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Bloch oscillations in Fermi gases

M. Rodríguez[1] and P. Törmä[2]
[1]*Laboratory of Computational Engineering, P. O. Box 9203, Helsinki University of Technology, FIN-02015 HUT, Finland*
[2]*Department of Physics, NanoScience Center, University of Jyväskylä, P.O.Box 35, FIN-40014 JYU, Finland*
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The possibility of Bloch oscillations for a degenerate and superfluid Fermi gas of atoms in an optical lattice is considered. For a one-component degenerate gas the oscillations are suppressed for high temperatures and band fillings. For a two-component gas, Landau criterion is used for specifying the regime where robust Bloch oscillations of the superfluid may be observed. We show how the amplitude of Bloch oscillations varies along the crossover from BCS to Bose-Einstein condensation.

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The experimental realization of optical lattices for bosonic atoms has led to several landmark experiments [1–4]. Very recently similar potentials have become available for trapping the *fermionic* isotopes as well [5,6]. An increase in the superfluid transition temperature when using potentials created by standing light waves has been predicted [7]. For trapped cold atoms, the famous BCS-BEC (Bose-Einstein Condensation) crossover problem [8–10] could be studied by tuning the interaction strength between the atoms using Feshbach resonances [6,11,12]. In optical lattices the whole BCS-BEC crossover could be scanned experimentally also in an even simpler way by modulating the light intensity. We consider Bloch oscillations in these systems and show that they can be used as a tool for studying the crossover.

Bloch oscillations are a pure quantum phenomenon occurring in a periodic potential. They have never been observed in a natural lattice for electrons as predicted in Ref. [13] because the scattering time of the electrons by lattice defects or impurities is much shorter than the Bloch period. However, Bloch oscillations have recently been observed in semiconductor superlattices [14] for quasiparticles penetrating the cores of a vortex lattice in a cuprate superconductor [15] and for periodic optical systems such as waveguide arrays [16]. Also cold bosonic atoms and superfluids in optical lattices have been shown to be clean and controllable systems well suited for the observation of Bloch oscillations [2–4].

Several novel aspects of the physics of Bloch oscillations arise for fermionic atoms in optical lattices.

(i) Impurity scattering can be made negligible, and the particle number controlled at will to produce any band filling. Even when Bloch oscillations were originally proposed for fermions, the effect of the Fermi sea had not played a major role. Due to impurity and defect scattering, the studies of transport in presence of a constant force have focused on drift velocities rather than oscillations. In this paper we generalize the semiclassical single-particle description of Bloch oscillations to arbitrary band fillings.

(ii) The possibility of an oscillating fermionic superfluid becomes relevant. We use the Landau criterion for the optical lattice imposing the Cooper pair size to be of the order of or smaller than the lattice spacing. For solid-state systems, the Cooper pair radius is usually much larger than the lattice spacing and periodicity irrelevant for the superfluid, therefore the system is treated as homogeneous when calculating supercurrents. Here we calculate the superfluid velocity in the *periodic* potential.

Two complementary approaches for quantum transport in periodic potentials have been experimentally realized for bosonic atoms: spectroscopy of Wannier-Stark ladder resonances [3] and direct observation of Bloch oscillations in momentum space [2,4]. The constant force inducing the transport is realized by chirping the frequency difference between the optical lattice beams. After applying the force for a given time, the lattice beams are switched off [2,4] and the lattice quasimomenta are directly mapped to the momenta of the free particles. In this way, the atomic cloud velocity at different times can be measured. The velocity shows Bloch oscillations whose amplitude depends on the bandwidth. Here we consider the case of fermionic atoms instead of bosons, and calculate how the amplitude of the oscillations is affected by the temperature and the Fermi sea using the tight-binding approximation. We also consider interacting fermions and show that pairing, leading to smoothening of the Fermi edge, suppresses the Bloch oscillation amplitude.

Using six counterpropagating laser beams of wavelength λ, an isotropic three-dimensional (3D) simple cubic lattice potential can be created which is of the form

$$V(\mathbf{r}) = V_0\left[\cos^2\left(\frac{\pi x}{a}\right) + \cos^2\left(\frac{\pi y}{a}\right) + \cos^2\left(\frac{\pi z}{a}\right)\right], \quad (1)$$

where V_0 is proportional to the laser intensity and $a=\lambda/2$. With the Bloch ansatz the Schrödinger equation leads to a band structure in the energy spectrum $\varepsilon_n(\mathbf{k})$. One-component degenerate Fermi gas at low temperatures can be considered as *noninteracting* since *p*-wave scattering is negligible and *s*-wave scattering is suppressed by Fermi statistics. We are interested in high enough values of V_0 such that tunneling is small and tight-binding approximation can be applied. The dispersion relation for the lowest band becomes $\varepsilon(\mathbf{k})=J[3-\cos(k_xa)-\cos(k_ya)-\cos(k_za)]$, where the bandwidth $J=(2/\sqrt{\pi})E_R(V_0/E_R)^{3/4}\exp(-2\sqrt{V_0/E_R})$ is obtained using the WKB approximation and $E_R=h^2/(8ma^2)$ is the recoil energy of the lattice [7].

In a two-component Fermi gas, atoms in two different hyperfine states $(\downarrow,\uparrow)$ may interact with each other. The interaction can be assumed pointlike, characterized by a scattering length a_S. The system Hamiltonian $\hat{H}=\Sigma_\alpha \int d^3\mathbf{r}\hat{\psi}_\alpha^\dagger(\mathbf{r}) \times(T+V)\hat{\psi}_\alpha(\mathbf{r})-|g|\int d^3\mathbf{r}\hat{\psi}_\uparrow^\dagger\hat{\psi}_\downarrow^\dagger\hat{\psi}_\downarrow\hat{\psi}_\uparrow$, where $g=4\pi\hbar^2 a_S/m$, can then be mapped to the attractive Hubbard model $\hat{H} =J\Sigma_{\langle i,j\rangle\sigma}\hat{c}_{i\sigma}^\dagger\hat{c}_{j\sigma}-U\Sigma_j\hat{c}_{j\uparrow}^\dagger\hat{c}_{j\downarrow}^\dagger\hat{c}_{j\downarrow}\hat{c}_{j\uparrow}$, where $U=E_R\sqrt{8\pi}|a_s|/a(V_0/E_R)^{3/4}$. The ground-state solution of this Hubbard Hamiltonian corresponds to a superfluid which is of BCS or BEC type depending on the parameters J and U. For optical lattices, the BCS ($J\gg U$) to BEC ($U\gg J$) crossover can be controlled by V_0 alone. One-band description is used in the Hubbard model also in the case of strong interactions [17]. We define the limits of the one-band approximation for the physical potential, Eq. (1), by demanding the lowest band gap to be bigger than the effective interaction U (note that $U>|g|$ for the parameters of interest). The band gap can be estimated by approximating the cosine potential well by a quadratic one. Demanding the corresponding harmonic-oscillator energy to be greater than U gives the condition $V_0/E_R<1/(4\pi^2)(a/|a_s|)^4$. Since $a>|a_S|$ (lattice period bigger than the scattering length) is imposed by considering on-site interactions only, the condition is easily valid in general, and for the parameters of Fig. 1 in particular. Estimates made using exact numerical band gaps in 1D support this argument.

Bloch oscillations for a single atom can be characterized considering the mean velocity of a particle in a Bloch state $\mathbf{v}(n,\mathbf{k})=\langle n,\mathbf{k}|\dot{\mathbf{r}}|n,\mathbf{k}\rangle$ given by

$$\mathbf{v}(n,\mathbf{k}) = \frac{1}{\hbar}\nabla_{\mathbf{k}}\varepsilon_n(\mathbf{k}). \qquad (2)$$

When a particle in the Bloch state $|n,\mathbf{k}_0\rangle$ is adiabatically affected by a constant external force $\mathbf{F}=F_x\hat{\mathbf{x}}$ weak enough not to induce interband transitions, it evolves up to a phase factor into the state $|n,\mathbf{k}(t)\rangle$ according to $\mathbf{k}(t)=\mathbf{k}_0+\mathbf{F}t/\hbar$. The time evolution has a period $\tau_B=h/(|F_x|a)$, corresponding to the time required for the quasimomentum to scan the whole Brillouin zone. If the force is applied adiabatically, it provides momentum to the system but not energy because the effective mass [given by $m(\varepsilon)^{-1}=(1/\hbar^2)(\partial^2\varepsilon/\partial k^2)$] is not always positive. For optical lattices the force (or tilt $V=-\mathbf{F}\cdot\mathbf{r}$ term in the Hamiltonian) can be realized by accelerating the lattice [2–4]. Using the tight-binding dispersion relation the velocity of an atom oscillates like

$$v_x(t) = \frac{Ja}{\hbar}\sin\left(k_{0x}a + \frac{F_x ta}{\hbar}\right). \qquad (3)$$

For cold bosonic atoms and condensates [2,4] nearly all of the population is in the lowest mode of the optical potential, Eq. (3) therefore describes the oscillation of the whole gas. We generalize the result for the case when many momentum states of the band (at $T=0$, the states with wave vector $|\mathbf{k}|\leq k_F$) are occupied. We calculate the velocity of the whole gas as the average over the normalized temperature-dependent distribution function (the Fermi distribution f) of the particles:

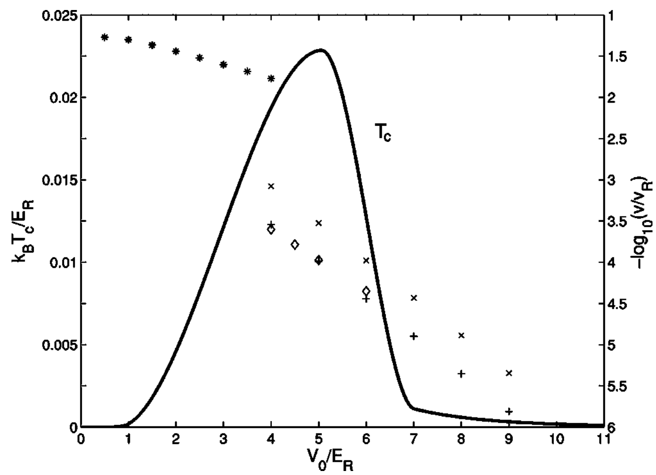

FIG. 1. The solid line shows the transition temperature (units on the left axis) as a function of the lattice depth V_0. The points represent the amplitude of the velocity Bloch oscillations (units on the right axis) in recoil velocity units (v_R). The *normal-state* oscillation amplitude at $T=\frac{2}{3}T_c^{max}$ is denoted by *. The *superfluid* oscillation amplitude at $T=0$ is denoted by $\diamond$. Note the difference in the magnitude of the normal state (*) and the superfluid ($\diamond$) oscillation amplitudes. The amplitude of the superfluid velocity oscillations in the bosonic limit, Eq. (13), is marked by $\times$ for pair size $l=a/3$ and by + for $l=a/4$. All magnitudes are given in recoil energy units $E_R=h^2/(8ma^2)$. The results are for ^{6}Li atoms in hyperfine states with scattering length $a_s=-2.5\times10^3 a_0$ in a half filled ($\mathbf{k_F}=1/2\mathbf{k_L}$) 3D CO_2 laser lattice ($a=10^5 a_0$).

$$\langle v_x(t)\rangle = \frac{1}{\hbar}\sum_{\mathbf{k}_0} f(\mathbf{k}_0)\nabla_{k_{0x}}\varepsilon\left(\mathbf{k}_0 + \frac{\mathbf{F}t}{\hbar}\right). \qquad (4)$$

Using the tight-binding dispersion relation for the Bloch energies, Eq. (4) reduces at $T=0$ to

$$\langle v_x(t)\rangle = \frac{Ja}{\hbar}\frac{\sin(k_{xF}a)}{k_{xF}a}\sin\left(\frac{F_x ta}{\hbar}\right). \qquad (5)$$

This shows that a macroscopic coherent oscillation, such as in Eq. (3), can still be observed if the band is not full, but the amplitude is suppressed by the band filling $k_{xF}a$, i.e., the effect of the Fermi sea can decrease the amplitude considerably compared to the bosonic case [2,4]. Also temperature can affect the amplitude of the oscillations. Using Eq. (4) we obtain that, compared to the $T=0$ case, the oscillation amplitude is nearly unchanged for $k_BT\leq 0.1J$, reduced by half for $k_BT=J$, and by one order of magnitude for $k_BT\sim 3J$.

The above results are valid for a one-component degenerate Fermi gas at low temperatures. In a two-component Fermi gas, atoms in the different hyperfine states interact with each other, which may lead to a superfluid state. Above T_c, weak interactions can be described by a mean-field shift in the chemical potential, leading to no qualitative changes in Bloch oscillations. Inelastic scattering and consequent damping of Bloch oscillations can be described, e.g., by balance equations [18]. In the following we consider the superfluid case where qualitative changes are expected.

In order to observe robust Bloch oscillations of a superfluid Fermi gas in the presence of momentum changing collisions, the critical velocity of the superfluid should not be reached before the edge of the Brillouin zone. A BCS superconductor can carry a persistent current q until a critical velocity $v_c=\Delta/p_F$. For higher current values, even at $T=0$, it is energetically favorable to break Cooper pairs and create a pair of quasiparticles [19]. This costs 2Δ in binding energy and decreases the Bloch energy ($\xi=\varepsilon-\mu$, where μ is the chemical potential) by $|\xi_{k_F+q}-\xi_{k_F-q}|\equiv 2|E_D|$. Therefore, for the current to be stable $|E_D|<\Delta$. This is the Landau criterion of superfluidity. For the tight-binding dispersion relation, we rewrite the condition as $J\sin(qa)\sin k_Fa<\Delta$. To complete a Bloch oscillation, $\sin(qa)$ should achieve its maximum value 1, i.e.,

$$\sin k_Fa < \Delta/J. \tag{6}$$

For weak coupling, Δ/J is given by the BCS theory, and in the attractive Hubbard model in the strong-coupling limit the gap at $T=0$ is given by $\Delta=\frac{1}{2}U$ for half filling [17]. Using these estimates, the relation (6) reduces for the parameters of Fig. 1 to $V_0/E_R>3.2$. That is, for $V_0/E_R>3.2$ Bloch oscillations of the superfluid could be observed. For $V_0/E_R<3.2$, the superfluid may break, and one has to apply the normal-state description. For this reason, we plot in Fig. 1 the superfluid oscillation amplitude for $V_0/E_R>4$ and the normal-state one for $V_0/E_R<4$.

To relate the Landau criterion to the Cooper pair size, we rewrite Eq. (6) in terms of the BCS coherence length $\xi_0=\hbar v_F/(\pi\Delta)$ and insert $J\sin(k_Fa)=\hbar v_F/a$ which yields $\xi_0<a/\pi$. The observation of robust Bloch oscillations is thus restricted to superfluids with BCS coherence length smaller than the lattice periodicity. This is the intermediate strong-coupling regime.

For calculating the superfluid velocity a space-dependent description of the superfluid has to be used. We combine the BCS ansatz with the Bloch ansatz for the lattice potential using the Bogoliubov–de Gennes (BdG) equations [20]. As given by the Landau criterion above, the interesting regime is the intermediate strong-coupling one. Note that even in the strong-coupling limit, the algebra of the BCS theory can be applied to all coupling strengths [9,21] together with an extra definition for the chemical potential which in the weak-coupling limit is given just by the Fermi energy of the non-interacting gas. The BdG equations are

$$\begin{pmatrix} H(\mathbf{r})-\mu & \Delta(\mathbf{r}) \\ \Delta(\mathbf{r})^* & -[H(\mathbf{r})-\mu] \end{pmatrix}\begin{pmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{pmatrix} = E\begin{pmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{pmatrix}. \tag{7}$$

When the external potential is periodic, one can use the Bloch ansatz for u and v because, by self-consistency, the Hartree and pairing fields are also periodic. We obtain

$$u_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}\tilde{u}_{\mathbf{k}}\phi_{\mathbf{k}}(\mathbf{r}), \quad v_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}\tilde{v}_{\mathbf{k}}\phi_{\mathbf{k}}(\mathbf{r}) \tag{8}$$

$$\Delta(\mathbf{r}) = \sum_{\mathbf{k}} |g|[1-2f(E_{\mathbf{k}})]u_{\mathbf{k}}(\mathbf{r})v_{\mathbf{k}}^*(\mathbf{r}), \tag{9}$$

where $\phi_{\mathbf{k}}$ are the fully periodic part of the Bloch functions, such that $[H(\mathbf{r})-\mu]\phi_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}}=\xi_{\mathbf{k}}\phi_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}}$.

To describe Bloch oscillations we impose the adiabatic condition, that is, momenta evolve according to $\mathbf{k}\to\mathbf{k}+\mathbf{F}t/\hbar\equiv\mathbf{k}+\mathbf{q}$, i.e., we consider BCS state with a drift (again only in x direction). The solutions of the BdG equations take the form

$$u_{\mathbf{k}}^{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}e^{i\mathbf{q}\cdot\mathbf{r}}\tilde{u}_{\mathbf{k}}^{\mathbf{q}}\phi_{\mathbf{k}+\mathbf{q}}(\mathbf{r}); \quad v_{\mathbf{k}}^{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}e^{-i\mathbf{q}\cdot\mathbf{r}}\tilde{v}_{\mathbf{k}}^{\mathbf{q}}\phi_{\mathbf{k}-\mathbf{q}}(\mathbf{r}),$$
$$\Delta^{\mathbf{q}}(\mathbf{r}) = \sum_{\mathbf{k}} |g|[1-2f(E_{\mathbf{k}}^{+\mathbf{q}})]u_{\mathbf{k}}^{\mathbf{q}}(\mathbf{r})v_{\mathbf{k}}^{\mathbf{q}*}(\mathbf{r}), \tag{10}$$

$$E_{\mathbf{k}}^{\mathbf{q}} = (\xi_{\mathbf{k}+\mathbf{q}}-\xi_{\mathbf{q}-\mathbf{k}})/2 \pm \sqrt{(\xi_{\mathbf{k}+\mathbf{q}}+\xi_{\mathbf{q}-\mathbf{k}})^2/4+|\Delta^{\mathbf{q}}|^2}$$
$$\equiv E_D \pm \sqrt{E_A^2+|\Delta^{\mathbf{q}}|^2},$$

where $2E_D$ is the energy difference and E_A the average energy. The $\pm$ holds for the particle and hole branch, respectively, and the particle branch eigenfunctions are $|\tilde{u}_{\mathbf{k}}^{\mathbf{q}}|^2, |\tilde{v}_{\mathbf{k}}^{\mathbf{q}}|^2=(1\pm E_A/\sqrt{E_A^2+|\Delta^{\mathbf{q}}|^2})/2$. The Hamiltonian transformed under the Bogoliubov transformation leading to Eq. (7) has to be positive definite. This means that one should use the solutions for which $E_{\mathbf{k}}^{+\mathbf{q}}>0$, i.e., $\min(\sqrt{E_A^2+|\Delta^{\mathbf{q}}|^2})=|\Delta^{\mathbf{q}}|>|E_D(\mathbf{k}')|$, where $\mathbf{k}'$ minimizes E_A^2. Remarkably, this condition is closely related to the Landau criterion $|\Delta|>E_D(\mathbf{k_F})$.

In the BCS ansatz, a common momentum $\mathbf{q}$ can be added to all particles, leading to correlations of the type $\langle c_{\mathbf{k}+\mathbf{q}}^\dagger c_{-\mathbf{k}+\mathbf{q}}^\dagger\rangle$. The momentum per pair becomes $2\mathbf{q}$. One can formally calculate this obvious result also by using the plane-wave ansatz $u_{\mathbf{k}}=|u_{\mathbf{k}}|e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}}$, $v_{\mathbf{k}}=|v_{\mathbf{k}}|e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}}$ [20] [Eq. (10) with $\phi=1$] and introducing an (unnormalized) order-parameter wave function $\Delta^{\mathbf{q}}(\mathbf{r})=e^{i2\mathbf{q}\cdot\mathbf{r}}C$, where C is given by Eq. (10) to be a constant in $\mathbf{r}$. Expectation values such as momentum ($\mathbf{p}=-i\partial/\partial\mathbf{r}$) can be calculated: $\langle\mathbf{p}\rangle=\langle\Delta^{\mathbf{q}}(\mathbf{r})|-i\partial/\partial\mathbf{r}|\Delta^{\mathbf{q}}(\mathbf{r})\rangle/\langle\Delta^{\mathbf{q}}(\mathbf{r})|\Delta^{\mathbf{q}}(\mathbf{r})\rangle=2\mathbf{q}$. The order-parameter wave function is defined in the spirit of (but not with a one-to-one correspondence to) the Ginzburg-Landau theory with a space-dependent wave function whose absolute value equals the gap. In case of Fermionic atoms the Ginzburg-Landau approach has been used to describe harmonic confinement [22] and vortices [23]. For the periodic potential we introduce the order-parameter wave function in the form $\Delta^{\mathbf{q}}(\mathbf{r})=\sum_{\mathbf{k}}\Delta_{\mathbf{k}}^{\mathbf{q}}(\mathbf{r})$, where using Eq. (10),

$$\Delta_{\mathbf{k}}^{\mathbf{q}}(\mathbf{r}) = F(\mathbf{k},\mathbf{q})\,\phi_{\mathbf{k}+\mathbf{q}}e^{i(\mathbf{k}+\mathbf{q})\mathbf{r}}\phi_{\mathbf{k}-\mathbf{q}}^\dagger e^{-i(\mathbf{k}-\mathbf{q})\mathbf{r}} \tag{11}$$

and $F(\mathbf{k},\mathbf{q})=|g|[1-2f(E_{\mathbf{k}}^{+\mathbf{q}})]\tilde{u}_{\mathbf{k}}^{\mathbf{q}}\tilde{v}_{\mathbf{k}}^{\mathbf{q}*}$. We calculate the superfluid velocity using $\langle\mathbf{v}_S\rangle=N\langle\Delta^{\mathbf{q}}(\mathbf{r})|\dot{\mathbf{r}}|\Delta^{\mathbf{q}}(\mathbf{r})\rangle$, where $\mathcal{N}=\langle\Delta^{\mathbf{q}}(\mathbf{r})|\Delta^{\mathbf{q}}(\mathbf{r})\rangle^{-1}$. Using $\langle\dot{\mathbf{r}}\rangle_{\phi_{\mathbf{k}-\mathbf{q}}^\dagger e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}}}=-(1/\hbar)(d/d\mathbf{k})\xi_{\mathbf{k}-\mathbf{q}}$ and the tight-binding energy dispersion relation the superfluid velocity becomes

$$\langle v_{xS}\rangle = \mathcal{N}\sum_{\mathbf{k}} |F(\mathbf{k},\mathbf{q})|^2 \frac{Ja}{\hbar}\cos k_x a \sin qa$$
$$= \frac{Ja}{\hbar}\sin(qa)\mathcal{N}\sum_{\mathbf{k}} \left| \frac{[1-2f(E_{\mathbf{k}}^{\mathbf{q}})]\Delta^{\mathbf{q}}}{\sqrt{E_A^2+|\Delta^{\mathbf{q}}|^2}} \right|^2 \cos k_x a. \quad (12)$$

This shows that also the superfluid oscillates [the term $\sin(qa)$, $q=Ft/\hbar$] with the same frequency as the normal-state gas, but with a different amplitude. The superfluid oscillation amplitude for selected parameters is shown in Fig. 1. We have also calculated the thermal quasiparticle contribution but it turns out to be negligible for half filling.

In the limit of large V_0, the interacting fermions form composite bosons, and one could describe the center-of-mass movement of the composite particle by defining $J^*=J(m \to 2m)$. In order to give a simple estimate for the effect of the Fermi statistics, we interpret $|F(\mathbf{k},\mathbf{q})|^2 \sim |F(\mathbf{k})|^2$ in Eq. (12) as reflecting the internal wave function of the pair in the composite boson limit, cf. Refs. [8,9]. The average velocity for the bosons becomes $\langle v_{xB}\rangle \propto (J^*a/\hbar)\sin qa \Sigma_{\mathbf{k}}|F(\mathbf{k})|^2 \cos k_x a$. If the pairs were extremely strongly bound, the internal wave function in real space is a δ-function, corresponding to a constant in k-space. This means $\langle v_B\rangle=0$ since the cosine integration in Eq. (12) would extend to the whole k-space with equal weight, i.e., there are no empty states in the Brillouin zone as required for Bloch oscillations. For on-site pairs, we use $|F(r)|^2 \propto \exp(-r^2/l^2)$ leading to $|F(k)|^2 \propto \mathcal{N}\exp(-l^2k^2/4)$, therefore the suppression factor for the Bloch oscillations becomes $S \sim \mathcal{N}\int dk \exp(-l^2k^2/4)\cos ka$, where l is the pair size. As a rough estimate for the average velocity we thus obtain

$$\langle v_{xB}\rangle \sim S\frac{J^*a}{\hbar}\sin\left(\frac{F_x ta}{\hbar}\right). \quad (13)$$

This is shown in Fig. 1 for pair sizes $l=a/3$ and $l=a/4$. It gives an order-of-magnitude estimate, approaching the results given by the BCS algebra.

In summary, we have considered Bloch oscillations of fermionic atoms in optical lattices for the BCS-BEC crossover regime. Oscillations in the velocity of the atoms could be observed as in the experiments for bosons [2,4]. We show that the amplitude of the oscillations decreases when the crossover is scanned, in general, due to the shrinking of the bandwidth. However, the change from the normal- to the superfluid-state description leads to a drastic change in the amplitude. This is due to smoothening of the Fermi edge by pairing. Bloch oscillations could be used for exploring pairing correlations since any localization in space (pair size) leads to broadening in momentum which suppresses the amplitude in the same way as band filling in the noninteracting gas. Even at $T \gg T_c$, the effect of collisions on Bloch oscillations can be studied producing information useful for applications of Bloch oscillations such as production of terahertz radiation [24,18]. Observation of oscillating fermionic atoms in optical lattices would contribute to the quest for a steadily driven fermionic Bloch oscillator.

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