Engineering squeezed states in high-Q cavities

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While it has been possible to build fields in high-Q cavities with a high degree of squeezing for some years, the engineering of arbitrary squeezed states in these cavities has only recently been addressed [Phys. Rev. A **68**, 061801(R) (2003)]. The present work examines the question of how to squeeze any given cavity-field state and, particularly, how to generate the squeezed displaced number state and the squeezed macroscopic quantum superposition in a a high-Q cavity.

Statistical properties of squeezed states of light have been widely investigated and the possibility of applying squeezing properties to the understanding of fundamental physical phenomena, as well as to solving technological problems, has been recognized [1]. As far as fundamental phenomena are concerned, the antibunching or sub-Poissonian photon statistic related to squeezed states has revealed unequivocal features of the quantum nature of light [2]. In addition, squeezed-state entanglements were recently employed for experimental demonstration of quantum teleportation of optical coherent states [3]. In technology, an improvement of the signal-to-noise ratio in optical communication has been proposed by reducing the quantum fluctuations in one quadrature component of the field at the expense of the amplified fluctuations in another component [4]. Moreover, the possibility of using the quadrature component with reduced quantum noise of a squeezed state as a pointer for the measurement of weak signals has been suggested for the detection of gravitational waves as well as for sensitive interferometric and spectroscopic measurements [5].

Although squeezed light is mainly supplied by nonlinear optical media as running waves, through backward [6] or forward [7] four-wave mixing and parametric down-conversion [8], the dynamics of the Jaynes-Cummings model (JCM) of atom-field interaction leads to standing squeezed states of the electromagnetic field in cavity quantum electrodynamics (QED) or the motional degree of freedom in ion traps [9]. Whereas cavity-field squeezing in the JCM is rather modest, about 20% for low average photon number, squeezing of up to 75% can be obtained with selective atomic measurements [10]. However, such squeezed states (and those obtained in the schemes employing atom-field interactions [11]) have not resulted from the unitary evolution $S(\xi)|\Psi\rangle$; in other words, the experimenter is not able to squeeze any desired state $|\Psi\rangle$ previously prepared in the cavity [$S(\xi)$ stands for the squeeze operator and ξ for a set of group parameters]. In the present proposal we consider exactly the question of how to squeeze any given cavity-field state $|\Psi\rangle$ and, in particular, how to generate (i) a squeezed displaced number state (SDNS) and (ii) a squeezed Schrödinger-cat-like state (SSCS) in a high-Q cavity.

The SDNS $|\xi;\alpha;n\rangle$ is obtained by the action of the displacement operator $D(\alpha)=\exp\left[\frac{1}{2}(\alpha^* a-\alpha a^\dagger)\right]$, followed by the squeeze operator $S(\xi)=\exp\left[\frac{1}{2}(\xi^* a^2-\xi a^{\dagger 2})\right]$, on the number state $|\xi;\alpha;n\rangle\equiv S(\xi)D(\alpha)|n\rangle$. It is readily seen that the SDNS contains various special cases such as the number state ($\xi=\alpha=0$), coherent state ($\xi=n=0$), squeezed number state ($\alpha=0$), displaced number state ($\xi=0$), and so on. Therefore, the SDNS allows a unified approach incorporating all these states, and their properties. Although the statistical properties of the SDNS are well known [12], the generation of SDNS in a cavity has not been reported yet. Recently, we showed how to achieve an effective quadratic Hamiltonian leading to the parametric frequency conversion process in cavity QED [13], opening the way for generation of cavity SDNS.

To avoid experimental complications stemming from introducing a nonlinear crystal inside a cavity, the squeeze operator is built from the dispersive interaction of the cavity mode with a driven three-level atom [13]. As sketched in Fig. 1, the atomic system is in the ladder configuration, where an intermediate atomic level ($|i\rangle$) lies between the ground ($|g\rangle$) and excited ($|e\rangle$) states. The quantized cavity mode of frequency ω couples dispersively both transitions $|g\rangle\leftrightarrow|i\rangle$ and $|e\rangle\leftrightarrow|i\rangle$, with coupling constants λ_g and λ_e, respectively, and detuning $\delta=|\omega-\omega_{\ell i}|(\ell=g,e)$. A classical field of frequency $\omega_0=2\omega+\Delta$ drives the atomic transition $|g\rangle\leftrightarrow|e\rangle$ dispersively, with coupling constant Ω. The transition $|g\rangle\leftrightarrow|e\rangle$ may be induced by applying a sufficiently strong electric field. While the quantum field promotes a two-photon interchange process, the classical driving field constitutes the source of the parametric amplification.

The Hamiltonian of our model, under the rotating wave approximation, is given by $H=H_0+V$, where

$$H_0=\hbar\omega a^\dagger a-\hbar\omega|g\rangle\langle g|+\hbar\delta|i\rangle\langle i|+\hbar\omega|e\rangle\langle e|, \qquad (1a)$$

$$V=\hbar(\lambda_g a|i\rangle\langle g|+\text{H.c.})+\hbar(\lambda_e a|e\rangle\langle i|+\text{H.c.})+\hbar(\Omega|e\rangle\langle g|e^{-i\omega_0 t}$$
$$+\text{H.c.}), \qquad (1b)$$

with $a^\dagger(a)$ standing for the creation (annihilation) operator of
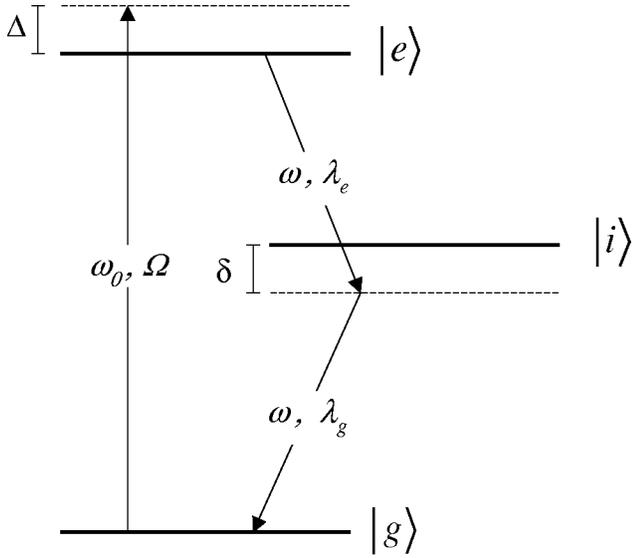
FIG. 1

FIG. 1. Energy-level diagram of the three-level atom for the parametric amplification scheme.

the quantized cavity mode. Writing H in the interaction picture [through the unitary transformation $U_0 = \exp(-iH_0 t/\hbar)$] and then applying the transformation $U = \exp[-i\delta t(|g\rangle\langle g| + |e\rangle\langle e|)]$, we obtain the Hamiltonian $\mathcal{H} = U_0^\dagger U^\dagger H U U_0 - H_0 - \hbar\delta(|g\rangle\langle g| + |e\rangle\langle e|)$. If the dispersive transitions are sufficiently detuned, i.e., $\delta \gg |\lambda_g|, |\lambda_e|, |\Omega|$, we obtain the adiabatic solutions for the transition operators σ_{ig} and σ_{ie} ($\sigma_{kl} \equiv |k\rangle\langle l|, k, l = g, i, e$) by setting $d\sigma_{ig}/dt = d\sigma_{ei}/dt = 0$; solving the resulting system, and inserting these adiabatic solutions for σ_{ig} and σ_{ei} into $\mathcal{H}$ (for more details, see [13]), the Hamiltonian becomes

$$
\begin{aligned}
\mathcal{H} \approx & -\hbar\delta(\sigma_{gg} + \sigma_{ee}) + \hbar(\Omega e^{-i\Delta t}\sigma_{eg} + \text{H.c.}) - \frac{\hbar}{\delta}\bigg\{(2a^\dagger a + 1) \\
& \times \bigg[|\lambda_g|^2\sigma_{gg} - (|\lambda_g|^2 + |\lambda_e|^2)\sigma_{ii} + |\lambda_e|^2\sigma_{ee} \\
& + \frac{|\lambda_g|^2 + |\lambda_e|^2}{2\delta}(\Omega e^{-i\Delta t}\sigma_{eg} + \text{H.c.})\bigg]\bigg\} - \frac{\hbar}{\delta}\bigg\{2(\lambda_g\lambda_e a^2\sigma_{eg} \\
& + \text{H.c.}) + \frac{1}{\delta}(\lambda_g\lambda_e\Omega^* e^{i\Delta t}a^2 + \text{H.c.})(\sigma_{gg} + \sigma_{ee} - 2\sigma_{ii})\bigg\}.
\end{aligned}
\tag{2}
$$

The state vector associated with the Hamiltonian (2) can be written using

$$
|\Psi(t)\rangle = |g\rangle|\Phi_g(t)\rangle + |i\rangle|\Phi_i(t)\rangle + |e\rangle|\Phi_e(t)\rangle,
\tag{3}
$$

where $|\Phi_\ell(t)\rangle = \int (d^2\alpha/\pi)\mathcal{A}_\ell(\alpha.t)|\alpha\rangle$ for $\ell = g, i, e$, the complex quantity α standing for the eigenvalues of a, and

$\mathcal{A}_\ell(\alpha, t) = \langle\alpha, \ell|\Psi(t)\rangle$ represents the set of expansion coefficients for $|\Phi_\ell(t)\rangle$ in the basis of coherent states $\{|\alpha\rangle\}$. Using the orthogonality of the atomic states and Eqs. (2) and (3), we obtain the uncoupled time-dependent (TD) Schrödinger equation for the atomic subspace $|i\rangle$ (in the Schrödinger picture):

$$
i\hbar\frac{d}{dt}|\Phi_i(t)\rangle = \mathcal{H}_i|\Phi_i(t)\rangle,
\tag{4}
$$

$$
\mathcal{H}_i = \hbar\varpi a^\dagger a + \hbar(\xi e^{-i\upsilon t}a^{\dagger 2} + \xi^* e^{i\upsilon t}a^2)
\tag{5}
$$

where $\varpi = \omega + \chi[\chi = 2(|\lambda_g|^2 + |\lambda_e|^2)/\delta]$ stands for the effective frequency of the cavity mode, while $\xi = 2\Omega\lambda_g^*\lambda_e^*/\delta^2 = |\xi|e^{-\Theta}$ and $\upsilon = 2\omega + \Delta$ are the effective amplitude and frequency of the parametric amplification field. For substance $\{|g\rangle, |e\rangle\}$ there is a TD Schrödinger equation which couples the fundamental and excited atomic states. Therefore, when we initially prepare the atom in the intermediate level $|i\rangle$, the dynamics of the atom-field dispersive interactions, governed by the effective Hamiltonian (5), results in a cavity mode with shifted frequency submitted to a parametric amplification process.

For the present purpose we consider the resonant regime, where the classical driving field has the same frequency ϖ as the effective cavity mode, so that $\upsilon = 2\varpi$ (i.e., $\Delta = 2\chi$). (A treatment of the off-resonant interaction between the effective cavity mode and the driving field was investigated in Ref. [13].) The evolution of the cavity-field state, in the interaction picture, is governed by a squeeze operator such as $|\Phi_i(t)\rangle = S(\xi, t)|\Phi_i(t_0)\rangle$, where

$$
S(\xi, t) = \exp[-i(\xi a^{\dagger 2} + \xi^* a^2)t].
\tag{6}
$$

The degree of squeezing in the resonant regime is determined by the factor $r(t) = 2|\xi|t$, while the squeeze angle is given by $\varphi = \pi/2 - \Theta$. For a specific cavity mode and atomic system, the parameter $r(t)$ can be adjusted in accordance with the coupling strength Ω and the interaction time t.

With this squeeze operator, in hand, we are able to show how to engineer the two specific squeezed states already mentioned: (i) the SDNS $|\xi: \alpha: n\rangle \equiv S(\xi)D(\alpha)|n\rangle$ and (ii) the SSCS $S(\xi)[\mathcal{N}(|\alpha\rangle + e^{i\phi}|-\alpha\rangle)]$ ($\mathcal{N}$ being the normalization factor).

Starting with the SDNS, the first step is to prepare the cavity field in the Fock state $|n\rangle$, which can in principle be done by any of the proposals in Ref. [14]. However, we observe that multiple-of-2 number states $|n = 2m\rangle$ ($m = 1, 2, \ldots$) can be generated as a by-product of the present scheme with the driving field switched off. In fact, considering $\Omega = 0$ and disregarding state $|i\rangle$, the Hamiltonian (2) becomes

$$\tilde{\mathcal{H}} = -\hbar\left[\left(\delta + \frac{|\lambda_g|^2}{\delta}\right) + \frac{2|\lambda_g|^2}{\delta}a^\dagger a\right]\sigma_{gg}$$

$$-\hbar\left[\left(\delta + \frac{|\lambda_e|^2}{\delta}\right) + \frac{2|\lambda_e|^2}{\delta}a^\dagger a\right]\sigma_{ee}$$

$$+2\frac{\hbar}{\delta}(\lambda_g\lambda_e a^2\sigma_{eg} + \text{H.c.}). \qquad (7)$$

This Hamiltonian allows the transition $|n,e\rangle \leftrightarrow |n+2,g\rangle$. Assuming that the atom is prepared in the state $|e\rangle$, we obtain the following evolution:

$$e^{-i\tilde{\mathcal{H}}t/\hbar}|n,e\rangle = |\Upsilon|^2\left[\frac{e^{-it\eta_+}}{|\Upsilon|^2 + (\eta_+ - \Lambda)^2} + \frac{e^{-it\eta_-}}{|\Upsilon|^2 + (\eta_- - \Lambda)^2}\right]|n,e\rangle$$

$$+\Upsilon^*\left[\frac{(\eta_+ - \Lambda)^2 e^{-it\eta_+}}{|\Upsilon|^2 + (\eta_+ - \Lambda)^2} + \frac{(\eta_- - \Lambda)^2 e^{-it\eta_-}}{|\Upsilon|^2 + (\eta_- - \Lambda)^2}\right]$$

$$\times|n+2,g\rangle, \qquad (8)$$

where

$$\eta_\pm = \frac{1}{2}(\Lambda + \Xi \pm \sqrt{(\Lambda - \Xi)^2 + 4\Upsilon^2}), \qquad (9a)$$

$$\Lambda = \delta + \frac{|\lambda_e|^2}{\delta} + \frac{2|\lambda_e|^2}{\delta}n, \qquad (9b)$$

$$\Xi = \delta + \frac{|\lambda_g|^2}{\delta} + \frac{2|\lambda_g|^2}{\delta}(n+2), \qquad (9c)$$

$$\Upsilon = \frac{2\lambda_g\lambda_e}{\delta}\sqrt{(n+2)(n+1)}. \qquad (9d)$$

If the initial cavity state is $|n\rangle$, the probability of detecting the atomic level $|g\rangle$ is given by $P_{g,n}(t) = \|\langle g|e^{-i\tilde{\mathcal{H}}t/\hbar}|n,e\rangle\|^2$. In order to prepare the state $|n+2\rangle$, the atom-field interaction time must be adjusted to maximize $P_{g,n}$, that is, $t = \pi(\eta_+ - \eta_-)^{-1}$, and the success probability is given by

$$P_{g,n} = |\Upsilon|^2\left\{\left(\frac{(\eta_+ - \Lambda)^2}{|\Upsilon|^2 + (\eta_+ - \Lambda)^2}\right)^2 + \left(\frac{(\eta_- - \Lambda)^2}{|\Upsilon|^2 + (\eta_- - \Lambda)^2}\right)^2\right.$$

$$\left. - \frac{(\eta_- - \Lambda)^2(\eta_+ - \Lambda)^2}{[|\Upsilon|^2 + (\eta_- - \Lambda)^2][|\Upsilon|^2 + (\eta_+ - \Lambda)^2]}\right\}.$$

Therefore, starting with an empty cavity and passing a stream of m three-level atoms through it with an adequately adjusted interaction time for each atom $t_k = \pi(\eta_+ - \eta_-)^{-1}$ (where the subscript k indicates the kth atom), we have a probabilistic technique for building multiple-of-2 number states $|n=2m\rangle$. Each atom is supposed to be detected in the state $|g\rangle$ before the subsequent atom enters the cavity.

To illustrate this technique, we will consider typical parameter values which follow from Rydberg states where the intermediate state $|i\rangle$ [an $(n-1)P_{3/2}$ level] is nearly halfway between $|g\rangle$ [an $(n-1)S_{1/2}$ level] and $|e\rangle$ (an $nS_{1/2}$ level), namely, $|\lambda_g| \sim |\lambda_e| \sim 7 \times 10^5$ s^{-1} [18], and we will assume the detuning $|\delta| \sim 1 \times 10^7$ s^{-1}. We show in Table I the interaction

TABLE I. Interaction time (t_k) and the probability ($p_{g,k}$) of detecting the state $|g\rangle$ to engineer the number state $|n=2m\rangle$ when m atoms are passed through the cavity. The total probability of success in building the state $|n=2m\rangle$ is $\mathcal{P} = \Pi_{k=1}^m P_{g,k}$.

m	n	$t_k(10^{-5}$ s$)$	$P_{g,m}$
1	2	0.9254	0.6
2	4	0.4446	0.8
3	6	0.2879	0.9

time and the probability of successfully building the states $|2\rangle, |4\rangle$, and $|6\rangle$ by passing one, two, and three atoms, respectively, through the cavity.

After, preparing the initial number state, the displacement operator is implemented by connecting a microwave source to the cavity [15]. The prepared cavity field $|n\rangle$ is displaced when the microwave source is turning on, the amount of displacement being adjusted by varying the time interval of injection of the classical microwave field. Finally, a driven three-level atom (in this step $\Omega \neq 0$) prepared in the intermediate state $|i\rangle$ is sent through the cavity to accomplish the squeezing operation. Particular cases of the SDNS, such as the squeezed vacuum $|\xi;0;0\rangle \equiv S(\xi)|0\rangle$ or the squeezed coherent state (SCS) $|\xi;\alpha;0\rangle \equiv S(\xi)|\alpha\rangle$ are easily engineered by sending just one driven three-level atom through a cavity initially prepared in the vacuum or the coherent state, respectively. The degree of squeezing will be discussed further.

As a second application, we consider the squeezed Schrödinger-cat-like state. The Schrödinger-cat-like state $|\Psi\rangle$ is easily generated by sending a two-level atom across a cavity (initially in a prepared coherent state $|\beta\rangle$) sandwiched by two Ramsey zones, as reported in [16]. Properly adjusting the atom-field interaction in the Ramsey zones and in the cavity, it is possible to obtain the state [16]

$$|\Psi^\pm\rangle = \mathcal{N}_\pm(c_g|\alpha\rangle \pm c_e|-\alpha\rangle), \qquad (10)$$

where $\alpha = i\beta, \mathcal{N}_\pm$ is the normalization factor, and the $+(-)$ sign occurs if the atom is detected in state $|g\rangle(|e\rangle)$. Finally, following the scheme proposed here, the SSCS is achieved by sending a driven three-level atom through the cavity.

Let us discuss briefly the degree of squeezing achievable with the present proposal, focusing our attention on the SCS and on the squeezed number state. For an initial coherent state prepared in the cavity, the variance of the squeezed quadrature is given by $\Delta X = e^{-2r}/4$ [17]. Assuming typical parameter values (cited above) and the coupling strength $\Omega \sim 7 \times 10^5$ s^{-1}, we obtain $|\xi| \sim 6.8 \times 10^3$ s^{-1}. For an atom-field interaction time about $t \sim 10^{-4}$ s (which is one order of magnitude smaller than the usual decay time of the open cavities used in experiments [19]) we get the squeezing factor $r(t) \sim 1.36$, resulting in a variance in the squeezed quadrature of $\Delta X \sim 1.6 \times 10^{-2}$ for the SCS. This represents a high degree of squeezing, around 93 % with the passage of just one driven three-level atom. For the squeezed number state— considering the above parameters and an initial number state $n=2$, which can be generated with the passage of just one atom through the cavity, as described above—we obtain a

variance in the squeezed quadrature $\Delta X \sim 8 \times 10^{-2}$, representing a degree of squeezing around 67 %.

We note that for weakly damped systems, such as fields trapped in realistic high-Q cavities, the lifetime of the squeezing is of the order of the relaxation time of the cavity, a result which is valid even at absolute zero [20]. Therefore, the dissipative mechanism of the cavity plays a much milder role in the lifetime of the squeezing than in decoherence phenomena [21]. Regarding atomic decay, note that for Rydberg levels the damping effects can be safely neglected on typical interaction time scales. A straightforward estimate of the fidelity of the prepared states under the damping effects can be made through the phenomenological operator technique, as described in [22].

In conclusion, we have shown how to engineer some squeezed states of the radiation field in cavity QED, based on the interaction of the field with a driven three-level atom. Particular states, such as the squeezed vacuum and the squeezed coherent state, are easily engineered by sending just on atom through the cavity, making our proposal attractive for experimental implementation. To build the SDNS, an intermediate step is needed to prepare the number state, as described previously, which clearly makes the SDNS less attractive for experimental implementation. The SSCS is accomplished be sending two atoms through the cavity, the first (a two-level atom interacting dispersively with the cavity mode) to prepare the Schrödinger-cat state, as in Ref. [16] and the second (as shown above) to execute the squeezing operation. Finally, we would like to underline that up to 93 % degree of squeezing of a field state, initially prepared in the coherent state, may be achieved by passing a single three-level atom through the cavity. This high degree of squeezing is crucial to the building of truly mesoscopic superpositions with a large average photon number and also a large "distance" in phase space between the centers of the quasiprobability distribution of the individual states composing the prepared superposition [23].

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