$

(j is the degeneracy index for the energy eigenstates) where

$$\rho_{B(t_0)}(E) \equiv \frac{\sum_j |\langle E;j|\psi,t_0\rangle_{B(t_0)}|^2}{\sum_j \int dE|\langle E;j|\psi,t_0\rangle_{B(t_0)}|^2}$$

is the unity normalized energy spectrum of the universe at time t_0 characterized by $B(t_0)$.

We now show that Eqs. (4.6) can only be satisfied if $\rho_{B_1(t_0)}(E)$, $\rho_{B_{-1}(t_0)}(E)$ have disjoint support in E. We apply Schwarz's inequality

$$\left[\int d\mathbf{x} f^2(\mathbf{x}) \int d\mathbf{x} g^2(\mathbf{x})\right]^{1/2} \geqslant \int d\mathbf{x} f(\mathbf{x}) g(\mathbf{x})$$

[where $\int d\mathbf{x} f^2(\mathbf{x}) \approx 0$, $\int d\mathbf{x} g^2(\mathbf{x}) \leqslant 1$, and $d\mathbf{x} = dB(t)dE$] to the product of the two equations in each of Eqs. (4.6a), (4.6b), and (4.6c) expressed in the form (4.7b), obtaining

$$0 \approx \int_{\Sigma_i} \frac{dB(t)}{\sqrt{2\pi\lambda(t-t_0)}} \int dE e^{-[1/\lambda(t-t_0)][B(t) - B_1(t_0) - 2\lambda(t-t_0)E]^2} e^{-[1/4\lambda(t-t_0)][B(t) - B_{-1}(t_0) - 2\lambda(t-t_0)E]^2} \rho^{1/2}_{B_1(t_0)}(E)\rho^{1/2}_{B_{-1}(t_0)}(E), \quad (4.8)$$

where Σ_i is any one of Σ_1, Σ_{-1}, Σ_0. Adding the three expressions (4.8) together, the range of $B(t)$ becomes the whole line so the integral over $B(t)$ can then be performed, resulting in

$$0 \approx e^{-[1/8\lambda(t-t_0)][B_1(t_0) - B_{-1}(t_0)]^2} \int dE \rho^{1/2}_{B_1(t_0)}(E)\rho^{1/2}_{B_{-1}(t_0)}(E). \quad (4.9)$$

No matter how large is $[B_1(t_0)-B_{-1}(t_0)]^2$, for large enough t the exponential in Eq. (4.9) is ≈ 1. Thus the integral must vanish and so, for each E, one (or both) of $\rho_{B_1(t_0)}(E)$, $\rho_{B_{-1}(t_0)}(E)$ must vanish (or be negligibly small).

However, there is no mechanism whereby a measurement "splits" the energy spectrum of the universe (whether the universe is just the apparatus or the real universe) into disjoint sets associated with each outcome. We conclude that the premise of this argument that the energy-driven collapse mechanism will produce macroscopic states, each corresponding to a single recorded experimental outcome (and not produce a superposition of such states), is false.

V. SECULAR DECREASE IN ENERGY BANDWIDTH AND CONSEQUENCES

In what follows, for definiteness we shall consider that universe refers to the actual universe, and universes describe an ensemble of which only one (that we inhabit) is actually realized.

If one expands the initial state vector of the universe $|\psi,0\rangle$ in energy eigenstates, $|\psi,0\rangle = \int dE C_j(E)|E;j\rangle$ (j is the degeneracy index), the state vector evolution (2.1) of a single universe characterized by $B(t)$ may be written using Eq. (2.5) as

$$|\psi,t\rangle_B = \sum_j \int dE C_j(E) e^{-iEt} e^{-\lambda t[E - \{B(t)/2\lambda t\}]^2}|E;j\rangle. \quad (5.1)$$

Thus, for *each* state vector, the initial energy spectrum $\Sigma_j|C_j(E)|^2$ is increasingly narrowed as time wears on by being multiplied by a Gaussian of energy spread $\sim(\lambda t)^{-1/2}$ [and mean value $B(t)/2\lambda t$].

Consequently, rapid time evolution of anything in *any* universe is restricted. Suppose that the universe has evolved for time T, and we consider physical phenomena over a relatively short-time interval thereafter, $(0, t-T)$, where $t-T \ll T$. Then the Gaussian in Eq. (5.1) is not much different at time t than at time T over this time interval [i.e., $\mathcal{T} \equiv (\lambda t)^{1/2} \approx (\lambda T)^{1/2}$ and $B(t)/2\mathcal{T}^2 \approx B(T)/2\mathcal{T}^2$], so Eq. (5.1) becomes approximately

$$|\psi,t\rangle_B \approx e^{-iH(t-T)}|\psi,T\rangle_B. \quad (5.2)$$

Since the (approximate) evolution (5.2) has the usual form and $|\psi,T\rangle_B$ has energy spread no greater than $\approx \hbar/\mathcal{T}$, according to the usual time-energy uncertainty relation we expect that the characteristic evolution time of any physical system in the so-restricted universe, e.g., any "pulse" behavior, can be no shorter than $\mathcal{T}$.

In Sec. V A we consider the ensemble of universes. The above argument for the restriction on pulse behavior is made more precise, in the expression, Eq. (5.4c), for the ensemble expectation value of any operator V. Also, from this expression, it is shown that the ensemble energy spectrum at any time t is unchanged from that at time 0. However, the energy spread restriction $\Delta E \lessapprox \hbar/\mathcal{T}$ on each universe does manifest itself, in the (approximate) vanishing of the density-matrix off-diagonal energy basis elements when their difference exceeds $\approx \Delta E$. It is also shown that were there a noninteracting subsystem present in each of the ensemble of universes (one may imagine that a physical system is present in $|\psi,0\rangle$ which somehow remains isolated for all time), then the ensemble of just these subsystems also obeys Eq. (5.4c).

However, we only have access to one universe [one $B(t)$], not the ensemble. In Sec. V B it is argued that, in a *single* universe, when the rest of the universe is traced over, Eq. (5.4c) holds to a good approximation for a subsystem which is recently isolated, i.e., one which may have interacted with the rest of the universe in the past but is presently not interacting with it. This allows us to discuss experiments in the single universe which are "turned on" at time T, two examples of which are treated in Sec. IV. Since it is a single universe, if collapse takes place properly, the result of an experiment should entail a unique outcome, but we shall see that this is not the case.

A. Ensemble behavior

Here we show that the ensemble of universes has the same energy spectrum as $|\psi,0\rangle$. [Even though the energy spread of each universe is narrowed to $\lessapprox \hbar/\mathcal{T}$, each universe's energy spread has a different mean value $\sim B(t)$, and these means are spread out over the whole real line.] However, the pulse constraint discussed above, since it holds for all state vectors, holds for the ensemble.

To see this formally, we write Eq. (2.1) in still another way, in terms of the state vector $|\psi,t\rangle$, which evolves from $|\psi,0\rangle$ under ordinary Schrödinger dynamics:

$$|\psi,t\rangle_B = e^{-(1/4\lambda t)[B(t) - 2\lambda t H]^2}|\psi,t\rangle \quad (5.3a)$$

$$= \frac{1}{\sqrt{4\pi}} \int_{-\infty}^{\infty} d\eta e^{-\eta^2/4} e^{-iB\eta/\sqrt{4\lambda t}} e^{i\sqrt{\lambda t}\eta H}|\psi,t\rangle \quad (5.3b)$$

$$=\frac{1}{\sqrt{4\pi}}\int_{-\infty}^{\infty}d\eta e^{-\eta^2/4}e^{-iB\eta/\sqrt{4\lambda t}}|\psi,t-\sqrt{\lambda}t\eta\rangle, \quad (5.3c)$$

where the Fourier transform of the Gaussian has been employed in Eq. (5.3b) to obtain an exponent linear in H. Thus, according to Eq. (5.3c), $|\psi,t\rangle_B$ may be viewed as a "time-smearing" superposition of $|\psi,t\rangle$'s over a range of t of order $\mathcal{T}$, with a Gaussian weight and a B-dependent phase.

The ensemble average $\langle V\rangle$, of any physical quantity V, is found by putting Eq. (5.3c) into Eq. (2.4):

$$\langle V\rangle(t)=\frac{1}{\sqrt{2\pi\lambda t}}\int_{-\infty}^{\infty}dB_B\langle\psi,t|V|\psi,t\rangle_B \quad (5.4a)$$

$$=\frac{1}{4\pi}\int_{-\infty}^{\infty}d\eta\int_{-\infty}^{\infty}d\eta' e^{-\eta^2/4}e^{-\eta'^2/4}\langle\psi,t-\sqrt{\lambda}t\eta'|V|\psi,t-\sqrt{\lambda}t\eta\rangle \times\frac{1}{\sqrt{2\pi\lambda t}}\int_{-\infty}^{\infty}dBe^{-iB(\eta-\eta')/\sqrt{4\lambda t}} \quad (5.4b)$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\eta e^{-\eta^2/2}\langle\psi,t-\sqrt{\lambda}t\eta|V|\psi,t-\sqrt{\lambda}t\eta\rangle. \quad (5.4c)$$

Equation (5.4c) makes precise the discussion in the introduction above. Any pulselike behavior of $\langle V\rangle(t)$ in standard quantum theory is "smeared" over a time interval $\approx\mathcal{T}$ [with $\mathcal{T}\approx(\lambda T)^{1/2}$ for $|t-T|\ll T$].

If $V=F(H)$ is an arbitrary function of the energy, it follows from Eq. (5.4c) that

$$\langle F(H)\rangle(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\eta e^{-\eta^2/2}\times\langle\psi,0|e^{iH(t-\sqrt{\lambda}t\eta)}F(H)e^{-iH(t-\sqrt{\lambda}t\eta)}|\psi,0\rangle =\langle\psi,0|F(H)|\psi,0\rangle, \quad (5.5)$$

i.e., the energy distribution of the ensemble of state vectors is always equal to its initial energy distribution.

However, that is not all there is to say with regard to the energy. The density matrix in the energy basis may be written, considering Eq. (5.4c), as

$$\langle E'|\rho|E\rangle=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\eta e^{-\eta^2/2}\langle E'|\psi,t-\sqrt{\lambda}t\eta\rangle\langle\psi,t-\sqrt{\lambda}t\eta|E\rangle \quad (5.6a)$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\eta e^{-\eta^2/2}e^{i(E'-E)\sqrt{\lambda}t\eta}\langle E'|\psi,t\rangle\langle\psi,t|E\rangle \quad (5.6b)$$

$$=e^{-\lambda t(E'-E)^2/2}\langle E'|\psi,t\rangle\langle\psi,t|E\rangle. \quad (5.6c)$$

In Eq. (5.6c), the pure density matrix of standard quantum theory is multiplied by a Gaussian, so that $\langle E'|\rho|E\rangle\approx 0$ if $|E'-E|>\hbar/T$. Thus one may expect that behavior in standard quantum theory which depends upon a coherent superposition of energy states with a spread $>\hbar/\mathcal{T}$ will be significantly altered under energy-driven collapse.

Next, consider the behavior of a noninteracting subsystem. Denote the subsystem by the subscript 1 and the rest of the universe by the subscript 2. The initial state is $|\psi,0\rangle=|\psi_1,0\rangle|\psi_2,0\rangle$ and $H=H_1+H_2$. For the ensemble expectation value of a quantity V_1, which depends only upon variables of subsystem 1, Eq. (5.4c) yields

$$\langle V_1\rangle(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\eta e^{-\eta^2/2}\langle\psi_1,t-\sqrt{\lambda}t\eta|V_1|\psi_1,t-\sqrt{\lambda}t\eta\rangle \times\langle\psi_2,t-\sqrt{\lambda}t\eta|\psi_2,t-\sqrt{\lambda}t\eta\rangle \quad (5.7a)$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\eta e^{-\eta^2/2}\langle\psi_1,t-\sqrt{\lambda}t\eta|V_1|\psi_1,t-\sqrt{\lambda}t\eta\rangle. \quad (5.7b)$$

In standard quantum theory, the state vector of two disconnected systems describes them as evolving separately. This is not the case with energy-driven collapse: the dynamics entangles the systems [through the quadratic dependence on H in the Gaussian in Eq. (2.1)]. However, for the ensemble, the density-matrix description has the two systems evolving independently. (This interesting result is implicit in the work of Refs. [14,17].) Since Eq. (5.7b) has the same form as Eq. (5.4c), the consequences of Eq. (5.4c) (smeared pulse behavior, unchanging energy spectrum, vanishing of sufficiently separated off-diagonal elements of the density matrix in the energy representation) hold for the subsystem as well. In particular, for the ensemble, under the secular energy narrowing to $\Delta E\approx(\lambda t)^{-1/2}$, the energy spread allowed to the whole universe is the same energy spread allowed to a noninteracting subsystem

B. Individual behavior

Now we wish to consider a single universe, characterized by $B(t)$, and an experiment prepared to be performed at time $\approx T$. For this to make sense, the state vector $|\psi,t_0\rangle_{B(t_0)}$ at some time $t_0<T$ must be compatible with both its function and history.

It is required that the apparatus, an isolated subsystem of the universe, properly performs the experiment under the standard quantum theory evolution. Thus we hypothesize that $|\psi,t_0\rangle_{B(t_0)}=|\psi_1,t_0\rangle|\psi_2,t_0\rangle$, where $|\psi_1,t_0\rangle$ describes the subsystem containing the apparatus and $|\psi_2,t_0\rangle$ describes the rest of the universe. That a reasonable initial state of the universe, $|\psi,0\rangle$, could have evolved to such a direct product is most unlikely, unless energy-driven collapse does indeed take place [so the huge superposition corresponding to all the possible universes that $|\psi,0\rangle$ evolves to under standard quantum theory is collapsed under evolution by $B(t)$ to one of them], so this is entailed by the hypothesis.

Account must also be taken of the state vector's evolution since time 0 (so its energy spectrum is appropriately narrow). One could consider the evolution of the state vector from

$|\psi,t_0\rangle_{B(t_0)}$ to $|\psi,t\rangle_{B(t)}$ using Eq. (4.1), but that would not make explicit the limited spectrum of $|\psi,t_0\rangle_{B(t_0)}$. Accordingly, we may employ Eq. (4.4) to display the state vector $|\psi,t\rangle_{B(t)}$ in terms of $|\psi,0\rangle$: actually, we shall utilize the form (5.3c) which is equivalent to Eq. (4.4) (except for the normalization factor).

Now, although the integral over η in Eq. (5.3c) ranges over $(-\infty,\infty)$ so that the state vector $|\psi,t\rangle_B$ is formally a superposition of state vectors $|\psi,t'\rangle$ for all t', because of the weighting factor exp $(-\eta^2/4)$ in practice the contribution of $|\psi,t'\rangle$ for $t' \ll t-k\mathcal{T}$ and $t' \gg t+k\mathcal{T}$ is negligible [where k is chosen so that exp $(-k^2/4)\approx 0$ to the accuracy one wishes]. With the above assumption of the unentangled nature of the subsystem, the state vector of the universe in standard quantum theory may be written as $|\psi_1,t'\rangle|\psi_2,t'\rangle$ over the interval $T-k\mathcal{T}\leq t' \leq T+k\mathcal{T}$. It then follows from putting the state vector expression, Eq. (5.3c), into Eq. (2.3) that

$$\begin{aligned}\langle V_1\rangle_B(t) = \frac{1}{4\pi_B\langle\psi,t|\psi,t\rangle_B}\int d\eta e^{-\eta^2/4}\int d\eta' e^{-\eta'^2/4} \\ \times e^{-iB(t)(\eta-\eta')/\sqrt{4\lambda t}}\langle\psi_1,t-\sqrt{\lambda t}\eta'|V_1|\psi_1,t-\sqrt{\lambda t}\eta\rangle \\ \times\langle\psi_2,t-\sqrt{\lambda t}\eta'|\psi_2,t-\sqrt{\lambda t}\eta\rangle. \qquad (5.8)\end{aligned}$$

The universe is a very big place, with lots going on. Accordingly, we expect that $|\psi_2,t\rangle$ is orthogonal to $|\psi_2,t'\rangle$ for t' just slightly different from t (we assume that T is large enough so that the time scale for the universe to change in standard quantum theory is much shorter than $\mathcal{T}$). That is, it is an excellent approximation to take $\langle\psi_2,t-\sqrt{\lambda t}\eta'|\psi_2,t-\sqrt{\lambda t}\eta\rangle\approx c(t)\delta(\eta-\eta')$. Then Eq. (5.8) becomes

$$\langle V_1\rangle_B(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\eta e^{-\eta^2/2}\langle\psi_1,t-\sqrt{\lambda t}\eta|V_1|\psi_1,t-\sqrt{\lambda t}\eta\rangle, \qquad (5.9)$$

using $\langle 1\rangle_B(t)=1$ [which follows from Eq. (2.3)] to select the overall normalization factor.

Although this is a single universe, we have arrived at the density-matrix description of Eq. (5.9) because the evolution has entangled the subsystem with the rest of the universe which is traced over. When the apparatus is included in the state vector description, if proper collapse occurs, just one of the macroscopically distinguishable outcomes of the experiment should be described by the state vector. However, because one does not know the state of the rest of the universe (which may have a decisive influence on the experiment's outcome), one must use the density-matrix description. Then, the density matrix should be diagonal in the basis describing the different outcomes of the experiment, and the probabilities associated with these diagonal states should be interpreted as giving the statistics of these actualized outcomes. If, however, the diagonal states of the density matrix turn out to be inappropriate for the theory (as occurs for the examples in Sec. VI, where each state has more energy spread than is allowed in the universe at time T), one is forced to the alternative explanation. It is that proper collapse does not occur: the diagonal density matrix form is there because the state vector describes an apparatus in a superposition of different measurement results entangled with the rest of the universe (which, when the latter is traced over, has the result of canceling the off-diagonal density-matrix elements) as happens in standard quantum theory.

Because of the importance of Eq. (5.9) to the argument in Sec. VI, we shall obtain it another way. Write $\langle V_1\rangle_B(t)$ in Eq. (5.8) in the form

$$\begin{aligned}\langle V_1\rangle_B(t)\sim\int dE_2dE_1'dE_1|\langle E_2|\psi_2,t\rangle|^2\langle\psi_1,t|E_1'\rangle\langle E_1'|V_1|E_1\rangle \\ \times\langle E_1|\psi_1,t\rangle e^{-(1/4\lambda t)[B(t)-2\lambda t(E_2+E_1')]^2} \\ \times e^{-(1/4\lambda t)[B(t)-2\lambda t(E_2+E_1)]^2}, \qquad (5.10)\end{aligned}$$

which is obtained by expressing the state vectors in Eq. (5.8) in the energy basis, extracting the η dependence using $\langle E|\psi,t-\sqrt{\lambda t}\eta\rangle=\exp\, iE\sqrt{\lambda t}\eta\langle E|\psi,t\rangle$ and performing the integrals over η, η'. Since $|\psi_2,t\rangle$ describes the universe (minus the small subsystem 1), we may take $|\langle E_2|\psi_2,t\rangle|^2$ in Eq. (5.10) to be approximately constant over the ranges of E_1, E_1', E_2 where the Gaussian exponents are small. Then the integral over E_2 results in

$$\begin{aligned}\langle V_1\rangle_B(t)\sim\int dE_1'dE_1\langle\psi_1,t|E_1'\rangle\langle E_1'|V_1|E_1\rangle \\ \times\langle E_1|\psi_1,t\rangle e^{-(\lambda t/2)(E_1'-E_1)^2}. \qquad (5.11)\end{aligned}$$

Note that $B(t)$ has disappeared from Eq. (5.11). Putting

$$e^{-(\lambda t/2)(E_1'-E_1)^2}=\frac{1}{\sqrt{2\pi}}\int d\eta e^{-\eta^2/2}e^{-i\sqrt{\lambda t}\eta(E_1'-E_1)}$$

into Eq. (5.11) and choosing the normalization factor so that $\langle 1\rangle_B(t)=1$ results in Eq. (5.9).

Since Eq. (5.9) has the same form as Eq. (5.4c), the consequences of Eq. (5.4c) (smeared pulse behavior, unchanging energy spectrum, approximate vanishing of sufficiently separated off-diagonal elements of the density matrix in the energy representation) hold in this case as well.

Lastly, we note that Eq. (5.9) also describes the situation where the initial state vector can be written as $|\psi,0\rangle=|\psi_1,0\rangle|\psi_2,0\rangle$ where subsystem 1, under standard quantum theory, does not interact with the rest of the universe and describes the apparatus set to turn on at time T. It is reassuring to see that, under energy-driven collapse, whether the apparatus somehow miraculously appears at the start of the universe or is constructed at a later time, it has the same description (which, of course, is a property of standard quantum theory).

VI. TWO EXPERIMENTS

We now analyze two different microscopic phenomena under energy-driven collapse, the precession of a spin 1/2 particle in a magnetic field, and excitation of a bound state followed by its decay. Each is considered to be an isolated system in a single universe, so Eq. (5.9) applies. The energy spectrum in standard quantum theory in the former case consists of two values, in the latter it is spread over a continuum. In both cases the energy spread (characteristic time) is chosen $\gg \hbar/T$ ($\ll \mathcal{T}$) so that the behavior expected from standard quantum theory is appreciably altered. Under energy-driven collapse, the spin 1/2 particle does not precess, while the excitation/decay products do not have an exponential time distribution and have a characteristic time $\mathcal{T}$.

If an apparatus were to properly interact with the microscopic system, it ought to record this altered behavior. However, when we apply Eq. (5.9) to the combined system +apparatus, we find that the density matrix describes the apparatus as recording precession in the first case and recording exponential decay in the second case.

A. Spin precession

Since application of Eq. (5.9) to an experimental situation has to respect the terms under which it was obtained, we need to arrange that the experiment commences in the neighborhood of time T. Therefore, in our model, the state vector $|\psi_1,t\rangle$ describes a (one-dimensional) "photon" (spatial coordinate Q, momentum P) which, at time T, switches on a "magnetic field," i.e., gives a spin (described by the Pauli matrices σ) the energy $(\epsilon/2)\sigma_3$. (A similar experimental situation was discussed by Finkelstein [23] in the context of criticizing an energy-collapsing density-matrix model proposed by Milburn [22]. His criticism is similar to the one given here.)

1. Standard quantum theory treatment

We first discuss this model in standard quantum theory [once we have obtained $|\psi_1,t\rangle$, we shall use it in Eq. (5.9) and see what happens under energy-driven collapse].

The Hamiltonian is

$$H = P + \Theta(Q)(\epsilon/2)\sigma_3. \tag{6.1}$$

In Eq. (6.1), for simplicity, we have set the energy of the photon to be P rather than $|P|$: then, from the Heisenberg equation, $dQ/dt=1$, the photon packet can only move to the right. We shall suppose that the part of the apparatus (which we do not model) which produces the photon makes its initial state a narrowly localized packet. The step function $\Theta(Q)$ acts like a switch, giving the spin-up/-down states the energy difference ϵ (chosen large enough so that $\epsilon \gg \hbar/T$) after the photon packet passes the origin at time $\approx T$.

The energy eigenstates are

$$\langle q|E_\pm\rangle = e^{ikq}[\Theta(-q)e^{\pm i(\epsilon/2)q} + \Theta(q)]|\pm\rangle, \tag{6.2}$$

with energy eigenvalues $E_\pm = k \pm \epsilon/2$, so the general solution of Schrödinger's equation is the superposition (writing $s \equiv t - T$),

$$\langle q|\psi_1,t\rangle = \sum_{n=\pm}\int dk e^{i[k+n(\epsilon/2)](q-s)} \times [\Theta(-q) + \Theta(q)e^{-in(\epsilon/2)}]qf_n(k)|n\rangle. \tag{6.3}$$

By choosing $f_+ = a(2\pi^3/\sigma^2)^{-1/4}\exp-[k+(\epsilon/2)]^2\sigma^2$ and $f_- = b(2\pi^3/\sigma^2)^{-1/4}\exp-[k-(\epsilon/2)]^2\sigma^2$, where a and b are complex constants, $|a|^2+|b|^2=1$, we obtain a wave function satisfying the correct initial conditions:

$$\langle q|\psi_1,t\rangle = \frac{1}{(2\pi\sigma^2)^{1/4}}e^{-(q-s)^2/4\sigma^2}[(a|+\rangle + b|-\rangle)\Theta(-q) + (ae^{-i(\epsilon/2)q}|+\rangle + be^{i(\epsilon/2)q}|-\rangle)\Theta(q)]. \tag{6.4}$$

According to Eq. (6.4) the spin does not precess for $s<0$, while the photon packet travels toward the switch at $q=0$. If the packet is sufficiently narrow, $\sigma\epsilon \ll 1$ (which we assume), although the $\sim\Theta(q)$ terms in Eq. (6.4) are spatially, not temporally, dependent, it nonetheless follows from Eq. (6.4) that the spin precesses once the photon passes the switch, since then

$$e^{-(q-s)^2/4\sigma^2}e^{\pm i(\epsilon/2)q} = e^{-(q-s)^2/4\sigma^2}e^{\pm i(\epsilon/2)(q-s)}e^{\pm i(\epsilon/2)s} \approx e^{-(q-s)^2/4\sigma^2}e^{\pm i(\epsilon/2)s}.$$

Using Eq. (6.4) we calculate

$$\langle\psi_1,t|\sigma_1|\psi_1,t\rangle = (a^*b + ab^*)\Phi\left(-\frac{s}{\sigma}\right) + e^{-\epsilon^2\sigma^2/2} \times \left[a^*be^{i\epsilon s}\Phi\left(\frac{s}{\sigma} + i\epsilon\sigma\right) + ab^*e^{-i\epsilon s}\Phi\left(\frac{s}{\sigma} - i\epsilon\sigma\right)\right] \tag{6.5a}$$

$$\approx 2ab\left[\Phi\left(-\frac{s}{\sigma}\right) + \cos(\epsilon s)\Phi\left(\frac{s}{\sigma}\right)\right], \tag{6.5b}$$

where, for simplicity, we have choosen a and b to be real in Eq. (6.5b), so that the spin is initially in the x-z plane, and we have utilized $\sigma\epsilon \ll 1$). $\Phi(x) \equiv (2\pi)^{-1/2}\int_{-\infty}^{x} dy \exp-(y^2/2)$ is the so-called normal distribution function, sort of a gradual step function, making its transition from 0 to 1 over the range, say, of $|x| \leq 2$ [since $\Phi(-2)\approx 0.02$, $\Phi(2)\approx 0.98$].

Equation (6.5) shows that the spin does not precess for large negative s/σ, that it does precess for large positive s/σ, and indicates that the transition takes place over, say, $|s|<2\sigma$. For $|s|>2\sigma$, the density matrix corresponding to the result (6.5b), readily obtained from Eq. (6.4), is that of pure precession:

$$\mathrm{Tr}_q\rho \approx [ae^{-i\epsilon s/2}|+\rangle + be^{i\epsilon s/2}|-\rangle][ae^{i\epsilon s/2}\langle+| + be^{-i\epsilon s/2}\langle-|]. \tag{6.6}$$

An apparatus which verifies the precession could consist of a clock with time resolution better than $\hbar/\epsilon$ which triggers a device to "instantaneously" (i.e., over a period better than the clock's resolution), nondestructively, and repeatedly measure the spin state at those times predicted by Eq. (6.6) when it comes around to point in a particular direction (the device axis). Then, each time it is measured, the spin will always be seen parallel to the device axis. Moreover suppose that, if the device sees the spin parallel to its axis for a preset number of spin revolutions, the apparatus automatically prints a $\surd$ to indicate that the spin precesses.

2. Energy-driven collapse treatment

We now put Eq. (6.5a) into Eq. (5.9), to see how the precession fares under energy-driven collapse:

$$\langle\sigma_1\rangle_B(t) = (a^*b + ab^*)\Phi\left(-\frac{s}{\sqrt{\sigma^2+\mathcal{T}^2}}\right) + e^{-\epsilon^2(\sigma^2+\mathcal{T}^2)/2}\left[a^*be^{i\epsilon s}\Phi\left(\frac{s+i\epsilon\sigma(\sigma+\mathcal{T})}{\sqrt{\sigma^2+\mathcal{T}^2}}\right) + ab^*e^{-i\epsilon s}\Phi\left(\frac{s-i\epsilon\sigma(\sigma+\mathcal{T})}{\sqrt{\sigma^2+\mathcal{T}^2}}\right)\right] \tag{6.7a}$$

$$\approx 2ab\left[\Phi\left(-\frac{s}{\mathcal{T}}\right) + e^{-\epsilon^2\mathcal{T}^2/2}\cos(\epsilon s)\Phi\left(\frac{s}{\mathcal{T}}\right)\right]. \tag{6.7b}$$

Again, as in Eq. (6.5b), in Eq. (6.7b) we have taken a and b to be real and we have utilized $\sigma\epsilon \ll 1$ as well as $\sigma \ll \mathcal{T}$. Equation (6.7b), compared to Eq. (6.5b), shows that the "switch over" takes a longer time, i.e., $|s| < 2\mathcal{T}$ instead of $|s| < 2\sigma$. This is because the photon packet is widened: the narrowed energy spectrum can no longer sustain a rapid transition.

For $|s| > 2\mathcal{T}$, the density matrix corresponding to the result (6.7b) may be written as

$$\mathrm{Tr}_q\rho \approx (1-e^{-\epsilon^2\mathcal{T}^2/2})[a^2|+\rangle\langle+| + b^2|-\rangle\langle-|] + e^{-\epsilon^2\mathcal{T}^2/2}[ae^{-i\epsilon s/2}|+\rangle + be^{i\epsilon s/2}|-\rangle] \times [ae^{i\epsilon s/2}\langle+| + be^{-i\epsilon s/2}\langle-|], \tag{6.8}$$

a mixture of spin up, spin down, and spin precessing. However, the precession part of the mixture becomes negligibly small if we take the energy separation $\epsilon \gg \hbar/\mathcal{T}$.

Therefore, in this case, if the apparatus we have discussed should function as described when it encounters the spin behavior given by Eq. (6.8), the repeated measurements should produce the result that the spin is sometimes parallel and sometimes antiparallel to the device axis, and the apparatus will not print a $\surd$.

One might, however, surmise that one could not come up with an apparatus governed by a clock whose resolution is better than $\hbar/\epsilon$ in a universe which suffers energy-driven collapse. But, if the clock's resolution is $\mathcal{T}$ or worse, the conclusion is the same.

Now, let us consider what happens according to energy-driven collapse when the apparatus is included in the state vector $|\psi_1, t\rangle$ and Eq. (5.9) is applied. Equation (5.9) gives the density matrix at time t as a superposition of pure states $|\psi_1, t-\eta\mathcal{T}\rangle\langle\psi_1, t-\eta\mathcal{T}|$, each of which describes the standard quantum evolution at a certain instant of time, in a $\approx\mathcal{T}$ neighborhood of t. Since, under the standard quantum evolution, there is precession which is measured by the apparatus, for $t-T$ sufficiently large, every apparatus in the superposition has printed out a $\surd$. Thus the density matrix at a large enough time t eventually describes that a $\surd$ is printed out with certainty.

So, we have a paradox. Analysis of the microscopic behavior indicates that there is no precession. Analysis of the microscopic system in interaction with the apparatus indicates that there is precession. In both cases, we have correctly applied Eq. (5.9) to the relevant situation.

To see what has gone wrong, consider the standard quantum theory state vector $|\psi_1, t-\eta\mathcal{T}\rangle$, which contributes to the density-matrix integral in Eq. (5.9) (for η of the order of a small integer). This state vector is the direct product of the apparatus state (which has recorded a certain number of spin detections parallel to its axis) including the clock which reads time $t-\eta\mathcal{T}$, and the narrow photon packet (which had earlier triggered the precession) at that time, and the spin state at that time. Since the clock is so precise, and the photon packet is so narrow, these states are orthogonal for fairly close values of η, each state describing a different clock time and a different photon packet location (as well as a different spin orientation which, however, is not responsible for the orthogonality). As a result, the integral in (5.9) gives a density matrix with the various distinguishable $|\psi_1, t-\eta\mathcal{T}\rangle\langle\psi_1, t-\eta\mathcal{T}|$ along the diagonal (and negligible off-diagonal elements). Then, as discussed following Eq. (5.9), if collapse occurs properly, a diagonal state should be interpreted as a possible outcome of the experiment in the single universe. However, such a state contains the localized photon packet of energy spread $\gg\mathcal{T}^{-1}$ (also, for the clock, a similar statement obtains), more energy spread than is allowed in the whole universe. Therefore, the density-matrix's diagonal states cannot be interpreted in this way. Instead, as discussed, they arise from a state vector which, as in standard quantum theory, describes a superposition of macroscopically different apparatus states entangled with the rest of the universe. The apparatus state does not measure the microscopic behavior and the state vector, which gives the standard quantum theory result, does not collapse properly.

B. Excitation of a bound state and its decay

Next, we consider a (one-dimensional) photon which at time $\approx T$ excites a bound state (located at x_0) which subsequently decays. The Hamiltonian is a modification of a well-known model of a two-state atom [24]:

$$H = \epsilon b^\dagger b + \int_{-\infty}^{\infty} dk\, k a_k^\dagger a_k + g\int_{-\infty}^{\infty} dk[a_k e^{ikx_0}b^\dagger + a_k^\dagger e^{-ikx_0}b]. \tag{6.9}$$

Here $b^\dagger$ creates the excited state of energy $\epsilon([b,b^\dagger]=1)$ and $a_k^\dagger$ creates a photon of momentum k $([a_k, a_{k'}^\dagger]=\delta(k-k'))$. The coupling constant $g\equiv(\Gamma/2\pi)^{1/2}$, where Γ turns out to be the bound-state lifetime. As in the preceding section, for simplicity, we choose the photon energy to be k rather than $|k|$, so that the photon only moves to the right but, also, the consequent unbounded energy spectrum allows the decay to be precisely exponential [25].

1. Standard quantum theory treatment

First, the analysis in standard quantum theory. The state vector has the form

$$|\psi,t\rangle = \beta(t)b^\dagger|0\rangle + \int_{-\infty}^{\infty} dk\alpha_k(t)a_k^\dagger|0\rangle, \qquad (6.10)$$

where $|0\rangle$ is the no-photon state and the ground state of the bound state. The Schrödinger equation implies

$$id\alpha_k/dt = g\beta e^{-ikx_0} + k\alpha_k, \quad id\beta/dt = \epsilon\beta + g\int_{-\infty}^{\infty} dk\alpha_k e^{ikx_0}. \qquad (6.11)$$

For insight, it is worth checking out the solution of Eq. (6.11) for decay without excitation [even though its initial conditions $\beta(T)=1$, $\alpha_k(T)=0$ are inappropriate for use in Eq. (5.9) if $\Gamma^{-1}<\mathcal{T}$, since then the initial state will have more energy spread than is allowed to the universe at time T]. Setting $s\equiv t-T$, the result for $s>0$ is

$$\alpha_k(t) = ge^{-ikx_0}\frac{-e^{-[(\Gamma/2)+i\epsilon]s}+e^{-iks}}{k-\epsilon+i(\Gamma/2)}, \qquad (6.12a)$$

$$\beta(t) = e^{-[(\Gamma/2)+i\epsilon]s}. \qquad (6.12b)$$

From Eqs. (6.12), the expectation value of the photon number density and the expectation value of the particle being in the excited state are

$$\langle\psi,t|a_k^\dagger a_k|\psi,t\rangle = \frac{2\pi\Gamma}{(k-\epsilon)^2+(\Gamma/2)^2} \times[1+e^{-\Gamma s}-2e^{-(\Gamma/2)s}\cos(k-\epsilon)s], \qquad (6.13a)$$

$$\langle\psi,t|b^\dagger b|\psi,t\rangle = e^{-\Gamma s}. \qquad (6.13b)$$

According to Eq. (6.13b), the decay is exponential. According to Eq. (6.13a), the photon distribution is Lorentzian for large s. From Eqs. (6.10) and (6.12a), the photon wave function in the position representation $|x\rangle=(2\pi)^{-1/2}\int dk\exp(-ikx)a_k^\dagger|0\rangle$ is found:

$$\langle x|\psi,t\rangle = i\Gamma^{1/2}e^{-i\epsilon[s-(x-x_0)]}e^{-(\Gamma/2)[s-(x-x_0)]} \times[\Theta(x-x_0-s)-\Theta(x-x_0)]. \qquad (6.14)$$

Thus the photon packet emerging from the decay has $|\langle x|\psi,t\rangle|^2$ taking on the value Γ at its leading edge $x=x_0+s$, exponentially falling to the value $\Gamma\exp-\Gamma s$ at its source, $x=x_0$, and vanishing elsewhere. (In this, and all the rest of the examples discussed, it can readily be checked that probability is conserved.)

In the case of interest, excitation of the bound state followed by decay, the initial conditions are $\beta(t)\approx 0$ for $s<0$, and an initial photon wave packet which reaches x_0 at $s\approx 0$. The solution of Eqs. (6.11) is then

$$\langle x|\psi,t\rangle = f[s-(x-x_0)] - i\Gamma^{1/2}\Theta(x-x_0)\beta(s-(x-x_0)), \qquad (6.15a)$$

$$\beta(s) = -i(\Gamma)^{1/2}e^{-[(\Gamma/2)+i\epsilon]s}\int_{-\infty}^{s} ds' f(s')e^{[(\Gamma/2)+i\epsilon]s'}, \qquad (6.15b)$$

where $f(z)$ describes the incident wave packet.

In what follows, we shall take the width of the incident wave packet σ to be much less than any time in the model, so $\sigma\epsilon\ll 1$, $\sigma\Gamma\ll 1$, and $\sigma\mathcal{T}^{-1}\ll 1$ can all be neglected compared to 1. In that case, we can also take $f(s)$ to be approximately the "square root of a δ function," $f(s)\approx\sigma^{1/2}\delta(s)$ (e.g., the square root of a normalized narrow Gaussian is a narrow unnormalized Gaussian). Then, the probabilities given by Eqs. (6.15) may be written as

$$|\langle x|\psi,t\rangle|^2 = |f[s-(x-x_0)]|^2 - \Gamma\sigma\Theta(x-x_0)\delta(s-(x-x_0)) + \Gamma^2\sigma\Theta(x-x_0)\Theta(s-(x-x_0))e^{-\Gamma[s-(x-x_0)]}, \qquad (6.16a)$$

$$|\beta(s)|^2 \approx \Gamma\sigma\Theta(s)e^{-\Gamma s}. \qquad (6.16b)$$

According to Eq. (6.16b), $|\beta(s)|^2=0$ for negative s, jumps to $\Gamma\sigma$ at $s=0$, and thereafter exponentially decays with lifetime Γ. According to Eq. (6.16a), the squared photon wave function consists of the initial wave packet, an interference term between it and the leading edge of the decaying packet and the decay product whose square vanishes for $x<x_0$, jumps at $x=x_0$ (when $s>0$) to $\Gamma^2\sigma\exp-\Gamma s$, exponentially rises as x increases to $\Gamma^2\sigma$ at $x=x_0+s$ and vanishes for $x>x_0+s$.

Suppose an accelerator produces a localized burst of many such photons and they hit many such bound states, spread out in a thin film of thickness σ so that, in the standard quantum theory description, the bound states are excited essentially simultaneously. Suppose a detector, with an accurate clock (of time resolution much better than Γ^{-1}) measures the time distribution of arrival of the resulting outgoing particles. (For use at the end of the following section, we shall assume the measurement is reasonably nondisturbing, so a detected particle is allowed to proceed beyond the detector.) Moreover, suppose the apparatus is designed to print out, at its leisure, the time spread of the decay products and whether the decay shape is exponential ($\mathcal{YES}$) or not ($\mathcal{NO}$). Then, a properly operating detector encountering many photons, each described by Eq. (6.16a), should print out Γ^{-1} and $\mathcal{YES}$.

2. Energy-driven collapse treatment

Now lets see what happens under energy-driven collapse. Using Eqs. (5.9) and (6.15), we calculate the expectation value of the photon position probability distribution and the expectation value of the occupation number of the excited state:

$$\langle|x\rangle\langle x|\rangle_B(t) = (2\pi)^{-1/2}\int_{-\infty}^{\infty} d\eta e^{-\eta^2/2}|\langle x|\psi,T-\mathcal{T}\eta\rangle|^2$$
$$= (2\pi\mathcal{T}^2)^{-1/2}e^{-(1/2\mathcal{T}^2)[s-(x-x_0)]^2}[1-\Gamma\sigma\Theta(x-x_0)] + \Gamma^2\sigma\Theta(x-x_0)\Phi[\{s-(x-x_0)/\mathcal{T}\}-\Gamma\mathcal{T}]$$

$$\times e^{-\Gamma[s-(x-x_0)]}e^{(1/2)(\Gamma\mathcal{T})^2}, \quad (6.17a)$$

$$\langle b^\dagger b\rangle_B(t) = (2\pi)^{-1/2}\int_{-\infty}^{\infty} d\eta e^{-\eta^2/2}|\beta(t-\mathcal{T}\eta)|^2$$
$$= \Gamma\sigma e^{-\Gamma s}e^{(1/2)(\Gamma\mathcal{T})^2}\Phi[(s/\mathcal{T})-\Gamma\mathcal{T}]. \quad (6.17b)$$

First consider the small $\mathcal{T}$ case, i.e., $\mathcal{T}\Gamma \ll 1$. From Eq. (6.17b), the bound-state excitation onset, governed by $\approx\Phi[s/\mathcal{T}]$, takes place over $\mathcal{T}$, because the incident photon wave-packet width is broadened from σ to $\mathcal{T}$. The decay is exponential with time constant Γ^{-1}. From Eq. (6.17a), the interference term [the term $\sim\Gamma\sigma\Theta(x-x_0)$] at the leading edge of the outgoing packet has likewise been broadened to a Gaussian of width $\mathcal{T}$. In other words, the outgoing packet's behavior is similar to that of the standard quantum theory (6.16b), except that its onset takes place over $\mathcal{T}$.

Of greatest interest is the large $\mathcal{T}$ case, $\mathcal{T}\Gamma \gg 1$. Utilizing the large x behavior $\Phi[-x]\to(2\pi)^{-1/2}x^{-1}\exp-(x^2/2)$, Eqs. (6.17) becomes

$$\langle |x\rangle\langle x|\rangle_B(t) \approx (2\pi\mathcal{T}^2)^{-1/2}e^{-(1/2\mathcal{T})^2[s-(x-x_0)]^2}$$
$$\times\left\{[1-\Gamma\sigma\Theta(x-x_0)]+\Gamma\sigma\Theta(x-x_0)\right.$$
$$\left.\times\left[1+\frac{s-(x-x_0)}{\Gamma\mathcal{T}^2}\right]\right\}, \quad (6.18a)$$

$$\langle b^\dagger b\rangle_B(t) \approx \sigma(2\pi\mathcal{T}^2)^{-1/2}e^{-(1/2\mathcal{T}^2)s^2}. \quad (6.18b)$$

From Eq. (6.18b), the decay of the bound state is no longer exponential. It is Gaussian with characteristic time $\mathcal{T}$. This is because the incident wave packet is Gaussian, not because the decay is Gaussian: Eqs. (6.18) behave just like the standard quantum theory description, Eqs. (6.15), with a broad Gaussian incident wave packet, of width $\mathcal{T}$, and a relatively rapid decay ($\Gamma^{-1}\ll\mathcal{T}$). Although the exponential decay is faster than $\mathcal{T}$, it does not appear in any equation since the decay is masked by the $\mathcal{T}$ behavior of the incident wave packet. Thus there is no violation of the stricture against "fast" pulse behavior.

From Eq. (6.18a), the interference term (second term in the small square bracket) and most of the outgoing packet (first term in the large square bracket) cancel, so that what remains has the shape of the incident packet plus a little "blip" (second term in the large square bracket), which is a little larger at the leading edge and a little smaller at the trailing edge.

While, in this model, the incident photon and the photon decay product cannot be separated [and Γ completely disappears from Eqs. (6.18)], if one wishes one can alter the model by adding to the Hamiltonian (6.9) another coupling term:

$$g'\int_{-\infty}^{\infty} dk[c_k e^{ikx_0}b^\dagger + c_k^\dagger e^{-ikx_0}b],$$

so that the excited state can decay to a c particle as well as an a particle. The decay product can be separated from the incident particle by making the decay go essentially only to the c particle: choose $g\sim\Gamma^{1/2}$ small enough ($\Gamma^{-1}\gg\mathcal{T}$) and $g'\sim\Gamma'^{1/2}$ large enough ($\Gamma'^{-1}\ll\mathcal{T}$), so the state becomes excited by the incident a particle, but decays to the c particle. Then, under energy-driven collapse, the large square bracket term in Eq. (6.18a) will describe the c-particle decay product.

Now, consider the experiment already discussed: in this case, a properly operating detector encountering many photons, each described by Eq. (6.18a), should print out $\mathcal{T}$ and $\mathcal{NO}$. Should the detector be incapable of responding with a time resolution not much better than $\mathcal{T}$, as one might suspect for a detector in a universe suffering energy-driven collapse, the printout will be the same.

Next, suppose the detector is included in the state vector. In the standard quantum description, the detector sees the results described in Eqs. (6.16) and will, at its leisure, print out Γ and $\mathcal{YES}$. To see what happens under energy-driven collapse, we apply Eq. (5.9). The density matrix is a superposition of pure states $|\psi,t-\mathcal{T}\eta\rangle\langle\psi,t-\mathcal{T}\eta|$, each of which satisfies the standard quantum description. Therefore, for t large enough, the density matrix describes with certainty that the apparatus prints out Γ and $\mathcal{YES}$.

Thus, as in the preceding section we have a paradox. There is a conflict between the result for the microscopic system alone, and the result for the apparatus interacting with the microscopic system, both correctly calculated using Eq. (5.9).

To see what has gone wrong, consider the nature of the density matrix at some time t during the measurement, calculated according to Eq. (5.9). Standard quantum theory does not produce collapse, so the state vector in standard quantum theory at time $t-\mathcal{T}\eta$ is a superposition of states, each describing an apparatus which has recorded a particular photon detection sequence in a direct product with the photons which have been nondestructively detected. Although the conclusion from each sequence is the same and leads to the same summarizing printout, since all sequences are different these are macroscopically distinguishable states. Under the time-smearing construction of Eq. (5.9), these states appear superposed in the resulting density matrix. If the density matrix is not diagonal in these states then, obviously, the superposition of such states which occurs in standard quantum theory has carried over to the energy-driven collapse theory, and collapse is not working. Suppose, then, that the density matrix is diagonal in these states. These are states for which the photons, resolved to better than Γ^{-1}, have more energy spread than is allowed in the universe (also, for the clock, a similar statement obtains). Thus again, as discussed following Eq. (5.9) and in the last section, the density matrix could not be the result of proper collapse.

C. Numerical values

We deliberately have not put numbers into these discussions of experiments. The point we are making, that proper collapse does not occur when the apparatus is included in the state vector, is a point of principle, not a conflict with experiment. However, it is worthwhile trying to find examples of altered phenomena predicted by the energy-driven collapse model, when the apparatus is not included in the state vector, which might be subject to an experimental test.

Hughston has made the interesting choice $\lambda = (G/\hbar^3 c^5)^{1/2}$=(Planck time)/$\hbar^2$ for which, with $T = 13.7 \times 10^9$ yr, one obtains $\mathcal{T} \equiv (\lambda T)^{1/2} \approx 1.5 \times 10^{-13}$ sec. Because timers, counters, oscilloscopes, printers, etc., do not operate at 10^{-13} sec, it is hard to come up with an experiment actually performed or even presently performable, for which the slight smearing over 10^{-13} sec has a practical detectable effect.

For example, there has been much work with light pulses of ≈ 1 to 100 fs (1 fs$=10^{-15}$ sec) which can be obtained from a mode-locked Ti:sappphire laser. In one such experiment, the cross correlation of the intensities of two such pulses is measured [26]. A 27 fs pulse (centered at 800 nm) is split into two pulses. One pulse is reshaped, the other delayed, and the two are recombined in a frequency-doubling crystal. The output intensity, filtered at twice the input frequency, is proportional to the intensity cross correlation of the two pulses at the delay time, and is measured by a photomultiplier tube. In one case, graphs of intensity cross correlation versus delay time show <54 fs structure. Nonetheless, the description under energy-driven collapse which causes a 150 fs "smearing" gives no different result than standard quantum theory because both pulses are similarly affected. The averaging of $|\psi_1, t-\mathcal{T}\eta\rangle\langle\psi_1, t-\mathcal{T}\eta|$ in Eq. (5.9) just means that the signal proportional to the cross correlation coming out of the frequency-doubling crystal will be spread over ≈ 150 fs instead of <54 fs. The photomultiplier measures its input intensity independent of the input pulse width, so the measurement result will be the same. Other experiments, such as those involving time-domain terahertz spectroscopy, are similarly configured, and thus similarly unaffected.

A direct measurement of pulse width, showing that a pulse less than $\mathcal{T}$ in width *can* be observed, *would* certainly be adequate to produce a discrepancy between the energy-driven collapse prediction and experiment. The fastest commercial oscilloscope of which I am aware, the Tektronix TDS 6604, operates at 6 GHz, with a 20 gigasample/sec rate, so if $\mathcal{T} \approx 10^{-10}$ sec, a discrepancy would be observed.

A similar situation prevails with regard to decay experiments such as discussed in Sec. VI B. For example, the π^0's lifetime of ≈ 0.1 fs, time dilated in the laboratory, to ≈ 5 fs (the pions had energy 7.1 GeV), has been measured by what may be thought of as a time-of-flight experiment [27]. 18 GeV protons bombarded platinum foils of various thicknesses, uniformly creating pions in a foil. If a moving pion has a short enough lifetime so that it decays to two γ's while it is still in the foil, there is a certain probability (largest when the γ's still have a lot of foil to travel through) that a γ will create an electron-positron pair by colliding with an atom in the foil. The positrons are detected, and the number of positrons produced in foils of various thicknesses can be related to the pion lifetime. Under energy-driven collapse, the observed results would be no different, since it is effectively distances that are measured. All energy-driven collapse requires is that the π^0 and decay product wave functions rise and fall over time $\geqslant \mathcal{T}$: there is no effect on the size of the *distance* traveled before decay or pair creation.

However, with regard to the spin precession discussion in Sec. VI A, it is presently just possible to obtain a magnetic field strong enough to observe an experimental discrepancy. An electron spin in a magnetic field precesses at 2.8 MHz/G. The largest pulsed magnetic fields at present, produced by machines in high magnetic-field laboratories around the world, are ≈ 70 T, corresponding to a frequency of $\approx 2 \times 10^{12}$ Hz. But, a magnetic field of 850 T, corresponding to a precession frequency of $\approx 2 \times 10^{13}$ Hz has been produced in a one-shot "self-destructive" magnet. Suppose a slug of matter were to be placed between the poles of such a magnet during its few milliseconds of operation, and the far infrared magnetic dipole radiation expected to be produced by the precessing electron spins were to be observed. The diminished radiation predicted by energy-driven collapse due to the diminished precession, compared to standard quantum theory's prediction of the precession, could be tested.

ACKNOWLEDGMENTS

I would especially like to thank Steve Adler for his hospitality at the Institute for Advanced Study at Princeton where this work was conceived, and for his many helpful comments on this paper. I would also like to thank Todd Brun, Brian Collett, Jerry Finkelstein, Larry Horwitz, Lane Hughston, Gordon Jones, Jim Ring, Ann Silversmith, and Don Stewart for useful remarks.

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