Entanglement for excitons in two quantum dots in a cavity injected with squeezed vacuum

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The dynamical behavior of entanglement for excitons in two quantum dots placed in a cavity driven by a broadband squeezed vacuum is studied in the low exciton density regime. It is found that the excitons in two quantum dots can evolve into a pure entangled state only when the dissipation of the cavity and exciton modes is negligible. When the injected field into the cavity is a squeezed vacuum, the excitons will be in a two-mode Gaussian entangled mixed state even in the long time limit. Analytical expressions are derived, showing the dependence of the entanglement property of the excitons in two quantum dots on the squeezing properties of the initial state prepared in the cavity and the strength of two-photon correlations of the squeezed vacuum injected into the cavity.

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Quantum entanglement constitutes the single most characteristic property that makes quantum mechanics distinct from any classical theory. Entangled states are not only used to test fundamental quantum-mechanical principles such as Bell's inequalities [1] but also play a central role in practical applications of the quantum information theory such as quantum computation [2], quantum teleportation [3], and quantum cryptography [4]. How to create an entangled state in a proper way is thus an important issue.

Recently, there has been growing interest in the quantum information properties of semiconductor quantum dots in the quest to implement quantum-dot scalable quantum computers. This interest is stimulated to some extent by recent experimental advances in the coherent observation and manipulation of quantum dots, including the demonstration of the quantum entanglement of excitons in a single dot [5], the observations of Rabi oscillations of excitons in a single dot [6], and the demonstration of a quantum dot as single-photon source [7]. Sanders *et al.* [8] have discussed the scalable solid-state quantum computer based on the electric dipole transitions within coupled single-electron quantum dots. Imamoglu [9] proposed a scheme to realize a controlled-NOT gate between two distant quantum dots via the cavity quantum electrodynamic (CQED) techniques. In a recent investigation by Liu *et al.* [10] into the generation of bipartite entangled coherent excitonic states in a system of two coupled quantum dots and CQED with dilute excitons, they found that bipartite maximally entangled coherent excitonic states can be generated when the initial cavity field is in an odd coherent state. This investigation was extended by Liu *et al.* [11] and Wang *et al.* [12] to study multipartite entanglement in a system of N quantum dots embedded in a cavity. It has been pointed out that at certain times, multipartite entangled excitonic coherent states or multiqubit W state can be created by initially preparing the cavity field in a superposition of coherent state or in the Fock state with one photon, respectively.

In this Brief Report, we study the generation of an entangled excitonic state in a system of two identical quantum dots embedded in a single-mode cavity, injected with a broadband squeezed vacuum in the low exciton density regime. The time-dependent Wigner function for excitons is solved analytically when the cavity field is initially in coherent squeezed state and the excitons in their vacuum state. It is found that if the dissipation of the cavity and exciton modes is neglected, the excitons in two quantum dots evolve into a pure entangled state periodically. When the dissipation of the cavity and exciton modes is included, the excitons in two quantum dots can be entangled and evolve into a two-mode mixed entangled state even in the long time limit if the field injected into the cavity is a squeezed vacuum. The dynamical behavior of the entanglement between excitons will turn out to be strongly dependent on the squeezing property of the initial field in the cavity and on the strength of two-photon correlations of the squeezed vacuum injected into the cavity.

The model under consideration consists of two quantum dots that are placed in a single-mode cavity. We assume that the radius R of each quantum dot is much larger than the Bohr radius a_B of excitons, but smaller than the wavelength λ of the cavity field, i.e., $a_B \ll R \leqslant \lambda$. For example, for Cds material, $a_B = 3$ nm, $R = 20$ nm, and $\lambda \approx 500$ nm, for GaAs, $a_B = 10$ nm, $R = 50$ nm, and $\lambda \approx 800$ nm [13], the above condition is satisfied. We also assume that there are only a few electrons excited from the valence-band to the conduction-band in each quantum dot. Then the exciton number in the ground state for each quantum dot is low, so that the mean distance between two excitons is much larger than the extension of an exciton. In this case, the excitons in each quantum dot can be approximated as a dilute boson gas and can then be described by boson operators [10–13]. In this approximation, all nonlinear interaction terms such as exciton-exciton interaction can be neglected. As the distance between the two quantum dots is also assumed to be large, the interaction between any two excitons can also be safely ignored. The excitons are assumed to be resonant with the eigenmode of the cavity field. In the rotating-wave approximation, the Hamiltonian for the system can be written as

$$H = H_0 + H_{int} = \omega a^\dagger a + \omega \sum_{j=1}^{2} b_j^\dagger b_j + \sum_{j=1}^{2} \sqrt{2} g_j (b_j^\dagger a + a^\dagger b_j), \qquad (1)$$

where $a^\dagger$ and a are the creation and annihilation operators of the cavity field with frequency ω, $b_j^\dagger$ and b_j represent the creation and annihilation operators of the excitons in the jth quantum dot. The parameters g_j are the coupling constants between the jth quantum dot and the cavity field. In order to simplify the calculation, the two dots are assumed to be identical and equally coupled to the cavity field: $g_1=g_2=g$. We also assume that the cavity mode is coupled to a broadband squeezed reservoir through the lossy output mirror of the cavity. Then the density operator $\rho(t)$ of the photon-exciton interaction system in the interaction picture obeys the equation

$$(d/dt)\,\rho=-i[H_{int},\rho]+L_a\rho+L_b\rho, \tag{2}$$

with

$$\begin{aligned}L_a\rho=&\gamma_a(N+1)(2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a)+\gamma_a N(2a^\dagger\rho a\\&-aa^\dagger\rho-\rho aa^\dagger)-\gamma_a M(2a^\dagger\rho a^\dagger-a^{\dagger 2}\rho-\rho a^{\dagger 2})\\&-\gamma_a M^*(2a\rho a-a^2\rho-\rho a^2),\\L_b\rho=&\gamma_e(N_e+1)\sum_{j=1}^{2}(2b_j\rho b_j^\dagger-b_j^\dagger b_j\rho-\rho b_j^\dagger b_j)\\&+\gamma_e N_e\sum_{j=1}^{2}(2b_j^\dagger\rho b_j-b_jb_j^\dagger\rho-\rho b_jb_j^\dagger).\end{aligned} \tag{3}$$

Here $L_b\rho$ and $L_a\rho$ denote, respectively, the dissipation of the exciton modes and the dissipation of the cavity mode which is coupled to a broadband squeezed vacuum [14]. The parameter γ_e represents the decay rate of the excitons, and γ_a is the decay rate of the cavity field, N and $M=|M|e^{i\theta}$ characterize the broadband squeezed vacuum injected into the cavity, such that $|M|^2\leqslant N(N+1)$. The equality condition holds for the minimum uncertainty squeezed states.

The master equation (2) can be solved analytically by use of the method of the characteristic function. Introducing two new boson operators $b_\pm$, which are defined as $b_\pm=(b_1\pm b_2)/\sqrt{2}$, the characteristic function for the field-exciton coupling system in the Wigner representation can be expressed as

$$\begin{aligned}\chi(\xi_a,\xi_+,\xi_+,t)=&\mathrm{Tr}[\rho\exp(-\xi_a^*a+\xi_a a^\dagger-\xi_+^*b_++\xi_+b_+^\dagger\\&-\xi_-^*b_-+\xi_-b_-^\dagger)].\end{aligned} \tag{4}$$

Using the standard operator correspondence we find that the characteristic function obeys

$$\begin{aligned}&\chi(\xi_a,\xi_+,\xi_-,t)\\&=\exp\Bigg\{t\Bigg\{-ig\left(\xi_+^*\frac{\partial}{\partial\xi_a^*}-\xi_a\frac{\partial}{\partial\xi_+}+\xi_a^*\frac{\partial}{\partial\xi_+^*}-\xi_+\frac{\partial}{\partial\xi_a}\right)\\&\quad-\gamma_a\Bigg[\xi_a\frac{\partial}{\partial\xi_a}+\xi_a^*\frac{\partial}{\partial\xi_a^*}+2(N+1)|\xi_a|^2\\&\quad-(M^*\xi_a^2+M\xi_a^{*2})\Bigg]-\gamma_e\sum_{j=\pm}\Bigg[\xi_j\frac{\partial}{\partial\xi_j}+\xi_j^*\frac{\partial}{\partial\xi_j^*}\\&\quad+2(N_e+1)|\xi_j|^2\Bigg]\Bigg\}\Bigg\}\chi(\xi_a,\xi_+,\xi_-,0).\end{aligned} \tag{5}$$

Assuming initially that the field is in a coherent squeezed state $|\Psi_F\rangle=\exp[\alpha_0(a-a^\dagger)]\exp[(r/2)(a^2e^{-i\phi}-a^{\dagger 2}e^{i\phi})]|0\rangle$ [14] and the excitons in the two quantum dots in vacuum states $|0_{b_1},0_{b_2}\rangle=|0_{b_+},0_{b_-}\rangle$, we can obtain the time-dependent characteristic function for the field-exciton coupling system explicitly as a Gaussian function when we set $\gamma_a=\gamma_e=\gamma$. Conversely, if we set $\gamma_a\neq\gamma_e$, the time-dependent characteristic function does not take the form of a Gaussian function, which is a key form to discuss the properties of purity and entanglement [16]. In particular, for a practical cavity, $\gamma_a\approx 10^{11}$ Hz [9], and for the large size quantum dots such as CdS or GaAs, the excitonic decay rate γ_e may also reach γ_a [13], so the assumption of $\gamma_a=\gamma_e$ is reasonable. In order to discuss the entanglement between the excitons in the two quantum dots, we only consider the case of $\gamma_a=\gamma_e$ in the following.

After having the characteristic function for the field-exciton coupling system, the corresponding Wigner function for the excitons in the bare excitonic representation can be expressed in a compact form as

$$W_e(\alpha_1,\alpha_2,t)=[1/(2\pi^2\sqrt{a_1a_2}a_3)]\exp(-\tfrac{1}{2}\alpha^\dagger D\alpha), \tag{6}$$

in which $\alpha^\dagger=\{[\alpha_1^*-(i\alpha_0/\sqrt{2})e^{-\gamma t}\sin gt]e^{-i\psi}$, $[\alpha_1+(i\alpha_0/\sqrt{2})e^{-\gamma t}\sin gt]e^{i\psi}$, $[\alpha_2^*-(i\alpha_0/\sqrt{2})e^{-\gamma t}\sin gt]e^{-i\psi}$, $[\alpha_2+(i\alpha_0/\sqrt{2})e^{-\gamma t}\sin gt]e^{i\psi}\}$, α_1 and α_2 are the complex parameters corresponding to the annihilation operators b_1 and b_2 of the excitons, respectively, $a_1=h_1+|h_2|$, $a_2=h_1-|h_2|$, $a_3=\frac{1}{2}[N_e(1-e^{-2\gamma t})+\frac{1}{2}]$, and

$$h_1=(N+N_e+1)f_1-(N-N_e)f_2+\tfrac{1}{2}e^{-2\gamma t}(\cosh 2r\sin^2 gt+\cos^2 gt), \tag{7}$$

$$h_2=e^{-i\theta}[|M|(f_2-f_1)+\tfrac{1}{2}\sin^2 gt\sinh 2re^{-2\gamma t}e^{i(\theta-\phi)}]=|h_2|e^{i2\psi}, \tag{8}$$

$$D=\frac{1}{4}\begin{pmatrix}1/a_1+1/a_2+1/a_3 & 1/a_1-1/a_2 & 1/a_1+1/a_2-1/a_3 & 1/a_1-1/a_2\\ 1/a_1-1/a_2 & 1/a_1+1/a_2+1/a_3 & 1/a_1-1/a_2 & 1/a_1+1/a_2-1/a_3\\ 1/a_1+1/a_2-1/a_3 & 1/a_1-1/a_2 & 1/a_1+1/a_2+1/a_3 & 1/a_1-1/a_2\\ 1/a_1-1/a_2 & 1/a_1+1/a_2-1/a_3 & 1/a_1-1/a_2 & 1/a_1+1/a_2+1/a_3\end{pmatrix}, \tag{9}$$

where $f_1=\frac{1}{2}(1-e^{-2\gamma t})$, $f_2=\mathrm{Re}\{[\gamma/2(\gamma-ig)](1-e^{-2(\gamma-ig)t})\}$.

The time-dependent Weyl-Wigner characteristic function for the excitons in the two quantum dots is defined as

$$\chi_e(\beta_1,\beta_2,t)=Tr_e[Tr_a(\rho(t))\exp(\beta_1 b_1^\dagger-\beta_1^* b_1+\beta_2 b_2^\dagger-\beta_2 b_2)]=\exp(-\tfrac{1}{2}\beta^\dagger V\beta), \quad (10)$$

where $\beta^\dagger=(\beta_1^*,\beta_1,\beta_2^*,\beta_2)$ is a four-vector and the matrix V is a 4×4 covariance matrix for the two modes. The matrix V can be obtained from the matrix D through the relation between the Weyl-Wigner characteristic function $\chi_e(\beta_1,\beta_2,t)$ and the Wigner function $W_e(\alpha_1,\alpha_2,t)$ as defined by Eq. (6). They are related by a Fourier transform which leads to $D=EV^{-1}E$, in which $E=\mathrm{diag}[1,-1,1,-1]$ is a diagonal 4×4 matrix. From this the matrix V can be expressed explicitly. Evidently, the matrices D and V are independent of the amplitude α_0 of the initial coherent squeezed field. This means that the properties of the excitons in two quantum dots such as the purity and the entanglement related to the matrices D and V are independent of α_0.

Gaussian operators which are projectors represent pure states [15]. The condition for a Gaussian Wigner function to represent a pure state may be written concisely as $\sqrt{\det D}=4$. The Gaussian operator corresponding to Eq. (6) is a projector provided that

$$a_3\sqrt{a_1 a_2}=\tfrac{1}{8}. \quad (11)$$

From this condition, we can find that only if $\gamma=0$ and $gt=n\pi$ or $(n+1/2)\pi(n=0,1,2,\dots)$, the excitons evolve into a pure state. This means that the pure state for the excitons in two quantum dots can only be generated when the dissipation is absent and the field is decoupled from the excitons. The presence of the dissipation always produces a mixed state for excitons no matter whether or not the cavity is injected with squeezed vacuum. However, the separability of the resulting mixed state for the excitons depends strongly on the property of the field injected into the cavity.

A separability criterion for two-mode Gaussian states has been established by Simon [16]. It relies on the partial transposition map acting on the two-party state. It has been shown that this criterion is equivalent to judging whether or not the quantum state is P representable. A two-mode Gaussian state is P representable, and hence, separable, if and only if $V-\frac{1}{2}I\geqslant0$, where I is the 4×4 unit matrix and V is the covariance matrix for the Weyl-Wigner characteristic function defined in Eq. (10). The two-mode state for the excitons described by Eq. (6) is therefore separable if and only if all the eigenvalues of $V-\frac{1}{2}I$ are non-negative. The four eigenvalues are

$$\lambda_{1,2}=N_e(1-e^{-2\gamma t}), \quad \lambda_\pm=h_1\pm|h_2|-\tfrac{1}{2}. \quad (12)$$

The eigenvalues λ_1 and λ_2 are degenerate. It is easily verified that the first three eigenvalues λ_1, λ_2, and λ_+ are positive, so they are not important for establishing the nonseparability of the state. The fourth eigenvalue λ_- can be negative to determine the nonseparability of the state.

First we consider that the coupling between the cavity field and the dots is strong (for example, the coupling constant $g\approx7.6\times10^{11}$ Hz may be possible to obtain in a system of large area quantum dots with cavity, and the decay rate $\gamma\approx10^{11}$ Hz [9,13]), so that the dissipation effect is neglected. In this case, λ_- is simplified to

$$\lambda_-=\tfrac{1}{2}(e^{-2r}-1)\sin^2 gt. \quad (13)$$

We note that, if the initial field in the cavity is a squeezed field (i.e., $r\neq0$), except at time points $gt=n\pi(n=0,1,2,\dots)$, λ_- is negative, which means that the excitons in two quantum dots are entangled periodically, intermediated by the interaction with the cavity field. At time points $gt=(n+1/2)\pi$, the entanglement between the excitons in two quantum dots reaches its maximum. This is similar to the case of two quantum dots interacting with a single-mode cavity field initially prepared in odd or even coherent states [10]. In fact, at $gt=n\pi$, the excitons in two quantum dots evolve into their ground state and the field evolve into its initial coherent squeezed state. And at $gt=(n+1/2)\pi$, the field evolves into a vacuum state and the excitons in the two quantum dots in a pure entangled state $\exp[(\alpha_0/\sqrt{2})(b_1+b_2-b_1^\dagger-b_2^\dagger)]\exp\{(r/4)[(b_1+b_2)^2e^{-i\phi}-(b_1^\dagger+b_2^\dagger)^2e^{i\phi}]\}\times|0_{b_1},0_{b_2}\rangle$. So at $gt=n\pi$ or $gt=(n+1/2)\pi$, the excitons in two dots are in pure states as mentioned above. The entanglement between the excitons in two quantum dots depends on the squeezing degree r of the initial field and is independent of the amplitude α_0. With increasing of the squeezing degree r of the initial field, the entanglement evidently increases. For $r=0$, which means that the initial field is in a coherent state, the excitons in two quantum dots cannot be entangled. That is to say, the appearance of entanglement between excitons in two quantum dots requires the existence of the nonclassical property of the cavity field, as pointed out by Kim *et al.* [17] in the case of a beam splitter.

Second we assume that the cavity field is initially in vacuum state, i.e., $r=0$ and $\alpha_0=0$, but the cavity mode is coupled to a broadband squeezed vacuum through lossy mirror. In this case, the eigenvalue λ_- becomes

$$\lambda_-=(N-|M|)(f_1-f_2)+N_e(f_1+f_2). \quad (14)$$

The second term in the above equation is always positive reflecting the thermal excitons is harmful for the entanglement between excitons in two quantum dots. Since $f_1-f_2\geqslant0$, λ_- may be negative only when $|M|>N$. Therefore, the appearance of the entanglement between excitons in two quantum dots requires that the two-photon correlation of the broadband squeezed vacuum injected into the cavity is nonclassical, since $|M|=N$ corresponds to the maximal classical correlations between pairs of photons. The linear interaction between the excitons and the cavity field transfers the nonclassical two-photon correlation into the entanglement between the excitons in two dots. The maximal entanglement happens at $|M|=\sqrt{N(N+1)}$, which, corresponding to the broadband squeezed vacuum being injected into the cavity, is an ideal squeezed vacuum. For $N_e=0$, except at $t=0$, the excitons in two quantum dots can be entangled if there exists a nonclassical two-photon correlation of the broadband

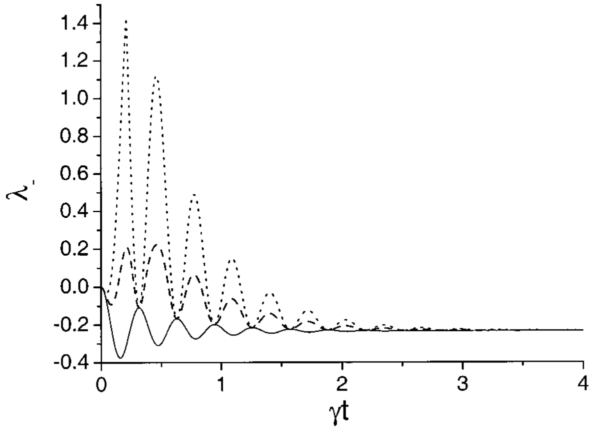

FIG. 1. λ_- vs γt for different relative phases $\theta-\phi$, in which $g=10\gamma$, $N=4$, $M=2\sqrt{5}$, and $r=1.0$, and $\theta-\phi=\pi$ (solid), $\pi/2$ (dashed), and 0 (dotted).

squeezed vacuum injected into the cavity. In the long time limit, the excitons are in a steady-state mixed entangled state. The negativity λ_- of this two-mode Gaussian entangled state in the long time limit is equal to $(N-|M|)g^2/2(\gamma^2+g^2)$. Increasing the coupling constant g, the entanglement increases. If g is much larger than the dissipation rate γ, $\lambda_-(\infty)$ approaches its saturation value $(N-|M|)/2$. Clark and Parkins [18] intensively discussed the relation of the pure state and the entangled state of two atoms with the squeezing state of the reservoir, and found that the entanglement between the two atoms only needs the two-photon correlation of pairs in the reservoir obeying $0<|M|\leqslant\sqrt{N(N+1)}$. But here the entanglement between the excitons in two dots requires $N<|M|\leqslant\sqrt{N(N+1)}$. This difference is caused by that here the excitons in two quantum dots are treated as dilute boson gas.

Figure 1 displays the time evolution of the negativity λ_- for the case in which the cavity field is initially in a squeezed vacuum and the cavity mode is also damped by a broadband squeezed vacuum. The long time behavior of λ_- is independent of the nature of the initial field and is governed by the property of the broadband squeezed vacuum injected into the cavity. The short time behavior of λ_- is decided by the properties of both the initial preparation in the cavity and the broadband squeezed vacuum injected into cavity. If the relative phase angle between the squeezing angles for the broadband squeezed vacuum and the initial coherent squeezed state in the cavity is chosen as $\theta-\phi=\pi$, then, for $N_e=0$, λ_- becomes

$$\lambda_-=(N-|M|)(f_1-f_2)+\tfrac{1}{2}(e^{-2r}-1)\sin^2 gt. \qquad (15)$$

The above equation is just a superposition of Eqs. (13) and (14). This is because at this relative phase angle, the linear interaction between the cavity field and the excitons in two dots transfers the nonclassical properties of the two fields into the entanglement for the excitons maximally. It appears that with an increasing squeezing degree of the initial cavity field, the maximum value of λ_- for short times can be larger than its steady-state value. The larger the squeezing degree r of the initial field is, the stronger the entanglement at short time. However, for $\theta-\phi\neq\pi$, the entanglement between the excitons in two dots may disappear. That is to say, whether the entanglement between the excitons in the short time region is enhanced by the initial coherent squeezed state is decided by the relative phase angle between the two squeezing angles θ and ϕ.

In summary, the dynamical behavior of entanglement for excitons in two quantum dots placed in a cavity driven by a broadband squeezed vacuum is studied in the low exciton density regime. The time-dependent Wigner function for excitons in two quantum dots is solved analytically when the cavity field is initially in a coherent squeezed state and the excitons in their vacuum state. When the dissipation of the cavity and exciton modes is included, the excitons in two quantum dots evolve into mixed states. When the injected field into the cavity is a squeezed vacuum, the excitons will be in a two-mode Gaussian entangled state even in the long time limit. In the short time region, the entanglement property of excitons in two quantum dots depends on the squeezing property of the initial state prepared in the cavity and the strength of the two-photon correlations of the squeezed vacuum injected into the cavity. But in the long time limit, the entanglement property of excitons is independent of the initial state prepared in the cavity.

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