High-energy expansion of Coulomb corrections to the e^+e^- photoproduction cross section

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The correction of order $1/\omega$ to the high-energy total e^+e^- photoproduction cross section in the electric field of a heavy atom is derived with the exact account of this field; ω is the photon energy. The consideration is based on the use of the quasiclassical electron Green's function in an external electric field. The next-to-leading correction to the cross section is discussed. The influence of screening on the Coulomb corrections is examined in the leading approximation. It turns out that the corresponding correction to the high-energy total cross section is ω independent. In the region where both produced particles are relativistic, the corrections to the high-energy asymptotics of the electron (positron) spectrum are derived. Our results for the total cross section are in good agreement with experimental data for photon energies down to a few MeV. In addition, the corrections to the bremsstrahlung spectrum are obtained from the corresponding results for pair production.

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I. INTRODUCTION

Knowledge of the photoabsorption cross sections is very important in various applications, see, e.g., Ref. [1]. The relevant processes are the atomic photoeffect, nuclear photoabsorption, incoherent and coherent photon scattering, and e^+e^- pair production. In the coherent processes, by definition, there is no excitation or ionization of an atom. The high-accuracy estimation of the corresponding cross sections is required. They have different dependence on the photon energy ω. At $\omega \gtrsim 10$ MeV, the cross section of e^+e^- pair production becomes dominant [2]. The coherent contribution σ_{coh} to the pair production cross section is roughly Z times larger than the incoherent one (Z is the atomic number), thereby being the most important for heavy atoms. Just the coherent pair production is considered below.

The theoretical and experimental investigation of the coherent pair production has a long history, see Ref. [2]. In the Born approximation, the cross section σ_B is known for arbitrary photon energy [3,4]. The account of the effect of screening is straightforward in this approximation and can be easily performed if the atomic form factor is known [5]. For heavy atoms it is necessary to take into account the Coulomb corrections σ_C,

$$\sigma_{coh} = \sigma_B + \sigma_C . \tag{1}$$

These corrections are higher-order terms of the perturbation theory with respect to the atomic field. The magnitude of σ_C depends on ω and the parameter $Z\alpha$ ($\alpha = 1/137$ is the fine-structure constant). The formal expression for σ_C, exact in $Z\alpha$ and ω, was derived by Øverbø *et al.* [6]. This expression has a very complicated form causing severe difficulties in computations. The difficulties grow as ω increases, so that numerical results in Ref. [6] were obtained only for $\omega < 5$ MeV.

In the high-energy region $\omega \gg m$ (m is the electron mass, $\hbar = c = 1$), the consideration is greatly simplified. As a result, a rather simple form was obtained in Ref. [7,8] for the Coulomb corrections in the leading approximation with respect to m/ω. However, the theoretical description of the Coulomb corrections at intermediate photon energies (5–100 MeV) has not been completed. At present, all estimates of σ_C in this region are based on the "bridging" expression derived by Øverbø [9]. This expression is actually an extrapolation of the results obtained for $\omega < 5$ MeV. It is based on some assumptions on the form of the asymptotic expansion of σ_C at high photon energy. It is commonly believed that the bridging expression has an accuracy providing the maximum error in σ_{coh} of the order of a few tens of percent.

Here we develop a description of e^+e^- pair production at intermediate photon energies by deriving the next-to-leading term of the high-energy expansion of σ_C. First we consider a pure Coulomb field and represent σ_C in the form

$$\sigma_C = \sigma_C^{(0)} + \sigma_C^{(1)} + \sigma_C^{(2)} + \cdots . \tag{2}$$

The term $\sigma_C^{(n)}$ has the form $(m/\omega)^n S^{(n)}(\ln \omega/m)$, where $S^{(n)}(x)$ is some polynomial. The ω independent term $\sigma_C^{(0)}$ corresponds to the result of Davies *et al.* [8]. In the present paper we derive the term $\sigma_C^{(1)}$. It turns out that $S^{(1)}$ is ω independent in contrast to a second-degree polynomial suggested by Øverbø [9]. We present an ansatz for $\sigma_C^{(2)}$, which provides a good agreement with available experimental data for $\omega > 5$ MeV.

The high-energy expansion of the Coulomb corrections to the spectrum has the form similar to Eq. (2). In the region $\varepsilon_\pm \gg m$, we derive the term $d\sigma_C^{(1)}/dx$, where ε_- and ε_+ are the electron and positron energy, respectively, $x = \varepsilon_-/\omega$. The term $d\sigma_C^{(1)}/dx$ may be important, e.g., for description of the development of electromagnetic showers in a medium. The correction found is antisymmetric with respect to the permutation $\varepsilon_+ \leftrightarrow \varepsilon_-$ and does not contribute to the total cross section. In fact, $\sigma_C^{(1)}$ originates from two energy regions $\varepsilon_+ \sim m$ and $\varepsilon_- \sim m$, where the spectrum is not known. However, our result for $\sigma_C^{(1)}$ allows us to claim that the spectrum

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in these regions differs drastically from the result obtained by Davies *et al.* [8] for $\varepsilon_\pm \gg m$, if the latter is formally applied at $\varepsilon_- \sim m$ or $\varepsilon_+ \sim m$.

The effect of screening on σ_C at $\omega \gg m$ is considered quantitatively. In the leading approximation, we find the corresponding correction $\sigma_C^{(scr)}$, which is ω independent similar to $\sigma_C^{(0)}$. So, for the atomic field, $\sigma_C^{(scr)}$ should be added to the right-hand side of Eq. (2). The screening correction to the spectrum is also obtained.

In this paper we present the explicit calculations of the corrections, which have been given without derivation in our recent work [10] and used for the detailed comparison of theory with experimental data.

II. GENERAL DISCUSSION

The cross section of e^+e^- pair production by a photon in an external field reads

$$d\sigma_{coh} = \frac{\alpha}{(2\pi)^4\omega} d\boldsymbol{p}\, d\boldsymbol{q}\, \delta(\omega - \varepsilon_+ - \varepsilon_-)|M|^2, \tag{3}$$

where $\varepsilon_- = \varepsilon_p = \sqrt{\boldsymbol{p}^2 + m^2}$, $\varepsilon_+ = \varepsilon_q$, and $\boldsymbol{p}$, $\boldsymbol{q}$ are the electron and positron momenta, respectively. The matrix element M has the form

$$M = \int d\boldsymbol{r}\, \overline{\psi}_{\boldsymbol{p}}^{(+)}(\boldsymbol{r}) \hat{e} \psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r}) \exp(i\boldsymbol{k}\cdot\boldsymbol{r}). \tag{4}$$

Here $\psi_{\boldsymbol{p}}^{(+)}$ and $\psi_{\boldsymbol{q}}^{(-)}$ are positive-energy and negative-energy solutions of the Dirac equation in the external field, e_μ is the photon polarization four-vector, $\boldsymbol{k}$ is the photon momentum, $\hat{e} = e_\mu \gamma^\mu$, γ^μ are the Dirac matrices. It is convenient to study various processes in external fields using the Green's function $G(\boldsymbol{r}_2, \boldsymbol{r}_1|\varepsilon)$ of the Dirac equation in this field. This Green's function can be represented in the form

$$G(\boldsymbol{r}_2,\boldsymbol{r}_1|\,\varepsilon) = \sum_n \frac{\psi_n^{(+)}(\boldsymbol{r}_2)\overline{\psi}_n^{(+)}(\boldsymbol{r}_1)}{\varepsilon - \varepsilon_n + i0} + \int \frac{d\boldsymbol{p}}{(2\pi)^3}\left[\frac{\psi_{\boldsymbol{p}}^{(+)}(\boldsymbol{r}_2)\overline{\psi}_{\boldsymbol{p}}^{(+)}(\boldsymbol{r}_1)}{\varepsilon - \varepsilon_p + i0} + \frac{\psi_{\boldsymbol{p}}^{(-)}(\boldsymbol{r}_2)\overline{\psi}_{\boldsymbol{p}}^{(-)}(\boldsymbol{r}_1)}{\varepsilon + \varepsilon_p - i0}\right], \tag{5}$$

where $\psi_n^{(+)}$ is the discrete-spectrum wave function, ε_n is the corresponding binding energy. The regularization of denominators in Eq. (5) corresponds to the Feynman rule. From Eq. (5),

$$\int d\Omega_{\boldsymbol{q}} \psi_{\boldsymbol{q}}^{(-)}(\boldsymbol{r}_2)\overline{\psi}_{\boldsymbol{q}}^{(-)}(\boldsymbol{r}_1) = -i\frac{(2\pi)^2}{q\varepsilon_q}\delta G(\boldsymbol{r}_2,\boldsymbol{r}_1|-\varepsilon_{\boldsymbol{q}}),$$

$$\int d\Omega_{\boldsymbol{p}} \psi_{\boldsymbol{p}}^{(+)}(\boldsymbol{r}_1)\overline{\psi}_{\boldsymbol{p}}^{(+)}(\boldsymbol{r}_2) = i\frac{(2\pi)^2}{p\varepsilon_{\boldsymbol{p}}}\delta G(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon_p), \tag{6}$$

where $\Omega_{\boldsymbol{p}}$ is the solid angle of $\boldsymbol{p}$, and $\delta G = G - \tilde{G}$. The function $\tilde{G}$ is obtained from Eq. (5) by the replacement $i0 \leftrightarrow -i0$.

Taking the integrals over $\Omega_{\boldsymbol{p}}$ and $\Omega_{\boldsymbol{q}}$ in Eq. (3), we obtain the electron spectrum, which is the cross-section differential with respect to the electron energy ε_-. Using relations (6), we express this spectrum via the Green's functions:

$$\frac{d\sigma_{coh}}{d\varepsilon_-} = \frac{\alpha}{\omega}\int\int d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\, \mathrm{Sp}\{\delta G(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon_-)\hat{e} \times \delta G(\boldsymbol{r}_1,\boldsymbol{r}_2|-\varepsilon_+)\,\hat{e}\}, \tag{7}$$

where $\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$ and $\varepsilon_+ = \omega - \varepsilon_-$ is the positron energy. Since the spectrum is independent of the photon polarization, here and below we assume $\boldsymbol{e}^* = \boldsymbol{e}$ (linear polarization).

Due to the optical theorem, the process of pair production is related to the process of Delbrück scattering (coherent scattering of a photon in the electric field of an atom via virtual electron-positron pairs). At zero scattering angle, the amplitude M_D of Delbrück scattering reads

$$M_D = 2i\alpha\int d\varepsilon \int\int d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\, \mathrm{Sp}\{G(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)\,\hat{e} \times G(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon - \omega)\,\hat{e}\}. \tag{8}$$

It is necessary to subtract, from the integrand in Eq. (8), the value of this integrand at zero external field $(Z\alpha = 0)$. Below, such a subtraction is assumed to be made.

It follows from Eqs. (7) and (8) and the analytical properties of the Green's function that

$$\frac{1}{\omega}\mathrm{Im}\, M_D = \sigma_{coh} + \sigma_{bf}. \tag{9}$$

Here

$$\sigma_{bf} = -\frac{2i\pi\alpha}{\omega}\int\int d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\sum_n \mathrm{Sp}\{\rho_n(\boldsymbol{r}_2,\boldsymbol{r}_1)\,\hat{e} \times \delta G(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon_n - \omega)\,\hat{e}\},$$

$$\rho_n(\boldsymbol{r}_2,\boldsymbol{r}_1) = \lim_{\varepsilon\to\varepsilon_n}(\varepsilon - \varepsilon_n)G(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon). \tag{10}$$

The quantity σ_{bf} coincides with the total cross section of the so-called bound-free pair production when an electron is produced in a bound state. In fact, due to the Pauli principle, there is no bound-free pair production on neutral atoms. Nevertheless, the term σ_{bf} should be kept in the right-hand side of Eq. (9). In a Coulomb field, the total cross section σ_{bf} was obtained in Ref. [11] for $\omega \gg m$. In this limit, $\sigma_{bf} \propto 1/m\omega$ and should be taken into account when using the relation (9) for the calculation of the corrections to σ_{coh}. The main contribution to σ_{bf} comes from the low-lying bound states [11] when screening can be neglected. So, in Eq. (9) we can use σ_{bf} obtained in Ref. [11].

It is convenient to represent $d\sigma_{coh}/d\varepsilon_-$ and M_D in another form using the Green's function $D(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)$ of the squared Dirac equation,

$$G(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)=\{\gamma^0[\varepsilon-V(\boldsymbol{r}_2)]-\boldsymbol{\gamma}\cdot\boldsymbol{p}_2+m\}D(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon),$$
$$\boldsymbol{p}_2=-i\boldsymbol{\nabla}_2. \tag{11}$$

According to Ref. [12], we can rewrite Eq. (7) in the form

$$\frac{d\sigma_{coh}}{d\varepsilon_-}=\frac{\alpha}{2\omega}\int\int d\boldsymbol{r}_1 d\boldsymbol{r}_2\, e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\mathrm{Sp}\{[(2\boldsymbol{e}\cdot\boldsymbol{p}_2-\hat{e}\hat{k})\times\delta D(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon_-)][(2\boldsymbol{e}\cdot\boldsymbol{p}_1+\hat{e}\hat{k})\times\delta D(\boldsymbol{r}_1,\boldsymbol{r}_2|-\varepsilon_+)]\}, \tag{12}$$

and Eq. (8) as

$$M_D=i\alpha\int d\varepsilon\int\int d\boldsymbol{r}_1 d\boldsymbol{r}_2 e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\,\mathrm{Sp}\{[(2\boldsymbol{e}\cdot\boldsymbol{p}_2-\hat{e}\hat{k})D(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)][(2\boldsymbol{e}\cdot\boldsymbol{p}_1+\hat{e}\hat{k})D(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon-\omega)]\}+2i\alpha\int d\varepsilon\int d\boldsymbol{r}\,\mathrm{Sp}D(\boldsymbol{r},\boldsymbol{r}|\varepsilon). \tag{13}$$

The last term in Eq. (13) is ω independent, and has no imaginary part. Therefore, it does not contribute to the relation (9). The details of the transformation of Eqs. (7) and (8) to Eqs. (12) and (13) are presented in Appendix A.

III. GREEN'S FUNCTION

To obtain the spectrum (7), (12) and the Delbrück scattering amplitude (8), (13) it is necessary to know the explicit form of the Green's function of the Dirac equation in the Coulomb potential $V(r)=-Z\alpha/r$. An integral representation for $G(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)$ has been obtained in Ref. [13]. For $|\varepsilon|>m$ it has the form

$$G(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)=-\frac{i}{4\pi r_2 r_1\kappa}\int_0^\infty ds\,\exp[2iZ\alpha s\,\lambda+i\kappa(r_2+r_1)\coth s]T,$$
$$T=[1-(\boldsymbol{\gamma}\cdot\boldsymbol{n}_2)(\boldsymbol{\gamma}\cdot\boldsymbol{n}_1)]\times\left[(\gamma^0\varepsilon+m)\frac{y}{2}\partial_y S_B-iZ\alpha\gamma^0\kappa\coth s\,S_B\right]+[1+(\boldsymbol{\gamma}\cdot\boldsymbol{n}_2)(\boldsymbol{\gamma}\cdot\boldsymbol{n}_1)](\gamma^0\varepsilon+m)S_A+imZ\alpha\gamma^0\boldsymbol{\gamma}\cdot(\boldsymbol{n}_2+\boldsymbol{n}_1)S_B+\frac{i\kappa^2(r_2-r_1)}{2\sinh^2 s}\boldsymbol{\gamma}\cdot(\boldsymbol{n}_2+\boldsymbol{n}_1)S_B-\kappa\coth s\,\boldsymbol{\gamma}\cdot(\boldsymbol{n}_2-\boldsymbol{n}_1)S_A. \tag{14}$$

In this formula

$$S_A=\sum_{l=1}^{\infty}e^{-i\pi\nu}J_{2\nu}(y)\,l[P_l'(x)+P_{l-1}'(x)],$$
$$S_B=\sum_{l=1}^{\infty}e^{-i\pi\nu}J_{2\nu}(y)[P_l'(x)-P_{l-1}'(x)],$$
$$\nu=\sqrt{l^2-(Z\alpha)^2},\quad \kappa=\sqrt{\varepsilon^2-m^2},\quad \lambda=\varepsilon/\kappa,$$
$$y=2\kappa\sqrt{r_1r_2}/\sinh s,\quad x=\boldsymbol{n}_1\cdot\boldsymbol{n}_2,\quad \boldsymbol{n}_{1,2}=\boldsymbol{r}_{1,2}/r_{1,2}, \tag{15}$$

$J_{2\nu}(y)$ are Bessel functions and $P_l(x)$ are Legendre polynomials, $P_l'(x)=\partial_x P_l(x)$. The Green's function $D(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)$ can be obtained from Eq. (14) by keeping in T the terms $\propto m$:

$$D(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)=-\frac{i}{4\pi r_2 r_1\kappa}\int_0^\infty ds\,\exp[2iZ\alpha s\,\lambda+i\kappa(r_2+r_1)\times\coth s]\left\{[1-(\boldsymbol{\gamma}\cdot\boldsymbol{n}_2)(\boldsymbol{\gamma}\cdot\boldsymbol{n}_1)]\frac{y}{2}\partial_y S_B+[1+(\boldsymbol{\gamma}\cdot\boldsymbol{n}_2)(\boldsymbol{\gamma}\cdot\boldsymbol{n}_1)]S_A+iZ\alpha\gamma^0\boldsymbol{\gamma}\cdot(\boldsymbol{n}_2+\boldsymbol{n}_1)S_B\right\}. \tag{16}$$

We are going to derive the high-energy asymptotic expansion of the spectrum in the region $\varepsilon_\pm\gg m$. For the first two terms of such an expansion the main contribution to the integral in Eqs. (7) and (12) is given by the region $r=|\boldsymbol{r}_2-\boldsymbol{r}_1|\sim\omega/m^2$, see Refs. [14,15]. Let us introduce the variable $\boldsymbol{\rho}$ as the component of $\boldsymbol{r}_1$ (or $\boldsymbol{r}_2$) perpendicular to $\boldsymbol{r}$:

$$\boldsymbol{\rho}=\frac{\boldsymbol{r}\times[\boldsymbol{r}_1\times\boldsymbol{r}_2]}{r^2}. \tag{17}$$

As shown in Ref. [15], the main contribution to the Coulomb corrections to the spectrum originates from the region $\rho\sim1/m$ and $\theta,\psi\sim m/\omega\ll1$, where θ is the angle between the vectors $\boldsymbol{r}_2$ and $-\boldsymbol{r}_1$, and ψ is the angle between the vectors $\boldsymbol{r}$ and $\boldsymbol{k}$. In this region we have $\theta\approx r\rho/r_1r_2$. The argument of the Legendre polynomials in Eq. (15) is $x=\boldsymbol{n}_1\cdot\boldsymbol{n}_2\approx-1+\theta^2/2$. Besides, the term $\kappa(r_2+r_1)\sim\omega^2/m^2\gg1$ in the exponents in Eqs. (14) and (16) is large, and the integral is determined by large s. Then $\coth s\approx1+2\exp(-2s)$, and from Eq. (16) we have $\exp(-2s)\sim1/\kappa r\sim m^2/\omega^2$. The argument y of the Bessel functions in $S_{A,B}$ can be estimated as $y\sim\kappa r/\sinh s\sim\omega/m\gg1$.

A simple method of the calculation of $S_{A,B}$ at $y\gg1$, $1+x\approx\theta^2/2\ll1$, and $y\theta\sim1$ has been formulated in the Appendix of Ref. [15]. It turns out that the leading term and the first correction are determined by values of $l\sim y\sim\omega/m$ in sums $S_{A,B}$. This fact is in agreement with the evident estimate $l\sim\varepsilon\rho\sim\omega/m\gg1$. In the same way as in Ref. [15] we obtain for $S_{A,B}$ with the first correction taken into account

$$S_A=-\frac{y^2}{8}J_0(y\theta/2)\left[1+i\frac{\pi(Z\alpha)^2}{y}\right],$$

$$S_B = -\frac{y}{2\theta} J_1(y\theta/2)\left[1 + i\frac{\pi(Z\alpha)^2}{y}\right]. \tag{18}$$

Let us pass in Eq. (16) from the integration over s to the integration over y, see Eq. (15). We have

$$\exp[2iZ\alpha\lambda s] \approx \left(\frac{4\kappa\sqrt{r_1 r_2}}{y}\right)^{2iZ\alpha\lambda}, \quad \coth s \approx 1 + \frac{y^2}{8\kappa^2 r_1 r_2}.$$

Then we obtain

$$D(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon) = \frac{ie^{i\kappa(r_1+r_2)}}{16\pi\kappa r_2 r_1}\int_0^\infty y\,dy\left(\frac{4\kappa\sqrt{r_1 r_2}}{y}\right)^{2iZ\alpha\lambda} \times \exp\left[\frac{i(r_2+r_1)y^2}{8\kappa r_1 r_2}\right]\Bigg\{[1+i\pi(Z\alpha)^2/y] \times\left[J_0(y\theta/2) + 2iZ\alpha\frac{J_1(y\theta/2)}{y\theta}\boldsymbol{\alpha}\cdot(\boldsymbol{n}_2+\boldsymbol{n}_1)\right] + \pi(Z\alpha)^2\frac{J_1(y\theta/2)}{y^2\theta}[\boldsymbol{n}_2\times\boldsymbol{n}_1]\cdot\boldsymbol{\Sigma}\Bigg\}, \tag{19}$$

where $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$, $\boldsymbol{\Sigma} = (i/2)[\boldsymbol{\gamma}\times\boldsymbol{\gamma}]$. This expression is the quasiclassical Green's function of the squared Dirac equation with the first correction taken into account. The leading term in this expression, as well as the corresponding expression for $G(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)$, has been derived in Refs. [14,15]. It is convenient to rewrite Eq. (19) in another form using the relations

$$\int_0^\infty q\,dq J_0(q\theta)g(q) = \int\frac{d\boldsymbol{q}}{2\pi}e^{i\boldsymbol{q}\cdot\boldsymbol{\theta}}g(q),$$

$$\int_0^\infty q\,dq J_1(q\theta)g(q) = -i\int\frac{d\boldsymbol{q}}{2\pi}\frac{(\boldsymbol{q}\cdot\boldsymbol{\theta})}{q\theta}e^{i\boldsymbol{q}\cdot\boldsymbol{\theta}}g(q), \tag{20}$$

where $g(q)$ is an arbitrary function, $\boldsymbol{q}$ and $\boldsymbol{\theta}$ are two-dimensional vectors. In our case we direct the vector $\boldsymbol{\theta}$ along $\boldsymbol{\rho}$ so that $\boldsymbol{\theta} = r\boldsymbol{\rho}/r_1 r_2$. Using Eq. (20) we have

$$D(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon) = \frac{ie^{i\kappa(r_1+r_2)}}{8\pi\kappa r_2 r_1}\int d\boldsymbol{q}\left(\frac{2\kappa\sqrt{r_1 r_2}}{q}\right)^{2iZ\alpha\lambda} \times\exp\left[\frac{i(r_2+r_1)q^2}{2\kappa r_1 r_2} + i\boldsymbol{q}\cdot\boldsymbol{\theta}\right]\Bigg\{\left[1+\frac{i\pi(Z\alpha)^2}{2q}\right] \times\left[1 + Z\alpha\frac{\boldsymbol{\alpha}\cdot\boldsymbol{q}}{q^2}\right] - \frac{i\pi(Z\alpha)^2}{4q^3}(\boldsymbol{r}/r)\cdot[\boldsymbol{q}\times\boldsymbol{\Sigma}]\Bigg\}. \tag{21}$$

The leading term of this formula has been obtained in Refs. [16,17]. Let us integrate by parts the term containing $\boldsymbol{\alpha}$ matrix and make the change of variable $\boldsymbol{q}\to\kappa(\boldsymbol{q}-\boldsymbol{\rho})$. We obtain

$$D(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon) = \frac{i\kappa e^{i\kappa r}}{8\pi^2 r_1 r_2}\int d\boldsymbol{q}\exp\left[i\frac{\kappa r q^2}{2r_1 r_2}\right]\left(\frac{2\sqrt{r_1 r_2}}{|\boldsymbol{q}-\boldsymbol{\rho}|}\right)^{2iZ\alpha\lambda} \times\Bigg\{\left(1+\frac{r}{2r_1 r_2\lambda}\boldsymbol{\alpha}\cdot\boldsymbol{q}\right)\left(1+i\frac{\pi(Z\alpha)^2}{2\kappa|\boldsymbol{q}-\boldsymbol{\rho}|}\right) - \frac{\pi(Z\alpha)^2}{4\kappa^2}(\gamma^0/\lambda - \boldsymbol{\gamma}\cdot\boldsymbol{r}/r)\frac{\boldsymbol{\gamma}\cdot(\boldsymbol{q}-\boldsymbol{\rho})}{|\boldsymbol{q}-\boldsymbol{\rho}|^3}\Bigg\}. \tag{22}$$

Note that in this formula and below we can set $\lambda = \mathrm{sgn}\,\varepsilon$.

It is easy to check that within our accuracy the contribution of the last term in braces vanishes after taking the trace in Eq. (12). Therefore, this term can be omitted in the problem under consideration. The remaining terms in Eq. (22) can be represented in the form

$$D(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon) = \left[1+\frac{\boldsymbol{\alpha}\cdot(\boldsymbol{p}_1+\boldsymbol{p}_2)}{2\varepsilon}\right]D^{(0)}(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon), \tag{23}$$

$$D^{(0)}(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon) = \frac{i\kappa e^{i\kappa r}}{8\pi^2 r_1 r_2}\int d\boldsymbol{q}\exp\left[i\frac{\kappa r q^2}{2r_1 r_2}\right] \times\left(\frac{2\sqrt{r_1 r_2}}{|\boldsymbol{q}-\boldsymbol{\rho}|}\right)^{2iZ\alpha\lambda}\left(1+i\frac{\pi(Z\alpha)^2}{2\kappa|\boldsymbol{q}-\boldsymbol{\rho}|}\right). \tag{24}$$

The function $D^{(0)}(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)$ is nothing but the quasiclassical Green's function of the Klein-Gordon equation in the Coulomb field. The function δD in Eq. (12) is defined as $\delta D = D - \tilde{D}$, where $\tilde{D}$ is obtained from Eq. (23) by the replacement $D^{(0)}\to D^{(0)*}$.

IV. COULOMB CORRECTIONS TO THE SPECTRUM

In this section we consider the Coulomb corrections to the spectrum, $d\sigma_C/dx$, for $\varepsilon_\pm \gg m$ taking into account terms of the order $m/\varepsilon_\pm$. According to Ref. [8], the higher-order terms of the perturbation theory with respect to the external field (Coulomb corrections) are not seriously modified by screening. However, this question has not been studied quantitatively so far. The influence of screening on Coulomb corrections is investigated in detail in Sec. VI. In the present section we calculate $d\sigma_C/d\varepsilon_-$ in a pure Coulomb field.

Substituting Eq. (23) in Eq. (12) and taking the trace, we obtain

$$\frac{d\sigma_C}{d\varepsilon_-} = \frac{4\alpha}{\omega}\mathrm{Re}\int\int d\boldsymbol{r}_1 d\boldsymbol{r}_2 e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\Bigg\{4[\boldsymbol{e}\cdot\boldsymbol{p}_2 D_-^{(0)}][\boldsymbol{e}\cdot\boldsymbol{p}_1 D_+^{(0)}] - \frac{\omega^2}{\varepsilon_-\varepsilon_+}[\boldsymbol{e}\cdot(\boldsymbol{p}_1+\boldsymbol{p}_2)D_-^{(0)}][\boldsymbol{e}\cdot(\boldsymbol{p}_1+\boldsymbol{p}_2)D_+^{(0)}]\Bigg\},$$
$$D_-^{(0)} = D^{(0)}(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon_-), \quad D_+^{(0)} = D^{(0)}(\boldsymbol{r}_1,\boldsymbol{r}_2|-\varepsilon_+). \tag{25}$$

The terms $\propto D^{(0)}D^{(0)*}$ are omitted in this formula since they do not contribute to the leading term and the correction we are interested in. Besides, we have integrated by parts the terms containing second derivatives of $D^{(0)}$. In this formula and below we assume the subtraction from the integrand of the terms of the order $(Z\alpha)^0$ and $(Z\alpha)^2$. Then we use the relation

$$(\boldsymbol{e}\cdot\boldsymbol{p}_{1,2})D^{(0)}(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)=\frac{i\kappa^2 e^{i\kappa r}}{8\pi^2 r_1 r_2}\int d\boldsymbol{q}\exp\left[i\frac{\kappa r q^2}{2r_1 r_2}\right]\times\left(\frac{2\sqrt{r_1 r_2}}{|\boldsymbol{q}-\boldsymbol{\rho}|}\right)^{2iZ\alpha\lambda}\left(1+i\frac{\pi(Z\alpha)^2}{2\kappa|\boldsymbol{q}-\boldsymbol{\rho}|}\right)\times\left(\mp\frac{\boldsymbol{e}\cdot\boldsymbol{r}}{r}+\frac{\boldsymbol{e}\cdot\boldsymbol{q}}{r_{1,2}}\right), \tag{26}$$

and pass from the variables $\boldsymbol{r}_{1,2}$ to the variables

$$\boldsymbol{r}=\boldsymbol{r}_2-\boldsymbol{r}_1,\quad \boldsymbol{\rho}=\frac{\boldsymbol{r}\times[\boldsymbol{r}_1\times\boldsymbol{r}_2]}{r^2},\quad z=-\frac{(\boldsymbol{r}\cdot\boldsymbol{r}_1)}{r^2}. \tag{27}$$

In terms of these variables $d\boldsymbol{r}_1 d\boldsymbol{r}_2=r\,d\boldsymbol{r}\,d\boldsymbol{\rho}\,dz$, and within our accuracy $r_1=rz$, $r_2=r(1-z)$. We obtain from Eq. (25)

$$\frac{d\sigma_C}{d\varepsilon_-}=-\frac{\alpha\varepsilon_-\varepsilon_+}{16\pi^4\omega}\mathrm{Re}\int\frac{d\boldsymbol{r}}{r^5}\int_0^1\frac{dz}{z^2(1-z)^2}\int\int\int\times d\boldsymbol{q}_-d\boldsymbol{q}_+d\boldsymbol{\rho}\left(\frac{Q_+}{Q_-}\right)^{2iZ\alpha}\exp\left[\frac{i\omega r}{2}\left(\psi^2-\frac{m^2}{\varepsilon_-\varepsilon_+}\right)+i\frac{\varepsilon_-q_-^2+\varepsilon_+q_+^2}{2rz(1-z)}\right]\left[1+\frac{i\pi(Z\alpha)^2}{2}\left(\frac{1}{\varepsilon_-Q_-}+\frac{1}{\varepsilon_+Q_+}\right)\right]\times\left\{4\varepsilon_-\varepsilon_+\left(\boldsymbol{e}\cdot\boldsymbol{r}+\frac{\boldsymbol{e}\cdot\boldsymbol{q}_-}{1-z}\right)\left(-\boldsymbol{e}\cdot\boldsymbol{r}+\frac{\boldsymbol{e}\cdot\boldsymbol{q}_+}{z}\right)-\frac{\omega^2}{z^2(1-z)^2}(\boldsymbol{e}\cdot\boldsymbol{q}_-)(\boldsymbol{e}\cdot\boldsymbol{q}_+)\right\}, \tag{28}$$

where $Q_\pm=|\boldsymbol{q}_\pm-\boldsymbol{\rho}|$ and ψ is the angle between vectors $\boldsymbol{r}$ and $\boldsymbol{k}$. Since $d\sigma_C/d\varepsilon_-$ is independent of the photon polarization, we can replace in Eq. (28) e^ie^j by $\frac{1}{2}\delta_\perp^{ij}=\frac{1}{2}(\delta^{ij}-k^ik^j/\omega^2)$. The integral over $\boldsymbol{\rho}$ can be taken with the help of the relations (see Appendix B)

$$f(Z\alpha)=\frac{1}{2\pi(Z\alpha)^2q^2}\int d\boldsymbol{\rho}\left[\left(\frac{Q_+}{Q_-}\right)^{2iZ\alpha}-1+2(Z\alpha)^2\ln^2\frac{Q_+}{Q_-}\right]=\mathrm{Re}[\psi(1+iZ\alpha)+C],$$

$$g(Z\alpha)=\frac{i}{4\pi q}\int\frac{d\boldsymbol{\rho}}{Q_+}\left[\left(\frac{Q_+}{Q_-}\right)^{2iZ\alpha}-1\right]=Z\alpha\frac{\Gamma(1-iZ\alpha)\Gamma(1/2+iZ\alpha)}{\Gamma(1+iZ\alpha)\Gamma(1/2-iZ\alpha)}, \tag{29}$$

where $\psi(t)=d\ln\Gamma(t)/dt$, $C=0.577\ldots$ is the Euler constant, $q=|\boldsymbol{q}_--\boldsymbol{q}_+|$. We have

$$\frac{d\sigma_C}{d\varepsilon_-}=-\frac{\alpha(Z\alpha)^2\varepsilon_-\varepsilon_+}{8\pi^2\omega}\mathrm{Re}\int_0^\infty\frac{dr}{r^3}\int_0^\infty\psi\,d\psi\int_0^1\frac{dz}{z^2(1-z)^2}\times\int\int d\boldsymbol{q}_-d\boldsymbol{q}_+\exp\left[\frac{i\omega r}{2}\left(\psi^2-\frac{m^2}{\varepsilon_-\varepsilon_+}\right)+i\frac{\varepsilon_-q_-^2+\varepsilon_+q_+^2}{2rz(1-z)}\right]\left[q^2f(Z\alpha)+\pi q\left(\frac{g(Z\alpha)}{\varepsilon_+}-\frac{g^*(Z\alpha)}{\varepsilon_-}\right)\right]\left\{4\varepsilon_-\varepsilon_+\left(-r^2\psi^2+\frac{\boldsymbol{q}_-\cdot\boldsymbol{q}_+}{z(1-z)}\right)-\frac{\omega^2}{z^2(1-z)^2}(\boldsymbol{q}_-\cdot\boldsymbol{q}_+)\right\}. \tag{30}$$

Passing to the variables $\tilde{\boldsymbol{q}}=\boldsymbol{q}_-+\boldsymbol{q}_+$, $\boldsymbol{q}=\boldsymbol{q}_--\boldsymbol{q}_+$, we take all integrals in the following order: $d\psi$, $d\tilde{\boldsymbol{q}}$, $d\boldsymbol{q}$, dr, dz. The final result for the Coulomb corrections to the spectrum reads

$$\frac{d\sigma_C^{(0)}}{dx}+\frac{d\sigma_C^{(1)}}{dx}=-4\sigma_0\left[\left(1-\frac{4}{3}x(1-x)\right)f(Z\alpha)-\frac{\pi^3(1-2x)m}{8x(1-x)\omega}\times\left(1-\frac{3}{2}x(1-x)\right)\mathrm{Re}\,g(Z\alpha)\right],$$

$$x=\varepsilon_-/\omega,\quad \sigma_0=\alpha(Z\alpha)^2/m^2. \tag{31}$$

In Eq. (31), the term $\propto f(Z\alpha)$ corresponds to the leading approximation $d\sigma_C^{(0)}/dx$ [8], the term $\propto\mathrm{Re}\,g(Z\alpha)$ is the first correction $d\sigma_C^{(1)}/dx$. In contrast to the leading term, this correction is antisymmetric with respect to the permutation $\varepsilon_+\leftrightarrow\varepsilon_-$ (or $x\leftrightarrow 1-x$) and, therefore, does not contribute to the total cross section. Besides, the correction is an odd function of $Z\alpha$ due to the charge-parity conservation and the antisymmetry mentioned above. The antisymmetric contribution enhances the production of electrons at $x<1/2$ and suppresses it at $x>1/2$. Evidently, the opposite situation occurs for positrons. Qualitatively, such a behavior of the spectrum takes place for any ω being the most pronounced at low photon energy [6]. At intermediate photon energies, the spectrum (31) essentially differs from that given by the leading approximation. We illustrate this statement in Fig. 1, where $\sigma_0^{-1}d\sigma_C/dx$ with correction (solid line) and without correction (dashed line) are plotted for $Z=82$ and $\omega=50$ MeV.

Due to the antisymmetry of $d\sigma_C^{(1)}/dx$ at $\varepsilon_\pm\gg m$, the term $\sigma_C^{(1)}$ in the total cross section may originate only from the energy regions $\varepsilon_-\sim m$ and $\varepsilon_+=\omega-\varepsilon_-\sim m$. The quasiclassical approximation cannot be used directly in these regions, and another approach is needed to calculate the spectrum. We are going to do this elsewhere. However, for the total cross section, it is possible to overcome this difficulty by means of dispersion relations (see Sec. V).

As known (see, e.g., Ref. [18]), the spectrum of bremsstrahlung can be obtained from the spectrum of pair produc-

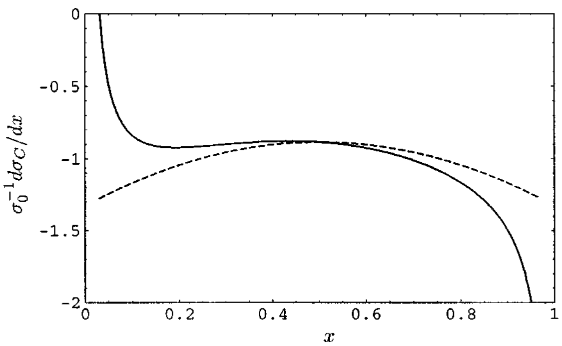

FIG. 1. The dependence of $\sigma_0^{-1} d\sigma_C/dx$ on x, see Eq. (31), for $Z=82$, $\omega=50$ MeV. Dashed curve, leading approximation; solid curve, first correction is taken into account.

tion. This can be performed by means of the substitution $\varepsilon_+ \to -\varepsilon$, $\omega \to -\omega'$, and $dx \to y\,dy$, where $y=\omega'/\varepsilon$, ω' is the energy of an emitted photon, ε is the initial electron energy. Using Eq. (31), we obtain for the Coulomb corrections to the bremsstrahlung spectrum

$$y\frac{d\sigma_C^\gamma}{dy}=-4\sigma_0\left[\left(y^2+\frac{4}{3}(1-y)\right)f(Z\alpha)-\frac{\pi^3(2-y)m}{8(1-y)\varepsilon}\times\left(y^2+\frac{3}{2}(1-y)\right)\mathrm{Re}\,g(Z\alpha)\right]. \quad (32)$$

This formula describes bremsstrahlung from electrons. For the spectrum of photons emitted by positrons, it is necessary to change the sign of $Z\alpha$ in Eq. (32). Our result (32) coincides with that obtained in Ref. [19] if the obvious mistake in the latter is corrected by changing

$$\frac{1}{\gamma}\to\frac{1}{2}\left(\frac{m}{\varepsilon}+\frac{m}{\varepsilon-\omega'}\right)=\frac{(2-y)m}{2(1-y)\varepsilon}$$

in Eq. (22) of Ref. [19]. The correction (32) is the most important at y close to unity, see Fig. 2, where $\sigma_0^{-1} y\,d\sigma_C^\gamma/dy$ with correction (solid line) and without correction (dashed line) are shown for $Z=82$ and $\varepsilon=50$ MeV.

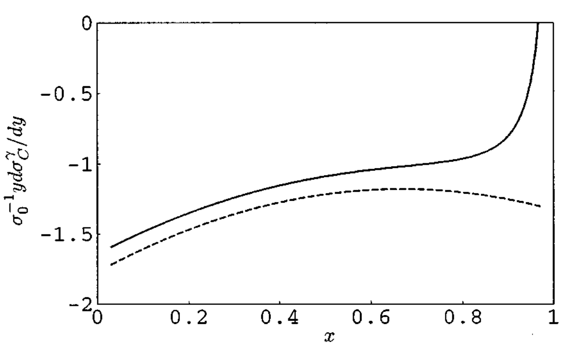

FIG. 2. The dependence of $\sigma_0^{-1} y\,d\sigma_C^\gamma/dy$ on y, see Eq. (31), for $Z=82$, $\varepsilon=50$ MeV. Dashed curve, leading approximation; solid curve, first correction is taken into account.

V. COULOMB CORRECTIONS TO THE TOTAL CROSS SECTION

In the leading approximation, the Coulomb corrections $\sigma_C^{(0)}$ to the total cross section of pair production for $\omega \gg m$ were obtained in Ref. [8]. Using this result and dispersion relations, the corresponding term $M_{DC}^{(0)}$ in the forward Delbrück scattering amplitude M_D was obtained in Ref. [20]. These two quantities read

$$\sigma_C^{(0)}=-\tfrac{28}{9}\,\sigma_0\,f(Z\alpha),\quad M_{DC}^{(0)}=-i\tfrac{28}{9}\,\omega\sigma_0\,f(Z\alpha), \quad (33)$$

where σ_0 and $f(Z\alpha)$ are defined in Eqs. (31) and (29), respectively.

In this section we derive the correction $\sigma_C^{(1)}$ by means of the relation (9). Starting from Eq. (13) and performing the same calculations as in the preceding section we obtain

$$M_{DC}^{(0)}+M_{DC}^{(1)}=-4i\omega\sigma_0\int_0^1 dx\left[\left(1-\frac{4}{3}x(1-x)\right)f(Z\alpha)-\frac{\pi^3 m}{8\omega}\left(1-\frac{3}{2}x(1-x)\right)\times\left(\frac{g^*(Z\alpha)}{x}-\frac{g(Z\alpha)}{1-x}\right)\right]. \quad (34)$$

Here the integration over x corresponds to the integration over ε/ω. After the integration the $M_{DC}^{(0)}$ in Eq. (34) coincides with that in Eq. (33). The integral in $M_{DC}^{(1)}$ is logarithmically divergent. Note that we have obtained the integrand in Eq. (33) under the conditions $x \gg m/\omega$ and $1-x \gg m/\omega$. Taking the integral from δ to $1-\delta$, where $\delta \gtrsim m/\omega$, we find within logarithmic accuracy that $\mathrm{Im}\,M_{DC}^{(1)}$ vanishes and

$$\mathrm{Re}\,M_{DC}^{(1)}=-\frac{\alpha(Z\alpha)^2\pi^3\,\mathrm{Im}\,g(Z\alpha)}{m}\ln\frac{\omega}{m}. \quad (35)$$

The quantity $\mathrm{Im}\,M_{DC}^{(1)}$ does not contain $\ln(\omega/m)$ and is determined by the regions of integration over ε, where $\varepsilon \sim m$ and $\omega-\varepsilon \sim m$, and, therefore, the quasiclassical approximation is invalid. Nevertheless, this quantity, which is related to $\sigma_C^{(1)}$ (9), can be obtained from the dispersion relation for M_D [20],

$$\mathrm{Re}M_D(\omega)=\frac{2}{\pi}\omega^2\,P\int_0^\infty \frac{\mathrm{Im}\,M_D(\omega')\,d\omega'}{\omega'(\omega'^2-\omega^2)}. \quad (36)$$

Using this relation, it can be easily checked that the high-energy asymptotics (35) unambiguously corresponds to the ω-independent high-energy asymptotics

$$\mathrm{Im}\,M_{DC}^{(1)}=\frac{\alpha(Z\alpha)^2\pi^4\,\mathrm{Im}\,g(Z\alpha)}{2m}. \quad (37)$$

Substituting Eq. (35) into Eq. (9) and using σ_{bf} from Ref. [11] in the form

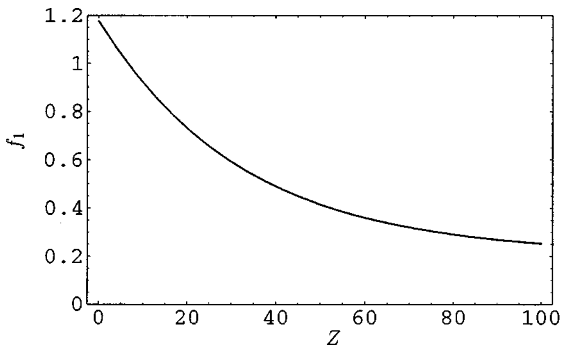

FIG. 3. The quantity f_1 as a function of Z.

$$\sigma_{bf}=4\pi\sigma_0\,(Z\alpha)^3\,f_1(Z\alpha)\,\frac{m}{\omega}, \qquad (38)$$

we have for $\sigma_C^{(1)}$

$$\sigma_C^{(1)}=\sigma_0\left[\frac{\pi^4}{2}\operatorname{Im} g(Z\alpha)-4\pi(Z\alpha)^3 f_1(Z\alpha)\right]\frac{m}{\omega}. \qquad (39)$$

The function $f_1(Z\alpha)$ is plotted in Fig. 3.

The quantity $(\omega/m)\sigma_C^{(1)}/\sigma_C^{(0)}$ is shown in Fig. 4 (solid curve). It is seen that this ratio is numerically large for any Z. Therefore, the term $\sigma_C^{(1)}$ gives a significant contribution to σ_C for intermediate photon energies. Dashed curve in Fig. 4 gives the same ratio when σ_{bf} in Eq. (39) is omitted. It is seen that the relative contribution of the term $\propto f_1(Z\alpha)$ in Eq. (39) is numerically small.

VI. SCREENING CORRECTIONS

In two previous sections the cross section of e^+e^- pair production has been considered for a pure Coulomb field. The difference $\delta V(r)$ between an atomic potential and a Coulomb potential of a nucleus leads to the modification of this cross section known as the effect of screening. In the Born approximation, this effect was studied long ago (see, e.g., Ref. [5]). Let us consider now $\sigma_C^{(scr)}$ characterizing the influence of screening on the Coulomb corrections. Recollect that the Coulomb corrections denote the higher-order terms of the perturbation theory with respect to the atomic field. So far it was only known that the correction $\sigma_C^{(scr)}$ is not large [8]. Here we consider this issue quantitatively.

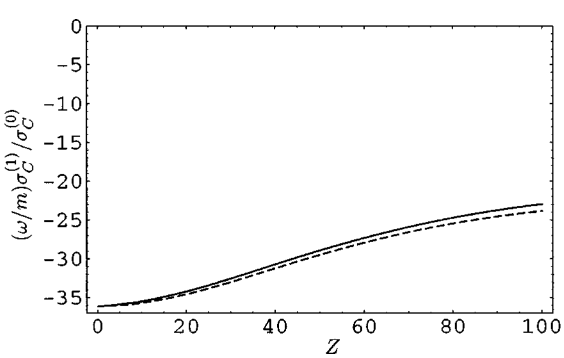

FIG. 4. The quantity $(\omega/m)\sigma_C^{(1)}/\sigma_C^{(0)}$ as a function of Z (solid curve). The dashed curve corresponds to the same quantity without the contribution of the bound-free pair production.

The quasiclassical Green's function $D^{(0)}(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)$ for an arbitrary localized potential $V(r)$ has been obtained in Ref. [21] with the first correction taken into account. The leading term has the form (see also Ref. [12])

$$D^{(0)}(\boldsymbol{r}_2,\boldsymbol{r}_1|\,\varepsilon)=\frac{i\kappa e^{i\kappa r}}{8\pi^2 r_1 r_2}\int d\boldsymbol{q}\times\exp\left[i\frac{\kappa r\, q^2}{2r_1r_2}-i\lambda r\int_0^1 dx V(\boldsymbol{r}_1+x\boldsymbol{r}-\boldsymbol{q})\right]. \qquad (40)$$

Substituting this formula into Eq. (25), we obtain [cf. Eq. (28)]

$$\frac{d\sigma_C}{d\varepsilon_-}=-\frac{\alpha\varepsilon_-\varepsilon_+}{16\pi^4\omega}\operatorname{Re}\int\frac{d\boldsymbol{r}}{r^5}\int_0^1\frac{dz}{z^2(1-z)^2}\int\int\int\times d\boldsymbol{q}_-d\boldsymbol{q}_+d\boldsymbol{\rho}\exp\left\{i\Phi+\frac{i\omega r}{2}\left(\psi^2-\frac{m^2}{\varepsilon_-\varepsilon_+}\right)+i\frac{\varepsilon_-q_-^2+\varepsilon_+q_+^2}{2rz(1-z)}\right\}\left\{4\varepsilon_-\varepsilon_+\left(\boldsymbol{e}\cdot\boldsymbol{r}+\frac{\boldsymbol{e}\cdot\boldsymbol{q}_-}{1-z}\right)\times\left(-\boldsymbol{e}\cdot\boldsymbol{r}+\frac{\boldsymbol{e}\cdot\boldsymbol{q}_+}{z}\right)-\frac{\omega^2}{z^2(1-z)^2}(\boldsymbol{e}\cdot\boldsymbol{q}_-)(\boldsymbol{e}\cdot\boldsymbol{q}_+)\right\},$$

$$\Phi=r\int_0^1 dx[V(\boldsymbol{r}_1+x\boldsymbol{r}-\boldsymbol{q}_+)-V(\boldsymbol{r}_1+x\boldsymbol{r}-\boldsymbol{q}_-)]. \qquad (41)$$

The phase Φ can be represented as

$$\Phi=2Z\alpha\ln(Q_+/Q_-)+\Phi^{(scr)}=2Z\alpha\ln(Q_+/Q_-)+r\int_0^1 dx[\delta V(\boldsymbol{r}_1+x\boldsymbol{r}-\boldsymbol{q}_+)-\delta V(\boldsymbol{r}_1+x\boldsymbol{r}-\boldsymbol{q}_-)]. \qquad (42)$$

As in the case of a pure Coulomb field, the main contribution to the Coulomb corrections comes from the region of integration $q_\pm\sim\rho\sim 1/m$. The main contribution to the integral over x in Eq. (42) comes from the narrow region around the point $x_0=-\boldsymbol{r}_1\cdot\boldsymbol{r}/r^2=z$, $\delta x=\rho/r\ll 1$. Therefore, it is possible to perform the integration in Eq. (42) from $-\infty$ to ∞. Thus we can estimate $\Phi^{(scr)}$ as $\Phi^{(scr)}\sim\rho\,\delta V(\rho)\sim Z\alpha\,\delta V(\rho)/V(\rho)\ll 1$. In our calculation of $\sigma_C^{(scr)}$, we retain the linear term of expansion in $\Phi^{(scr)}$. By definition,

$$\delta V(r)=\int\frac{d\boldsymbol{\Delta}}{(2\pi)^3}e^{i\boldsymbol{\Delta r}}F(\Delta)\frac{4\pi Z\alpha}{\Delta^2}, \qquad (43)$$

where $F(Q)$ is the atomic electron form factor. Substituting this formula to Eq. (42) and taking the integral over x from $-\infty$ to ∞, we obtain for $\Phi^{(scr)}$

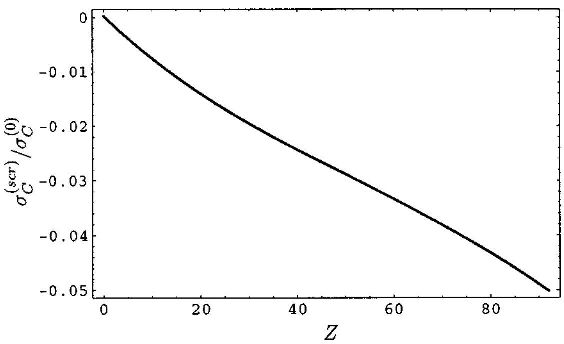

FIG. 5. The ratio $\sigma_C^{(scr)}/\sigma_C^{(0)}$ as a function of Z.

$$\Phi^{(scr)}=\int\frac{d\mathbf{\Delta}_\perp}{(2\pi)^2}e^{i\mathbf{\Delta}_\perp\boldsymbol{\rho}}F(\Delta_\perp)\frac{4\pi Z\alpha}{\Delta_\perp^2},\tag{44}$$

where $\mathbf{\Delta}_\perp$ is two-dimensional vector lying in the plane perpendicular to $\boldsymbol{r}$. Then we use the identity, see Eqs. (22) and (23) in Ref. [17],

$$\int d\boldsymbol{\rho}\left(\frac{|\boldsymbol{\rho}-\boldsymbol{q}_+|}{|\boldsymbol{\rho}-\boldsymbol{q}_-|}\right)^{2iZ\alpha}\exp[i\mathbf{\Delta}_\perp\cdot(\boldsymbol{\rho}-\boldsymbol{q}_\pm)]$$
$$=\frac{q^2}{4\Delta_\perp^2}\int d\boldsymbol{f}\left(\frac{f_+}{f_-}\right)^{2iZ\alpha}\exp[i\boldsymbol{q}\cdot\boldsymbol{f}_\pm/2],\tag{45}$$

where $\boldsymbol{q}=\boldsymbol{q}_--\boldsymbol{q}_+$ and $\boldsymbol{f}_\pm=\boldsymbol{f}\pm\mathbf{\Delta}_\perp$. Expanding Eq. (41) in $\Phi^{(scr)}$ and using the identity (45), we take the integrals over variables $\boldsymbol{q}_\pm$, $\boldsymbol{r}$, and z and obtain

$$\frac{d\sigma_C^{(scr)}}{dx}=\frac{8\alpha(Z\alpha)}{3\pi}\int\frac{d\mathbf{\Delta}_\perp}{\Delta_\perp^4}F(\Delta_\perp)\int\frac{d\boldsymbol{f}}{2\pi}\left(\frac{f_+}{f_-}\right)^{2iZ\alpha}$$
$$\times\left[\frac{R(\xi_-,a)}{f_-^2}-\frac{R(\xi_+,a)}{f_+^2}\right],$$

$$R(\mu,a)=\frac{(\mu-1)}{4\mu^2}\left\{\frac{1}{2\sqrt{\mu}}[18-6\mu+a(\mu^2+2\mu-3)]\right.$$
$$\left.\times\ln\left[\frac{\sqrt{\mu}+1}{\sqrt{\mu}-1}\right]-18-a(\mu-3)\right\},$$

$$\xi_\pm=1+16m^2/f_\pm^2,\quad a=6x(1-x).\tag{46}$$

Using the trick introduced in Sec. IX in Ref. [17], we rewrite this formula in another form. Let us multiply the integrand in Eq. (46) by

$$1\equiv\int_{-1}^{1}dy\,\delta\left(y-\frac{2\boldsymbol{f}\cdot\mathbf{\Delta}_\perp}{\boldsymbol{f}^2+\mathbf{\Delta}_\perp^2}\right)$$
$$=(\boldsymbol{f}^2+\mathbf{\Delta}_\perp^2)\int_{-1}^{1}\frac{dy}{|y|}\,\delta((\boldsymbol{f}-\mathbf{\Delta}_\perp/y)^2-\mathbf{\Delta}_\perp^2(1/y^2-1)),\tag{47}$$

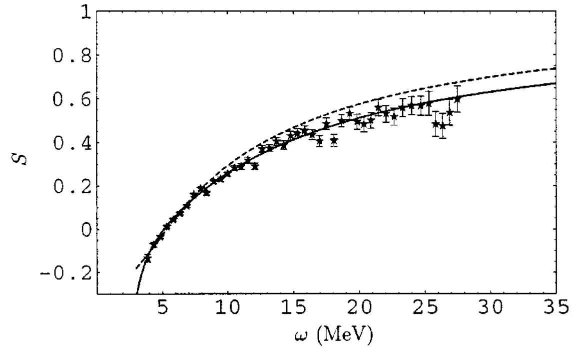

FIG. 6. The ω dependence of $S=(\sigma_{coh}-\sigma_B)/\sigma_C^{(0)}$ for Bi. Solid curve, our result; dashed curve, the result of Øverbø [9]; experimental data from Ref. [23].

change the order of integration over $\boldsymbol{f}$ and y, and make the shift $\boldsymbol{f}\to\boldsymbol{f}+\mathbf{\Delta}_\perp/y$. After that the integration over f can be done easily. Then we make the substitution $y=\tanh\tau$ and obtain

$$\frac{d\sigma_C^{(scr)}}{dx}=\frac{32}{3}\sigma_0m^2\int_0^\infty\frac{dQ}{Q^3}F(Q)\int_0^\infty\frac{d\tau}{\sinh\tau}\left[\frac{\sin(2Z\alpha\tau)}{2Z\alpha}-\tau\right]$$
$$\times\int_0^{2\pi}\frac{d\varphi}{2\pi}[e^{\tau}R(\mu_+,a)-e^{-\tau}R(\mu_-,a)],$$

$$\mu_\pm=1+\frac{8m^2e^{\pm\tau}\sinh^2\tau}{Q^2(\cosh\tau+\cos\varphi)}.\tag{48}$$

Integrating over x, we have

$$\sigma_C^{(scr)}=\frac{32}{3}\sigma_0m^2\int_0^\infty\frac{dQ}{Q^3}F(Q)\int_0^\infty\frac{d\tau}{\sinh\tau}\left[\frac{\sin(2Z\alpha\tau)}{2Z\alpha}-\tau\right]$$
$$\times\int_0^{2\pi}\frac{d\varphi}{2\pi}[e^{\tau}R(\mu_+,1)-e^{-\tau}R(\mu_-,1)].\tag{49}$$

Similar to $\sigma_C^{(0)}$, this correction is ω independent. Shown in Fig. 5 is the Z dependence of the ratio $\sigma_C^{(scr)}/\sigma_C^{(0)}$ calculated with the use of the form factors taken from Ref. [22]. As seen from Fig. 5, this ratio is approximately fitted by the linear function $\sigma_C^{(scr)}\approx-5.4\times10^{-4}Z\sigma_C^{(0)}$.

The corresponding correction to the bremsstrahlung spectrum is obtained from Eq. (48) by means of the same substitutions as in Sec. IV. So that the quantity $y^{-1}d\sigma_C^{\gamma(scr)}/dy$ is given by the right-hand side of Eq. (48) if we set $a=(y-1)/y^2$.

VII. ESTIMATION OF $\sigma_C^{(2)}$ FROM EXPERIMENTAL DATA

The most detailed and accurate experimental data have been obtained just in the region of intermediate photon energies. In this region, the first correction $\sigma_C^{(1)}$, obtained above, becomes large, see Fig. 4 for $(\omega/m)\sigma_C^{(1)}/\sigma_C^{(0)}$, and the next term $\sigma_C^{(2)}$ in the expansion (2) may be significant. Using the arguments similar to those presented by Davies

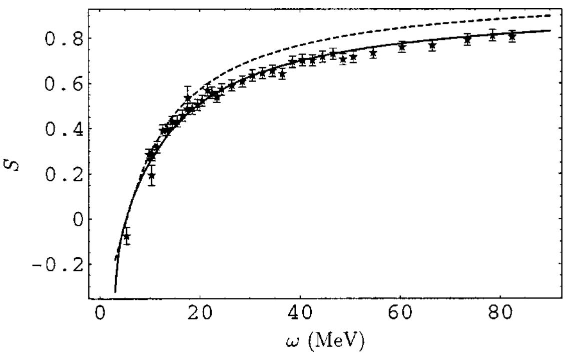

FIG. 7. Same as Fig. 6 but for Pb; experimental data from Refs. [24,25].

et al. [8] the following ansatz for $\sigma_C^{(2)}$ has been suggested in our recent paper [10]:

$$\sigma_C^{(2)}=\sigma_0[b\ \ln(\omega/2m)+c]\left(\frac{m}{\omega}\right)^2, \tag{50}$$

where b and c are some functions of $Z\alpha$. It was shown in Ref. [10] that experimental data for σ_{coh} are well described by the formula

$$\sigma_{coh}=\sigma_B+\sigma_C^{(0)}+\sigma_C^{(scr)}+\sigma_C^{(1)}+\sigma_C^{(2)}, \tag{51}$$

where $\sigma_C^{(0)}$, $\sigma_C^{(scr)}$, and $\sigma_C^{(1)}$ are from Eqs. (33), (49), and (39), respectively; $\sigma_C^{(2)}$ is given by Eq. (50) with $b=3.78(\omega/m)\sigma_0^{-1}\sigma_C^{(1)}$, $c=0$.

It is interesting to compare our predictions for the Coulomb corrections to the total cross section with the results of Øverbø [9]. Shown in Figs. 6 and 7 is the ratio $S=(\sigma_{coh}-\sigma_B)/\sigma_C^{(0)}$, which is the Coulomb corrections in units of $\sigma_C^{(0)}$, Eq. (33).

Our results are represented by solid curves, those of Øverbø are shown as dashed curves. The values of S extracted from the experimental data are also shown. The results for Bi are plotted in Fig. 6 with the experimental data taken from Ref. [23]. The results for Pb are plotted in Fig. 7 with the experimental data taken from Ref. [24,25]. It is seen that the difference between our results and those of Øverbø is small at relatively low energies and becomes noticeable as ω increases. According to our results, this difference tends to a constant $\sigma_C^{(scr)}/\sigma_C^{(0)}$ at $\omega\to\infty$. The experimental data are, on the whole, in a better agreement with our results than with those of Øverbø.

VIII. CONCLUSION

For the e^+e^- photoproduction, we have calculated the leading correction (31) to the electron spectrum in the region $\varepsilon_\pm \gg m$. This contribution noticeably modifies the spectrum at intermediate photon energy. It turns out that the correction is antisymmetric with respect to the permutation $\varepsilon_+\leftrightarrow\varepsilon_-$ and hence does not contribute to the total cross section. The leading correction to the total cross section, $\sigma_C^{(1)}$, originates from two regions $\varepsilon_+\sim m$ and $\varepsilon_-\sim m$. We have obtained $\sigma_C^{(1)}$ (39) using dispersion relations. In contrast to the form of the fit suggested by Øverbø [9], the quantity $\sigma_C^{(1)}$ does not contain any powers of $\ln(\omega/m)$. We have also performed the quantitative investigation of the influence of screening on the Coulomb corrections (48) and (49). It is important that $\sigma_C^{(scr)}$ does not vanish in the high-energy limit. We have suggested a form for the next-to-leading correction $\sigma_C^{(2)}$ to the total cross section. Altogether, the corrections found allow one to represent well the available experimental data.

Starting with the results obtained for the e^+e^- photoproduction spectrum, we have obtained the corresponding corrections to the bremsstrahlung spectrum as well.

ACKNOWLEDGMENTS

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APPENDIX A

In this appendix we perform the transformation from Eq. (8) to Eq. (13). The transformation from Eq. (7) to Eq. (12) is performed in the same way.

The Delbrück scattering amplitude (8) can be represented as follows:

$$M_D=2i\alpha\int d\varepsilon\int d\boldsymbol{r}_2\ \mathrm{Sp}\langle\boldsymbol{r}_2|[\hat{\mathcal{P}}-m]^{-1}\hat{e}[\hat{\mathcal{P}}-\hat{k}-m]^{-1}\hat{e}\,|\boldsymbol{r}_2\rangle. \tag{A1}$$

Here $\hat{\mathcal{P}}=\gamma^0[\varepsilon-V(\boldsymbol{r})]-\boldsymbol{\gamma}\cdot\boldsymbol{p}$ and $\boldsymbol{p}=-i\boldsymbol{\nabla}$. We represent the operator $[\hat{\mathcal{P}}-m]^{-1}$ in Eq. (A1) in the form

$$[\hat{\mathcal{P}}-m]^{-1}=[\hat{\mathcal{P}}^2-m^2]^{-1}(\hat{\mathcal{P}}+m)$$

and use the relation

$$(\hat{\mathcal{P}}+m)\hat{e}=[-\hat{e}(\hat{\mathcal{P}}-\hat{k}-m)-\hat{e}\hat{k}-2\boldsymbol{e}\cdot\boldsymbol{p}].$$

Then we obtain

$$M_D=2i\alpha\int d\varepsilon\int d\boldsymbol{r}_2\ \mathrm{Sp}\langle\boldsymbol{r}_2|\{-[\hat{\mathcal{P}}^2-m^2]^{-1}[2\boldsymbol{e}\cdot\boldsymbol{p}+\hat{e}\hat{k}][(\hat{\mathcal{P}}-\hat{k})^2-m^2]^{-1}(\hat{\mathcal{P}}-\hat{k}+m)\hat{e}-\hat{e}[\hat{\mathcal{P}}^2-m^2]^{-1}\hat{e}\}|\boldsymbol{r}_2\rangle. \tag{A2}$$

We obtain another expression for M_D using the representation

$$[\hat{\mathcal{P}}-\hat{k}-m]^{-1}=(\hat{\mathcal{P}}-\hat{k}+m)[(\hat{\mathcal{P}}-\hat{k})^2-m^2]^{-1}$$

and the relation

$$\hat{e}(\hat{\mathcal{P}}-\hat{k}+m)=[-\hat{e}(\hat{\mathcal{P}}-m)-\hat{e}\hat{k}-2\boldsymbol{e}\cdot\boldsymbol{p}].$$

We have

$$M_D = 2i\alpha\int d\varepsilon\int d\boldsymbol{r}_2\,\mathrm{Sp}\langle\boldsymbol{r}_2|\{-(\hat{\mathcal{P}}+m)[\hat{\mathcal{P}}^2-m^2]^{-1}[2\boldsymbol{e}\cdot\boldsymbol{p} + \hat{e}\hat{k}][(\hat{\mathcal{P}}-\hat{k})^2-m^2]^{-1}\hat{e} - \hat{e}[(\hat{\mathcal{P}}-\hat{k})^2-m^2]^{-1}\hat{e}\}|\boldsymbol{r}_2\rangle. \tag{A3}$$

The terms odd in m in Eqs. (A2) and (A3) vanish after taking a trace. It is evident that

$$\int d\varepsilon\int d\boldsymbol{r}_2\,\langle\boldsymbol{r}_2|[(\hat{\mathcal{P}}-\hat{k})^2-m^2]^{-1}|\boldsymbol{r}_2\rangle = \int d\varepsilon\int d\boldsymbol{r}_2\langle\boldsymbol{r}_2|[\hat{\mathcal{P}}^2-m^2]^{-1}|\boldsymbol{r}_2\rangle.$$

Taking a half sum of Eqs. (A2) and (A3) and using the identity $\int d\boldsymbol{x}\,\mathrm{Sp}\langle\boldsymbol{x}|A_1A_2|\boldsymbol{x}\rangle = \int d\boldsymbol{x}\,\mathrm{Sp}\langle\boldsymbol{x}|A_2A_1|\boldsymbol{x}\rangle$, valid for arbitrary operators A_1 and A_2, we obtain

$$M_D = i\alpha\int d\varepsilon\int d\boldsymbol{r}_2\,\mathrm{Sp}\langle\boldsymbol{r}_2|\{(2\boldsymbol{e}\cdot\boldsymbol{p}-\hat{e}\hat{k})[\hat{\mathcal{P}}^2-m^2]^{-1}[2\boldsymbol{e}\cdot\boldsymbol{p} + \hat{e}\hat{k}][(\hat{\mathcal{P}}-\hat{k})^2-m^2]^{-1} + 2[(\hat{\mathcal{P}}-\hat{k})^2-m^2]^{-1}\}|\boldsymbol{r}_2\rangle. \tag{A4}$$

This formula is equivalent to Eq. (13).

APPENDIX B

In this appendix we derive the formulas (29). In the integral for $f(Z\alpha)$ let us make the change of variables $\boldsymbol{\rho}\to(\boldsymbol{\rho}+\boldsymbol{q}_+ + \boldsymbol{q}_-)/2$:

$$f(Z\alpha) = \frac{1}{8\pi(Z\alpha)^2q^2}\int d\boldsymbol{\rho}\left[\left(\frac{|\boldsymbol{\rho}+\boldsymbol{q}|}{|\boldsymbol{\rho}-\boldsymbol{q}|}\right)^{2iZ\alpha} - 1 + 2(Z\alpha)^2\ln^2\frac{|\boldsymbol{\rho}+\boldsymbol{q}|}{|\boldsymbol{\rho}-\boldsymbol{q}|}\right]. \tag{B1}$$

Let us multiply the integrand in Eq. (46) by

$$1 \equiv \int_{-1}^{1} dy\,\delta\left(y - \frac{2\boldsymbol{\rho}\cdot\boldsymbol{q}}{\rho^2+q^2}\right) = (\boldsymbol{\rho}^2+\boldsymbol{q}^2)\int_{-1}^{1}\frac{dy}{|y|}\,\delta((\boldsymbol{\rho}-\boldsymbol{q}/y)^2 - q^2(1/y^2-1)), \tag{B2}$$

change the order of integration over $\boldsymbol{\rho}$ and y, and make the shift $\boldsymbol{\rho}\to\boldsymbol{\rho}+\boldsymbol{q}/y$. Then the integral over $\boldsymbol{\rho}$ becomes trivial and we have

$$f(Z\alpha) = \frac{1}{4(Z\alpha)^2}\int_{-1}^{1}\frac{dy}{|y|^3}\left[\left(\frac{1+y}{1-y}\right)^{iZ\alpha} - 1 + \frac{(Z\alpha)^2}{2}\ln^2\left(\frac{1+y}{1-y}\right)\right]. \tag{B3}$$

Then we make the substitution $y=\tanh\tau$ and obtain

$$f(Z\alpha) = \frac{1}{2(Z\alpha)^2}\int_0^\infty d\tau\frac{\cosh\tau}{\sinh^3\tau}[\cos(2Z\alpha\tau) - 1 + 2(Z\alpha)^2\tau^2] = \int_0^\infty\frac{d\tau e^{-\tau}}{\sinh\tau}[1-\cos(2Z\alpha\tau)] = \mathrm{Re}[\psi(1+iZ\alpha)+C]. \tag{B4}$$

In order to calculate the function $g(Z\alpha)$ we make a shift $\boldsymbol{\rho}\to\boldsymbol{\rho}+\boldsymbol{q}_+$ in Eq. (29) and use the exponential parametrization

$$A^\nu = \frac{e^{i\pi\nu/2}}{\Gamma(-\nu)}\int_0^\infty\frac{ds}{s^{1+\nu}}\exp[iAs]. \tag{B5}$$

Then we have

$$g(Z\alpha) = \frac{ie^{\pi Z\alpha/2}}{4\pi q\Gamma(iZ\alpha)}\int d\boldsymbol{\rho}\rho^{-1+2iZ\alpha}\int_0^\infty ds\,s^{-1+iZ\alpha} \times\{\exp[is(\boldsymbol{\rho}-\boldsymbol{q})^2] - \exp[is\rho^2]\}. \tag{B6}$$

Taking the integral over the angles of two-dimensional vector $\boldsymbol{\rho}$ and making the substitutions $\rho\to\rho/\sqrt{s}$, $s\to s/q^2$ we come to

$$g(Z\alpha) = \frac{ie^{\pi Z\alpha/2}}{2\Gamma(iZ\alpha)}\int_0^\infty d\rho\,\rho^{2iZ\alpha}\exp[i\rho^2]\int_0^\infty ds\,s^{-3/2} \times[e^{is}J_0(2\rho\sqrt{s}) - 1]. \tag{B7}$$

Taking the integrals over s and then over ρ we finally obtain

$$g(Z\alpha) = Z\alpha\frac{\Gamma(1-iZ\alpha)\Gamma(1/2+iZ\alpha)}{\Gamma(1+iZ\alpha)\Gamma(1/2-iZ\alpha)}. \tag{B8}$$

[1] J.H. Hubbell, Radiat. Phys. Chem. **59**, 113 (2000).
[2] J.H. Hubbell, H.A. Gimm, and I. Øverbø, J. Phys. Chem. Ref. Data **9**, 1023 (1980).
[3] H.A. Bethe and W. Heitler, Proc. R. Soc. London, Ser. A **146**, 83 (1934).
[4] G. Racah, Nuovo Cimento **11**, 461 (1934).
[5] R. Jost, J.M. Luttinger, and M. Slotnick, Phys. Rev. **80**, 189 (1950).
[6] I. Øverbø, K.J. Mork, and H.A. Olsen, Phys. Rev. **175**, 1978 (1968).
[7] H.A. Bethe and L.C. Maximon, Phys. Rev. **93**, 768 (1954).
[8] H. Davies, H.A. Bethe, and L.C. Maximon, Phys. Rev. **93**, 788 (1954).
[9] I. Øverbø, Phys. Lett. B **71**, 412 (1977).
[10] R.N. Lee, A.I. Milstein, and V.M. Strakhovenko, e-print hep-ph/0307388.
[11] A.I. Milstein and V.M. Strakhovenko, Zh. Éksp. Teor. Fiz. **103**, 1584 (1993) [JETP **76**, 775 (1993)].
[12] R.N. Lee and A.I. Milstein, Phys. Lett. A **198**, 217 (1995).
[13] A.I. Milstein and V.M. Strakhovenko, Phys. Lett. A **90**, 447 (1982).
[14] A.I. Milstein and V.M. Strakhovenko, Phys. Lett. A **95**, 135

(1983).
[15] A.I. Milstein and V.M. Strakhovenko, Zh. Éksp. Teor. Fiz. **85**, 14 (1983) [JETP **58**, 8 (1983)].
[16] R.N. Lee, A.I. Milstein, and V.M. Strakhovenko, Zh. Éksp. Teor. Fiz. **112**, 1921 (1997) [JETP **85**, 1049 (1997)].
[17] R.N. Lee, A.I. Milstein, and V.M. Strakhovenko, Phys. Rev. A **57**, 2325 (1998).
[18] V.B. Berestetski, E.M. Lifshits, and L.P. Pitayevsky, *Quantum Electrodynamics* (Pergamon, Oxford, 1982).
[19] V.N. Baier and V.M. Katkov, Dokl. Akad. Nauk (SSSR) **227**, 325 (1976) [Sov. Phys. Dokl. **21**, 150 (1976)].
[20] F. Rohrlich, Phys. Rev. **108**, 169 (1957).
[21] R.N. Lee, A.I. Milstein, and V.M. Strakhovenko, Zh. Éksp. Teor. Fiz. **117**, 75 (2000) [JETP **90**, 66 (2000)].
[22] J.H. Hubbell and I. Øverbø, J. Phys. Chem. Ref. Data **8**, 69 (1979).
[23] N.K. Sherman, C.K. Ross, and K.H. Lokan, Phys. Rev. C **21**, 2328 (1980).
[24] E.S. Rosenblum, E.F. Shrader, and R.M. Warner, Jr., Phys. Rev. **88**, 612 (1952).
[25] H. A. Gimm and J.H. Hubbell, Natl. Bur,. Stand. (U.S.) Misc. Publ. No. 968 (U.S. GPO, Washington, D.C., 1978).