Optimal processing of quantum information via *W*-type entangled coherent states

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Optimized probabilistic teleportation and remote symmetric entangling of an arbitrary logical qubit are studied using particular forms of *W*-type entangled coherent states. Of interest is the fact that, while the teleportation can alternatively be performed by the GHZ-type entangled coherent states, the remote symmetric entangling strictly requires those of the *W* type.

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I. INTRODUCTION

Quantum information can be carried by a quantum system with a finite set of discrete states each of which is defined just by a single-quantum number. The simplest cases concern a two-level atomic system, a spin-half particle, or a photon with two kinds of polarization, etc. that form the unit of quantum information, a qubit. Quantum information can also be coded in a state which is characterized by an infinite number of degrees of freedom. Examples are a position or momentum wave function of a microscopic particle or a state of a field. The latter version constitutes the so-called continuous-variable quantum information [1]. The continuous-variable approach promises to be more compact and more efficient in both coding and manipulating quantum information and thus has been developed rapidly during the last few years from both theoretical and experimental point of view (see, e.g., Ref. [2]). An intermediate, quite simple but very useful, way [3] for coding is to utilize superpositions of a finite number of macroscopically distinguishable states each of which is however embedded in an unbounded vector space. In this approach, instead of qubits, one deals with logical qubits. A logical qubit is regarded as a superposition of two continuous-variable states which are linearly independent but not necessarily orthogonal to each other. Since coherent states are presently of practical use and readily available from laser sources, an elegant choice for representing a logical qubit is to use two coherent states of an optical mode $|\alpha\rangle$ and $|-\alpha\rangle$ with α the complex coherent amplitude. The logical qubit therefore lives in a two-dimensional subspace of the mode's full Hilbert space of infinite dimension, and reads

$$|\Psi\rangle = x|\alpha\rangle + y|-\alpha\rangle, \tag{1}$$

where the coefficients x, y (assumed to be real for simplicity) obey the normalization condition

$$x^2 + y^2 + 2xyz = 1, \tag{2}$$

with

$$z = \langle\alpha|-\alpha\rangle = \exp(-2|\alpha|^2) \tag{3}$$

the overlap between $|\alpha\rangle$ and $|-\alpha\rangle$. The concrete value of $|\alpha|$ thus serves as a measure of nonorthogonality of the two coherent states being used.

To process quantum information encoded in logical qubits of the form (1) the so-called entangled coherent states (ECS's) [4] have proven very helpful. To date, however, only ECS's of the GHZ type [5],

$$|\mathrm{GHZ},\alpha\rangle_{1\ldots N} = c_1|\alpha,\alpha,\ldots,\alpha\rangle_{1\ldots N} + c_2|-\alpha,-\alpha,\ldots,-\alpha\rangle_{1\ldots N}, \tag{4}$$

with $c_{1,2}$ the normalization coefficients, have been utilized (see, e.g., Ref. [6]). In this work, by two explicit applications, we show that ECS's of the *W* type [7],

$$|W,\alpha\rangle_{1\ldots N} = a_1|\alpha,-\alpha,\ldots,-\alpha\rangle_{1\ldots N} + a_2|-\alpha,\alpha,\ldots,-\alpha\rangle_{1\ldots N} + \cdots + a_N|-\alpha,-\alpha,\ldots,\alpha\rangle_{1\ldots N}, \tag{5}$$

with $a_{1,2,\ldots,N}$ the normalization coefficients, are generally also useful and, in particular, there exist tasks for which the GHZ-type ECS's are not suitable but the *W*-type ones are. In this connection, we also notice that while *W* states have been invoked to for several kinds of quantum information processing in terms of qubits [8], none have been done for logical qubits exploiting *W*-type ECS's.

The present paper is structured as follows. In Sec. II we consider an optimized probabilistic protocol for Alice to teleport an unknown logical qubit to Clare with the assistance of Bob when the three parties share a prior three-mode *W*-type ECS. In Sec. III we address a problem of how Alice can do her best with a given logical qubit in such a way that the logical qubit is symmetrically entangled with a reference state between two remote collaborators Bob and Clare. We show that such a task can be accomplished by means of the *W* type but not of the GHZ-type ECS. Sec. IV discusses on experiment efficiency due to photodetector imperfection of the two protocols proposed in Secs. II and III. Finally, conclusion is drawn in Sec. V.

II. TELEPORTATION

Let Alice be given an unknown logical qubit $|\Psi\rangle_U = x|\alpha\rangle_U + y|-\alpha\rangle_U$ and share with two remote parties (Bob

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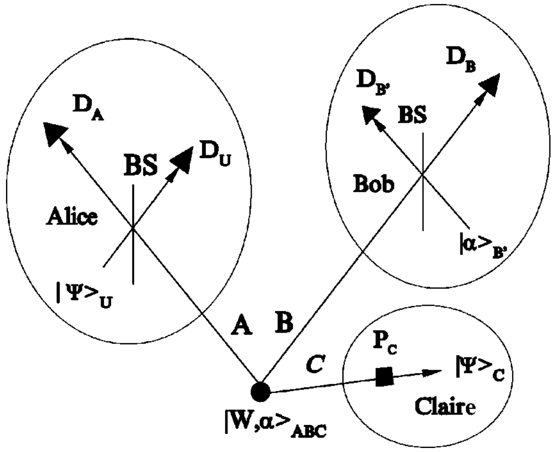

FIG. 1. Teleportation setup. Alice, Bob, and Claire share a prior W-type ECS $|W,\alpha\rangle_{ABC}$. BS's denote 50:50 beam splitters, $D_{A,U,B,B'}$ photodetectors, and P_C a phase shifter.

and Claire) a prior three-mode W-type ECS

$$|W,\alpha\rangle_{ABC}=a_1|\alpha,-\alpha,-\alpha\rangle_{ABC}+a_2|-\alpha,\alpha,-\alpha\rangle_{ABC}+a_3|-\alpha,-\alpha,\alpha\rangle_{ABC}, \tag{6}$$

where a_j are some coefficients (assumed to be real for simplicity) obeying the normalization condition

$$a_1^2+a_2^2+a_3^2+2z^2(a_1a_2+a_2a_3+a_3a_1)=1. \tag{7}$$

Alice's task is to teleport $|\Psi\rangle_U$ to either one of her two remote collaborators. Without loss of generality, we assume that Claire is the one who receives the teleported logical qubit. Even so, Bob would not be left jobless. In fact, his assistance is important in the following way. Bob prepares an ancilla in state $|\alpha\rangle_{B'}$ and mixes it with his mode B of the shared state $|W,\alpha\rangle_{ABC}$ by a 50:50 beam splitter. As for Alice, she does similarly as Bob but between the unknown state $|\Psi\rangle_U$ and her mode A of the state $|W,\alpha\rangle_{ABC}$ (see Fig. 1). After passing the beam splitters the initial total state $|\Psi\rangle_U|\alpha\rangle_{B'}|W,\alpha\rangle_{ABC}$ becomes (here the action of a 50:50 beam splitter reads as $|\alpha\rangle_i|\beta\rangle_j\to|(\alpha+\beta)/\sqrt{2}\rangle_i|(\alpha-\beta)/\sqrt{2}\rangle_j$ [9])

$$\begin{aligned}|\Phi\rangle_{UAB'BC}=&|0\rangle_A[|0\rangle_{B'}|\alpha\sqrt{2}\rangle_B\otimes(a_1x|\alpha\sqrt{2}\rangle_U|-\alpha\rangle_C\\&+a_3y|-\alpha\sqrt{2}\rangle_U|\alpha\rangle_C)+a_2y|-\alpha\sqrt{2}\rangle_U\\&\times|\alpha\sqrt{2}\rangle_{B'}|0\rangle_B|-\alpha\rangle_C]+|0\rangle_U[|0\rangle_{B'}|\alpha\sqrt{2}\rangle_B\\&\otimes(a_1y|-\alpha\sqrt{2}\rangle_A|-\alpha\rangle_C+a_3x|\alpha\sqrt{2}\rangle_A|\alpha\rangle_C)\\&+a_2x|\alpha\sqrt{2}\rangle_A|\alpha\sqrt{2}\rangle_{B'}|0\rangle_B|-\alpha\rangle_C].\end{aligned} \tag{8}$$

Inspecting Eq. (8) reveals that the coefficients a_1 and a_3 should be related as $a_1=\pm a_3$.

A. $a_1=a_3$

Let us first consider the situation with

$$a_1=a_3=a'. \tag{9}$$

Denoting by $n_{B'}$, n_B, n_A, and n_U the photon numbers counted by detectors $D_{B'}$, D_B, D_A, and D_U (see Fig. 1), respectively, and looking closer at Eq. (8) with the constraint (9) we recognize that the above-posed task succeeds if

$$n_{B'}=0, \quad n_B>0 \tag{10}$$

combined either with

$$n_A=0, \quad n_U=0,2,4,\ldots, \tag{11}$$

or with

$$n_U=0, \quad n_A=0,2,4,\ldots \tag{12}$$

and, fails otherwise. In case the outcomes (10) and (11) happen Claire gets a state which can easily be converted into $|\Psi\rangle_C$ by the action of the operator $\hat{P}_C(\pi)$ where $\hat{P}_j(\varphi)=\exp(-i\varphi\hat{a}_j^+\hat{a}_j)$ with $\hat{a}_{j=A,B,C}$ being the annihilation operator of photon in mode j. Alternatively, in case the outcomes (10) and (12) happen the state at Claire's location collapses exactly into $|\Psi\rangle_C$ without doing anything [or, as one could say, by the action of the operator $\hat{P}_C(0)$]. The total success probability is calculated to be

$$\begin{aligned}\Pi'=&\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}|_{B'}\langle 0|_B\langle n|_A\langle 0|_U\langle 2m|\Phi\rangle_{UAB'BC}|^2+\{U\leftrightarrow A\}\\&=2a'^2\exp(-2|\alpha|^2)[1-\exp(-2|\alpha|^2)]\cosh(2|\alpha|^2)\\&=a'^2(1-z)(1+z^2).\end{aligned} \tag{13}$$

The above obtained probability of success Π' does not depend on the coefficients of the logical qubit to be teleported, i.e., it is independent of x and y. Yet, it does depends on α [or, the same, on z due to Eq. (3)] and on a'. In the limit $|\alpha|\to\infty$ (i.e., $z\to 0$) Π' tends to a'^2; whereas in the limit $|\alpha|\to 0$ (i.e., $z\to 1$) Π' tends to 0 like $4a'^2|\alpha|^2$. For a given value of α (or, the same, of z) the quantum channel state $|W,\alpha\rangle_{ABC}$ could be tailored so as to maximize Π', i.e., to optimize the teleportation performance. Combining the conditions (7) and (9) yields

$$a'=a'_{\pm}=\frac{\pm\sqrt{\Delta}-2a_2z^2}{2(1+z^2)}, \tag{14}$$

where $\Delta=2[2a_2^2z^4+(1-a_2^2)(1+z^2)]$ and $a_2^2\leqslant(1+z^2)/(1+z^2-2z^4)$. Figure 2(a) displays $a'_{\pm}$ and $a'^2_{\pm}$ as a function of a_2 for a fixed value of z. For each given z, Π' reaches a maximal value $\Pi'_{\max}$ proportional to $a'^2_{+\max}=a'^2_{-\max}$ which corresponds to two values of a_2: $a_2=a_{2+}^{opt}<0$ and $a_2=a_{2-}^{opt}=-a_{2+}^{opt}>0$. The pair $\{a_{2+}^{opt},a'_{+\max}\}$ changes in accordance with z as seen in Fig. 2(b). Figure 3 illustrates how Π' depends simultaneously on both z and a_2 when a' takes on the value of a'_+ (upper plot) and when a' takes on the value of a'_- (lower plot). At this moment it is worth noting that the equally weighted W-type ECS with $a_1=a_2=a_3$ does the job,

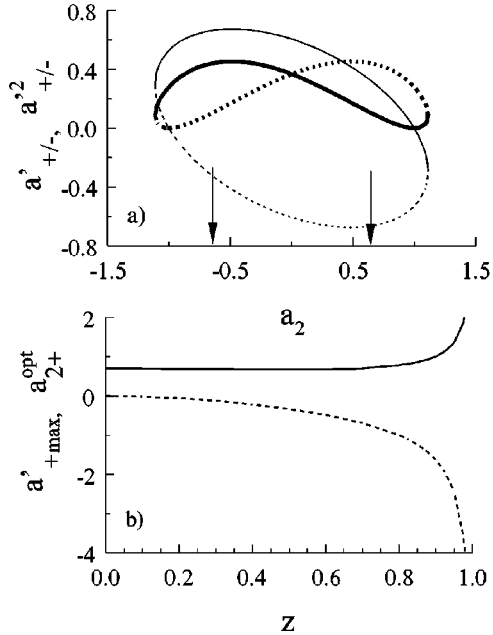

FIG. 2. (a) a'_+ (thin solid curve), a'^2_+ (thick solid curve), a'_- (thin dashed curve), and a'^2_- (thick dashed curve) as a function of a_2 for $z=0.6$. The left (right) arrow indicates the value of a_{2+}^{opt} ($a_{2-}^{opt}=-a_{2+}^{opt}$) at which $a'^2_+=a'^2_{+\max}$ ($a'^2_-=a'^2_{-\max}=a'^2_{+\max}$). (b) $a'_{+\max}=-a'_{-\max}$ (solid line) and $a_{2+}^{opt}=-a_{2-}^{opt}$ (dashed line) vs z.

of course. But such a symmetric entangled state leads to the success probability equal to $(1-z)(1+z^2)/[3(1+2z^2)] \leqslant 1/3$ which is not optimal.

B. $a_1=-a_3$

Now we consider the situation with

$$a_1=-a_3=a''. \tag{15}$$

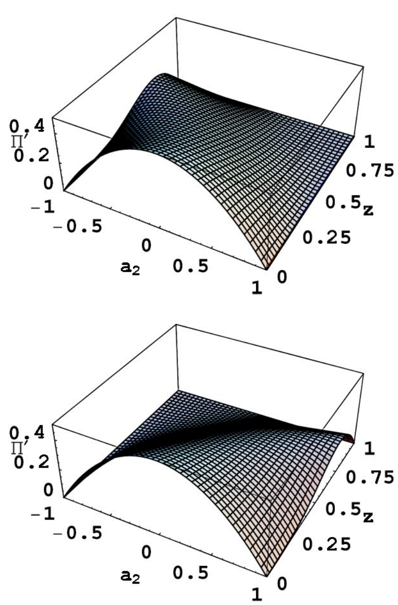

FIG. 3. The upper plot is the probability of successful teleportation $\Pi'(a'_+)$ and the lower one is $\Pi'(a'_-)$, as a function of a_2 and z.

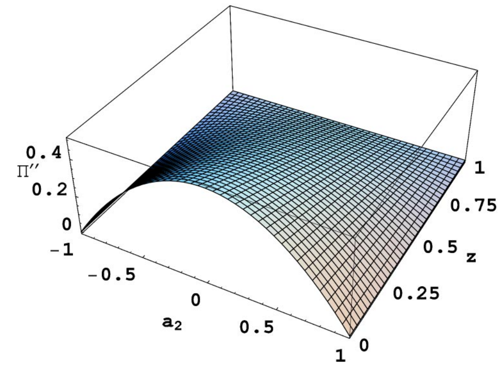

FIG. 4. The probability of successful teleportation Π'' as a function of a_2 and z.

Under this situation the teleportation succeeds if the outcomes (10) occur together with either

$$n_A=0, \quad n_U=1,3,5, \ldots \tag{16}$$

or

$$n_U=0, \quad n_A=1,3,5, \ldots, \tag{17}$$

and fails otherwise. To get the teleported state Claire needs just to apply $\hat{P}_C(\pi)$ or $\hat{P}_C(0)$ to her mode C of the shared W-type ECS depending on whether the outcomes (10) and (16) or (10) and (17) happen. The total success probability in this situation is

$$\begin{aligned}\Pi''=&\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}|{}_{B'}\langle 0|_B\langle n|_A\langle 0|_U\langle 2m+1|\Phi\rangle_{UAB'BC}|^2\\&+\{U\leftrightarrow A\}=2a''^2\exp(-2|\alpha|^2)\\&\times[1-\exp(-2|\alpha|^2)]\sinh(2|\alpha|^2)\\=&a''^2(1-z)(1-z^2),\end{aligned} \tag{18}$$

which tends to a''^2 in the limit $|\alpha|\to\infty$ and quickly to zero like $8a''^2|\alpha|^4$ in the limit $|\alpha|\to 0$. Combining the conditions (7) and (15) yields

$$a''^2=\frac{1-a_2^2}{2(1-z^2)} \tag{19}$$

with $a_2^2\leqslant 1$. Since, for any allowed $z\in[0,1]$, a''^2 is uniquely determined by a_2^2 and monotonically increased with decreasing a_2^2, the state $|W,\alpha\rangle_{ABC}$ could be engineered so that a_2^2 gets as small as possible in order to have as high as possible the success probability Π'', which is, on account of Eqs. (18) and (19), equal to $(1-a_2^2)(1-z)/2$. Figure 4 displays Π'' as a simultaneous function of both a_2 and z.

C. Discussion

As is clear from above, the proposed teleportation protocol fully fails if $n_B=0$. But what will happen if the outcomes (10) occur but in Eqs. (11) and (12) the numbers $n_{U,A}$ are odd or in Eqs. (16) and (17) the numbers $n_{U,A}$ are even? It is quite interesting to note that these happenings do not imply a full failure in the sense that Claire can still do something to obtain a state which though is not an exact replica of but may be very close to $|\Psi\rangle_C$. In fact, under those circumstances the state at Claire's [after a proper action of $\hat{P}_C(\pi)$ or $\hat{P}_C(0)$] collapses into $|\tilde{\Psi}\rangle_C = x|\alpha\rangle_C - y|-\alpha\rangle_C$. What Claire can do further is either apply to $|\tilde{\Psi}\rangle_C$ the displacement operator $\hat{D}_C(\gamma)=\exp(\gamma\hat{a}_C^+ - \gamma^*\hat{a}_C)$ with $\gamma = i\pi/2\alpha^*$ or the "phase" operator [10] $\hat{\Pi}_C^{(-)}(\phi)=\exp(-i\phi|-\alpha\rangle_{CC}\langle-\alpha|)$ with $\phi=\pi$ to obtain the state $|\Psi_1\rangle_C=\hat{D}_C(\gamma)|\tilde{\Psi}\rangle_C$ or $|\Psi_2\rangle_C = \hat{\Pi}_C^{(-)}(\pi)|\tilde{\Psi}\rangle_C$, respectively. Making use of the condition (2) the corresponding fidelities can be derived. As a result,

$$F_1 = |_C\langle\Psi_1|\Psi\rangle_C|^2 = \exp\left(-\frac{\pi^2}{4|\alpha|^2}\right), \tag{20}$$

$$F_2 = |_C\langle\Psi_2|\Psi\rangle_C|^2 = 1-2\exp(-2|\alpha|^2)[x+y+2(x+y)^2\exp(-2|\alpha|^2)]. \tag{21}$$

Although F_1 does not depend on the state to be teleported whereas F_2 depends on it, both F_1 and F_2 are exponentially becoming closer and closer to 1 when $|\alpha|$ is growing. Hence, $|\Psi_{1,2}\rangle_C$ can be looked upon as approximate states of $|\Psi\rangle_C$. The larger the value of $|\alpha|$ the smaller the difference between $|\Psi_{1,2}\rangle_C$ and $|\Psi\rangle_C$.

In order to proceed correctly to obtain the exact replica of $|\Psi\rangle_C$ or its approximate states $|\Psi_{1,2}\rangle_C$ Claire should distinguish four possibilities of Alice's measurement outcomes $\{n_A=0, n_U=$ odd/even; $n_U=0, n_A=$ odd/even$\}$ and two possibilities (Bob actually needs to count the photon number by detector D_B only) of Bob's measurement outcomes $\{n_B=0; n_B>0\}$. Therefore, the total classical communication cost is three bits, i.e., the same cost as in the protocol using GHZ-type ECS's [9].

The upper bound of the success probability of the protocol via W-type ECS's is $\Pi'_{\max}=\Pi''_{\max}=50\%$ which is achieved in the double limit $|\alpha|\to\infty$ (i.e., $z\to 0$) and $a_2\to 0$ [see Eqs. (13) and (18) or Figs. 3 and 4 as a visual aid]. This upper bound coincides with the maximally possible success probability for teleportation performed via GHZ-type ECS's [9]. So the W-type ECS is considered as an alternative way to realize teleportation, especially in case of lack of GHZ-type ECS's (there is no need to distill GHZ type from W-type states [11]). In the next section the W-type ECS exhibits its full power in a task in which GHZ-types ECS's turn out to be useless.

III. REMOTE SYMMETRIC ENTANGLING

This section is concerned with the remote symmetric entangling problem which is formulated as follows. Suppose that Alice possesses a logical qubit $|\Psi\rangle_U$ and she wishes to create entanglement between two far away collaborators, Bob and Claire, in such a way that the latter two share a symmetric entangled state either in the form

$$|\Xi_+\rangle_{BC} = N_+(|\Psi\rangle_B|\alpha\rangle_C + |\alpha\rangle_B|\Psi\rangle_C), \tag{22}$$

or in the form

$$|\Xi_-\rangle_{BC} = N_-(|\Psi\rangle_B|-\alpha\rangle_C + |-\alpha\rangle_B|\Psi\rangle_C), \tag{23}$$

where the normalization coefficients are determined by

$$N_+ = \frac{1}{\sqrt{2[1+(x+yz)^2]}} \tag{24}$$

and

$$N_- = \frac{1}{\sqrt{2[1+(xz+y)^2]}}. \tag{25}$$

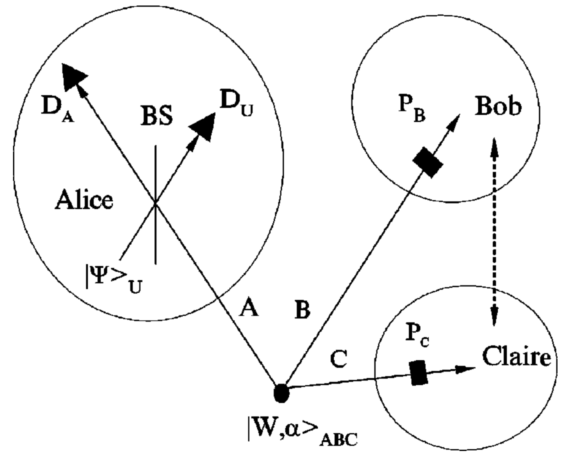

FIG. 5. Remote symmetric entangling setup. Alice, Bob, and Claire share a prior W-type ECS $|W,\alpha\rangle_{ABC}$. BS is a 50:50 beam splitter, $D_{A,U}$ photodetectors, and $P_{B,C}$ phase shifters. The dashed two-head arrow indicates a symmetric entanglement between Bob and Claire.

The task just mentioned may be called quantum symmetrization which can be exploited as an efficient way to stabilize logical-qubit-based quantum computation, similar to the usual qubit context [12]. At first thought, one might think of using something like a three-mode Fredkin gate to do the job [13]. This however requires a meeting together of all the collaborators at one location which is not the case we are addressing here. It is also verifiable that the GHZ-type ECS are not of any use for the symmetrization though they are for teleportation [9]. Interestingly enough, particular W-type ECS's come adequate as we shall demonstrate right now.

As in teleportation, let Alice, Bob, and Claire share beforehand a three-mode W-type ECS of the general form (6). For the task of symmetric entangling no ancillas are needed, and all Bob and Claire must provide themselves with are phase shifters (i.e., operators $\hat{P}_{B,C}(\varphi)$ already defined before) which might be in use. The remote entangling scheme is sketched in Fig. 5. After Alice mixes the logical qubit $|\Psi\rangle_U$ with her mode A of the shared W-type ECS by a 50:50

beam splitter, the total system will be described by the state $|\Theta\rangle_{UABC}$ which is of the form

$$|\Theta\rangle_{UABC}=|0\rangle_A[a_1x|\alpha\sqrt{2}\rangle_U|-\alpha\rangle_B|-\alpha\rangle_C+a_2y|-\alpha\sqrt{2}\rangle_U|\alpha\rangle_B|-\alpha\rangle_C+a_3y|-\alpha\sqrt{2}\rangle_U|-\alpha\rangle_B|\alpha\rangle_C]+|0\rangle_U[a_1y|-\alpha\sqrt{2}\rangle_A|-\alpha\rangle_B|-\alpha\rangle_C+a_2x|\alpha\sqrt{2}\rangle_A|\alpha\rangle_B|-\alpha\rangle_C+a_3x|\alpha\sqrt{2}\rangle_A|-\alpha\rangle_B|\alpha\rangle_C]. \tag{26}$$

Inspecting Eq. (26) reveals that the coefficients $a_{1,2,3}$ should be related as $\pm a_1=2a_2=2a_3$.

A. $a_1=2a_2=2a_3$

Let us first consider the constraint

$$a_1=2a_2=2a_3=2a \tag{27}$$

with which Eq. (26) becomes

$$a\{|0\rangle_A[(x|\alpha\sqrt{2}\rangle_U|-\alpha\rangle_B+y|-\alpha\sqrt{2}\rangle_U|\alpha\rangle_B)|-\alpha\rangle_C+|-\alpha\rangle_B(x|\alpha\sqrt{2}\rangle_U|-\alpha\rangle_C+y|-\alpha\sqrt{2}\rangle_U|\alpha\rangle_C)]+|0\rangle_U[(x|\alpha\sqrt{2}\rangle_A|\alpha\rangle_B+y|-\alpha\sqrt{2}\rangle_A|-\alpha\rangle_B)|-\alpha\rangle_C+|-\alpha\rangle_B(x|\alpha\sqrt{2}\rangle_A|\alpha\rangle_C+y|-\alpha\sqrt{2}\rangle_A|-\alpha\rangle_C)]\}. \tag{28}$$

To complete the quantum symmetrization Alice measures her output modes by detectors $D_{A,U}$ and analyzes the measurement outcomes. In the case of $\{n_A=0$ and $n_U=1,3,5,\ldots\}$ or $\{n_U=0$ and $n_A=1,3,5,\ldots\}$ the job fails. Yet, the two remaining possibilities of the measurement outcomes may smile with a success. Namely, if $\{n_A=0$ and $n_U=0,2,4,\ldots\}$ then, by application of $\hat{P}_B(\pi)\otimes\hat{P}_C(\pi)$, Bob and Claire are ready to share the symmetric entangled state (22). The probability of success is given by

$$\Pi_+=\frac{a^2}{N_+^2}\exp(-2|\alpha|^2)\cosh(2|\alpha|^2)=\frac{a^2}{2N_+^2}(1+z^2). \tag{29}$$

Alternatively, if $\{n_U=0$ and $n_A=0,2,4,\ldots\}$ then, without doing anything, Bob and Claire readily share the symmetric entangled state (23) with the success probability equal to

$$\Pi_-=\frac{a^2}{N_-^2}\exp(-2|\alpha|^2)\cosh(2|\alpha|^2)=\frac{a^2}{2N_-^2}(1+z^2). \tag{30}$$

To satisfy both conditions (7) and (27) simultaneously the quantity a is found to be

$$a^2=\frac{1}{2(3+5z^2)}. \tag{31}$$

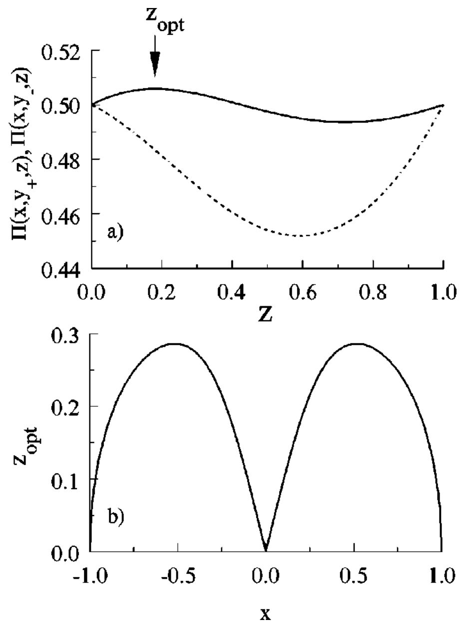

FIG. 6. (a) The probabilities of successful symmetric entangling $\Pi(x,y_+,z)=\Pi(-x,y_-,z)$, solid curve, and $\Pi(x,y_-,z)=\Pi(-x,y_+,z)$, dashed curve, as a function of z for $x=0.2$. (b) The optimal value of $z=z_{opt}$ vs x.

If we do not care which one of states (22) and (23) Bob and Claire obtain, i.e., our purpose is just to make Bob and Claire share the state $|\Psi\rangle$ in a symmetric manner, no matter it is entangled with state $|\alpha\rangle$ or state $|-\alpha\rangle$, then the two probabilities in Eqs. (29) and (30) will add to give $\Pi=\Pi_++\Pi_-$,

$$\Pi=\frac{1+z^2}{2(3+5z^2)}[4+(z^2-1)(x^2+y^2)]. \tag{32}$$

As is followed from Eq. (32), Π depends explicitly not only on $|\alpha|$ (through z) but also on x,y. Hence, the symmetric entangling we are considering is not universal in the sense that it works differently for different logical qubits. Then a question can be asked: which logical qubit is optimal? Or, in other words, what are the values of x,y, and z that maximize the success probability Π? To answer that question we pay attention to the following peculiar feature inherent to a logical qubit: x and y are themselves z dependent due to the normalization condition (2). For each z, a possible value of $x[x^2\leqslant 1/(1-z^2)]$ is associated with a pair of $y=y_\pm=-xz\pm\sqrt{1-x^2+x^2z^2}$ which depends not only on x but also on z. Curious is the fact that for a fixed value of x there exists an optimal $z=z_{opt}$ with which the success probability becomes maximal $\Pi=\Pi_{\max}>1/2$, as is illustrated by an arrow in Fig. 6(a), say, for $x=0.2$. The dependence of z_{opt} on x is drawn in Fig. 6(b).

B. $-a_1=2a_2=2a_3$

Now we turn to choose

$$-a_1=2a_2=2a_3=2\tilde{a}, \tag{33}$$

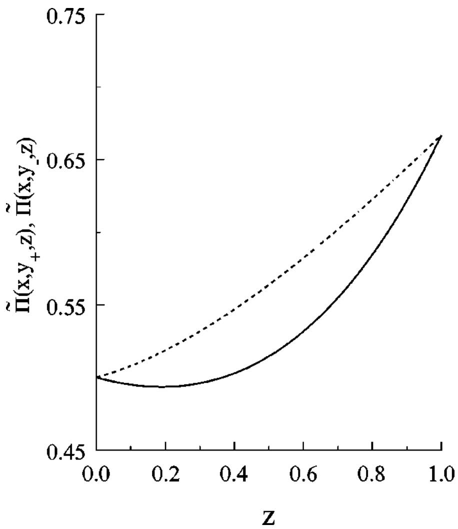

FIG. 7. The probabilities of successful symmetric entangling $\tilde{\Pi}(x,y_+,z)=\tilde{\Pi}(-x,y_-,z)$, solid curve, and $\tilde{\Pi}(x,y_-,z)=\tilde{\Pi}(-x,y_+,z)$, dashed curve, as a function of z for $x=0.2$.

which together with Eq. (7) determines $\tilde{a}$ as

$$\tilde{a}^2=\frac{1}{6(1-z^2)}. \tag{34}$$

The choice (33) can be rewritten as $a_1+a_2+a_3=0$ the qubit version of which was investigated in Ref. [14] where the associated state was called zero sum amplitude entangled state. Under this situation the state (26) turns out to be

$$\begin{aligned}\tilde{a}\{&|0\rangle_A[(-x|\alpha\sqrt{2}\rangle_U|-\alpha\rangle_B+y|-\alpha\sqrt{2}\rangle_U|\alpha\rangle_B)|-\alpha\rangle_C\\&+|-\alpha\rangle_B(-x|\alpha\sqrt{2}\rangle_U|-\alpha\rangle_C+y|-\alpha\sqrt{2}\rangle_U|\alpha\rangle_C)]\\&+|0\rangle_U[(x|\alpha\sqrt{2}\rangle_A|\alpha\rangle_B-y|-\alpha\sqrt{2}\rangle_A|-\alpha\rangle_B)|-\alpha\rangle_C\\&+|-\alpha\rangle_B(x|\alpha\sqrt{2}\rangle_A|\alpha\rangle_C-y|-\alpha\sqrt{2}\rangle_A|-\alpha\rangle_C)]\}.\end{aligned} \tag{35}$$

The symmetric entangling succeeds if $\{n_A=0$ and $n_U=1,3,5,\ldots\}$ followed by additional application of $\hat{P}_B(\pi)\otimes\hat{P}_C(\pi)$ or, if $\{n_U=0$ and $n_A=1,3,5,\ldots\}$ The total probability of success equals

$$\tilde{\Pi}=\tfrac{1}{6}[4+(z^2-1)(x^2+y^2)]. \tag{36}$$

At variant with the previous subsection, here both $\tilde{\Pi}(x,y_\pm,z)$ increase monotonically with z and the highest success probability can approach 2/3 when $z\to1$ (see Fig. 7).

C. Discussion

Two forms of W-type ECS's with their coefficients prepared so as to meet the condition (27) or (33) have been shown to accomplish the symmetric entangling between remote locations. For a better performance the condition (33) seems perhaps advantageous because of the relations $\tilde{\Pi}(x,y_\pm,z)\geqslant\Pi(x,y_\pm,z)$ which hold for any $z\in[0,1]$. In order for Bob and Claire to correctly decide whether or not they should apply the operators $\hat{P}_B(\pi)\otimes\hat{P}_C(\pi)$, two bits need to be publicly announced by Alice about her measurement outcomes. A remarkable thing to be noted here is that the equally weighted W-type ECS with $a_1=a_2=a_3$ does not suit the job of remote symmetric entangling though it does (though nonoptimally) for teleportation as demonstrated in the preceding section.

IV. EXPERIMENT EFFICIENCY

As a common matter associated with almost all schemes based on passive optics elements and photodetectors, it is of essence to reliably distinguish between neighboring numbers of detected photons. The fact why evenness and oddness are so decisive can be explicitly seen by rewriting the formulas in terms of (unnormalized) even/odd coherent states $|\alpha,\pm\rangle=|\alpha\rangle\pm|-\alpha\rangle$. For instance, under the conditions (9) and (10), Eq. (8) reduces to a form that can be cast into

$$|\Phi\rangle_{UAB'BC}\to|\Omega\rangle_{UAB'BC}+|\Omega\rangle_{AUB'BC^\pi}, \tag{37}$$

where

$$\begin{aligned}|\Omega\rangle_{UAB'BC}=&\tfrac{1}{2}a'|0\rangle_U|0\rangle_{B'}|\beta\rangle_B(|\beta,+\rangle_A|\Psi\rangle_C\\&+|\beta,-\rangle_A|\tilde{\Psi}\rangle_C)\end{aligned} \tag{38}$$

and

$$\begin{aligned}|\Omega\rangle_{AAB'BC^\pi}=&\tfrac{1}{2}a'|0\rangle_A|0\rangle_{B'}|\beta\rangle_B(|\beta,+\rangle_U\hat{P}_C(\pi)|\Psi\rangle_C\\&+|\beta,-\rangle_U\hat{P}_C(\pi)|\tilde{\Psi}\rangle_C),\end{aligned} \tag{39}$$

with $\beta=\alpha\sqrt{2}$. The entanglement between mode C and mode A (U) which now may be either in an even state $|\beta,+\rangle_{A(U)}$ or in an odd state $|\beta,-\rangle_{A(U)}$ clearly indicates the necessity of distinguishing between evenness and oddness of the counted photon number. This job is generally difficult but can be done in principle by quantum-nondemolition schemes [9,15].

So far it would have deemed all right if the photodetectors were perfect. However, though high-quantum-efficiency photon counting systems exist [16], perfect photodetectors have not been available yet in reality. We shall therefore study the problem of how experiment efficiency is sensitive to photodetector imperfection. The unavoidable imperfection may be due to several reasons such as dark counts and/or losses, etc. In what follows we shall take into account the effect due to losses. Let us assume that all our detectors have the same efficiency $\eta<1$. Such an imperfect detector could be modeled [17] by a perfect detector (with $\eta=1$) to be placed behind a lossless beam splitter BS_T of transmittivity $T=\eta$ at which the beam under consideration is superimposed on an ancilla mode prepared in the vacuum state. The part of the beam that is transmitted (with probability η) travels towards the perfect detector. The other part that is reflected (with probability $1-\eta$) is assumed to be fully absorbed by the environment. This implies that the state of the transmitted mode (which is of our interest) should be described by a reduced density matrix which is obtained by tracing out over the reflected mode. We now consider the teleportation when the condition (9) is met [the case with the condition (15) can be treated similarly and will not be presented here]. In case no photons of modes U and B' are registered, we need to deal with Eq. (38). Each of modes A and B is now split on a beam splitter $\mathrm{BS}_{T=\eta}$ before entering a respective perfect detector. For convenience, their transmitted modes are also called modes A and B, respectively. After tracing out over the reflected modes the state $|\Omega\rangle_{UAB'BC}$ becomes mixed and is characterized by the following reduced density matrix (in accordance with the action of the beam splitter $\mathrm{BS}_{T=\eta}$: $|\alpha\rangle_l|\delta\rangle_m \rightarrow |\sqrt{\eta}\alpha+\sqrt{1-\eta}\delta\rangle_l|\sqrt{1-\eta}\alpha-\sqrt{\eta}\delta\rangle_m$):

$$\begin{aligned}\rho_{UAB'BC}=&\tfrac{1}{4}a'^2|0\rangle_U\langle 0|\otimes|0\rangle_{B'}\langle 0|\otimes|\sqrt{\eta}\beta\rangle_B\langle\sqrt{\eta}\beta|\otimes[|\Psi\rangle_C\langle\Psi|\otimes(|\sqrt{\eta}\beta\rangle_A\langle\sqrt{\eta}\beta|\\&+q|\sqrt{\eta}\beta\rangle_A\langle-\sqrt{\eta}\beta|+q|-\sqrt{\eta}\beta\rangle_A\langle\sqrt{\eta}\beta|+|-\sqrt{\eta}\beta\rangle_A\langle-\sqrt{\eta}\beta|)+|\tilde{\Psi}\rangle_C\langle\tilde{\Psi}|\\&\otimes(|\sqrt{\eta}\beta\rangle_A\langle\sqrt{\eta}\beta|-q|\sqrt{\eta}\beta\rangle_A\langle-\sqrt{\eta}\beta|-q|-\sqrt{\eta}\beta\rangle_A\langle\sqrt{\eta}\beta|+|-\sqrt{\eta}\beta\rangle_A\langle-\sqrt{\eta}\beta|)\\&+|\Psi\rangle_C\langle\tilde{\Psi}|\otimes(|\sqrt{\eta}\beta\rangle_A\langle\sqrt{\eta}\beta|-q|\sqrt{\eta}\beta\rangle_A\langle-\sqrt{\eta}\beta|+q|-\sqrt{\eta}\beta\rangle_A\langle\sqrt{\eta}\beta|\\&-|-\sqrt{\eta}\beta\rangle_A\langle-\sqrt{\eta}\beta|)+|\tilde{\Psi}\rangle_C\langle\Psi|\otimes(|\sqrt{\eta}\beta\rangle_A\langle\sqrt{\eta}\beta|+q|\sqrt{\eta}\beta\rangle_A\langle-\sqrt{\eta}\beta|-q|-\sqrt{\eta}\beta\rangle_A\langle\sqrt{\eta}\beta|\\&-|-\sqrt{\eta}\beta\rangle_A\langle-\sqrt{\eta}\beta|)],\end{aligned}\tag{40}$$

where the important parameter

$$q=z^{2(1-\eta)}\tag{41}$$

depends not only on degree of the coherent state nonorthogonality but also on degree of detector imperfection. The probability $P_{\pm}$ that the perfect detectors behind the beam splitters $\mathrm{BS}_{T=\eta}$ register a nonzero photon number of mode B and an even/odd photon number of mode A are derived in the form

$$P_{\pm}=\tfrac{1}{2}a'^2(1-z^{\eta})(1\pm z^{2\eta}).\tag{42}$$

If an even/odd photon number is detected, the state of mode C turns out to be a mixed state which is characterized by the density matrix

$$\rho_{C,\pm}=\frac{1\pm q}{2}|\Psi\rangle_C\left\langle\Psi\right|+\frac{1\mp q}{2}\left|\tilde{\Psi}\right\rangle_C\langle\tilde{\Psi}|.\tag{43}$$

In case no photons of modes A and B' are registered, we need to deal with Eq. (39). By doing similar calculations as for Eq. (38) we get the following result. The probability that $n_{B'}>0$ and n_U is even/odd is given by the same formula (42). But in this case, if an even/odd photon number of mode U is detected, the state of mode C is described by the following density matrix:

$$\begin{aligned}\rho'_{C,\pm}=&\frac{1\pm q}{2}\hat{P}_C(\pi)|\Psi\rangle_C\langle\Psi|\hat{P}_C^{+}(\pi)\\&+\frac{1\mp q}{2}\hat{P}_C(\pi)|\tilde{\Psi}\rangle_C\langle\tilde{\Psi}|\hat{P}_C^{+}(\pi).\end{aligned}\tag{44}$$

Hence, up to the action of $\hat{P}_C(\pi)$, the density matrix $\rho_{C,\pm}$ results with the total probability $2P_{\pm}$. We see that, either an even or odd photon number is detected, Claire gets, instead of a pure state as in the case of perfect detectors, a statistical mixture of two unequally-weighted states, one which is desired ($|\Psi\rangle_C$ or $\hat{P}_C(\pi)|\Psi\rangle_C$) and the other which is undesired ($|\tilde{\Psi}\rangle_C$ or $\hat{P}_C(\pi)|\tilde{\Psi}\rangle_C$). Of course, for nearly perfect detectors ($1-\eta\ll 1$, $q\simeq 1$), detection of an even photon number seems good enough because in this case the probability for getting $|\Psi\rangle_C$ (or $\hat{P}_C(\pi)|\Psi\rangle_C$) is much higher than that for getting $|\tilde{\Psi}\rangle_C$ (or $\hat{P}_C(\pi)|\tilde{\Psi}\rangle_C$). In the limit $\eta\rightarrow 1$, we have $q\rightarrow 1$, $2P_+\rightarrow\Pi'$ [see Eq. (13)] and $\rho_{C,+}\rightarrow|\Psi\rangle_C\langle\Psi|$ (which is the right pure state), as it should be.

We can go along the same "technical" line as above to study the effect of detector imperfection for remote symmetric entangling. Below we only present the final result for the case when the coefficients satisfy the condition (27) and when $n_U=0$ and $n_A=0,2,4,\ldots$ are detected. Then, instead of Π_- given by Eq. (30), one has

$$\Pi_-\rightarrow\frac{1}{4}a^2\left(\frac{1+q}{N_-^2}+\frac{1-q}{\tilde{N}_-^2}\right)(1+z^{2\eta}),\tag{45}$$

with $\tilde{N}_-^2=1/[2(xz-y)^2]$, while Bob and Claire, instead of the pure entangled state $|\Xi_-\rangle_{BC}$ defined by Eq. (23), obtain the mixed state

$$\begin{aligned}|\Xi_-\rangle_{BC}\rightarrow\rho_{BC,-}=&\frac{1+q}{2}|\Xi_-\rangle_{BC}\\&\times\left\langle\Xi_-\right|+\frac{1-q}{2}\left|\tilde{\Xi}_-\right\rangle_{BC}\langle\tilde{\Xi}_-|,\end{aligned}\tag{46}$$

where $|\tilde{\Xi}_-\rangle_{BC}=\tilde{N}_-(|\tilde{\Psi}\rangle_B|-\alpha\rangle_C+|-\alpha\rangle_B|\tilde{\Psi}\rangle_C)$. Transparently, the results (45) and (46) recover those of perfect detectors which are respectively given by Eqs. (30) and (23).

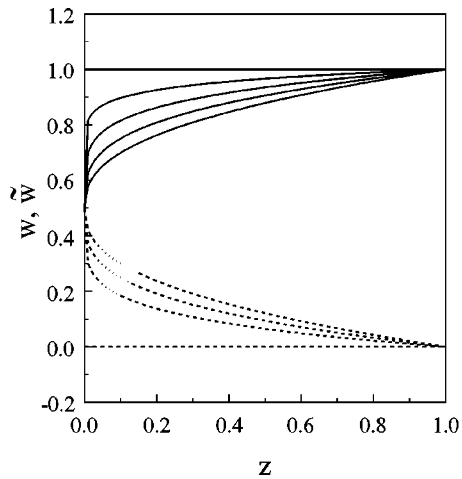

FIG. 8. Solid (dashed) curves represent the weight w ($\tilde{w}$) of $|\Psi\rangle_C$ and $|\Xi_-\rangle_{BC}$ ($|\tilde{\Psi}\rangle_C$ and $|\tilde{\Xi}_-\rangle_{BC}$) in $\rho_{C,-}$ and $\rho_{BC,-}$, respectively, as a function of z for various values of $\eta = 1$, 0.95, 0.9, 0.85, and 0.8 counted from top (bottom).

A common message followed from Eqs. (43), (44), and (46) is that in both teleportation and symmetric entangling the weights $w = (1+q)/2$ and $\tilde{w} = (1-q)/2$ of the desired state and the undesired state, respectively, are solely determined by the parameter q. In Fig. 8 we plot those weights in dependence on both z and η. This figure shows that, for a given degree of detector imperfection $\eta < 1$, the experiment efficiency (in this work we define experiment efficiency by the difference $w - \tilde{w}$: $w - \tilde{w} = 1$ means obtaining the desired pure state and $w - \tilde{w} = 0$ corresponds to a maximally mixed state) increases with z. Also, as it is, for $\eta = 1$ (perfect detectors), one identically has $w = 1$ and $\tilde{w} = 0$, irrespective of z.

V. CONCLUSION

In this work *W*-type entangled coherent states have been touched upon for the first time to perform two explicit tasks of processing quantum information encoded in logical qubits of the form of a linear superposition of two antiphased coherent states. The first task is teleportation which can also be done via states of the GHZ type. The second task is remote symmetric entangling which strictly requires use of *W*-type states. For each task the coefficients of the *W*-type entangled coherent state should be properly prepared so as to satisfy certain constraints, apart from the normalization condition. An optimal performance can be achieved by adjusting values of the conditioned coefficients. The effect of detector imperfection is also taken into account. While perfect detectors allow to get (probabilitically) an exact replica of the teleported state as well as a pure desired symmetric entangled state, imperfection of detectors results in a mixture between the desired state and its ‘‘complementary’’ (undesired) one. Detailed calculations signal that a given degree of detector imperfection ($\eta < 1$) could be ‘‘tolerated’’ by using entangled coherent states with a higher value of z so as the resulting (mixed) state would better resemble the desired (pure) state. This result can simply be gained by observing that the experiment efficiency defined by the difference $w - \tilde{w}$ coincides exactly with the single parameter $q = z^{2(1-\eta)}$ which, for a fixed $\eta < 1$, increases with z. Extension to the case of an arbitrary number of remote collaborators is straightforward. What remains problematic and is presently in progress is how to generate an appropriate *W*-type entangled coherent state in practice. Although generation schemes for *W*-states [18] and GHZ-type states [19] exist, those for *W*-type entangled coherent states have still to wait to be discovered.

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