# Analysis of resonances in Møller scattering in a laser field of relativistic radiation power

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Presently available laser sources can yield powers for which the ponderomotive energy $U_p$ of an electron can be equal to or even larger than the rest energy $mc^2$ of an electron. Therefore it has become of interest to consider fundamental radiation-induced or assisted processes in such powerful laser fields. In the present work we consider laser-assisted electron-electron scattering in such a field, assuming that the laser beam has linear polarization. We investigate in detail the angular and polarization dependence of the differential cross sections of the laser-assisted nonlinear processes as a function of the order $N$ of absorbed or emitted laser photons $\omega$. The present work is a continuation of our previous analysis of Compton scattering and of Mott scattering in a linearly polarized laser field [Phys. Rev. A **65**, 022712 (2002); **65**, 033408 (2002)].

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## I. INTRODUCTION

The relativistic treatment of electron-electron scattering in quantum electrodynamics is called Møller scattering [1]. Details on this process can be found in the older book by Heitler [2] and shorter accounts can be found in the books by Bjorken and Drell [3] and by Itzykson and Zuber [4]. With the advent of the laser, people became interested in investigating fundamental processes of quantum electrodynamics in the presence of a laser field. Such processes were, in particular, Compton scattering [5] and electron-atom scattering [6]. Surveys of early work can be found in Refs. [7,8]. Electron-electron scattering in a laser field was investigated for the first time by Oleĭnik [9] who showed that the effective cross sections of Møller scattering contain resonances connected with the discrete nature of the energy spectrum of the electron+plane electromagnetic wave system. Similar features can apparently be found in laser-assisted potential scattering of electrons [10]. The resonance phenomena in laser-assisted Møller scattering were reinvestigated in more detail, including considerations in the nonrelativistic limit, by Bös *et al.* [11] and by Borisov *et al.* [12] who included radiative corrections to eliminate the resonance divergences. A nonrelativistic treatment of electron-electron scattering in a laser field was presented by Bergou *et al.* [13] and by Krainov and Roshchupkin [14]. In all these calculations the laser field is represented by a monochromatic, circularly polarized electromagnetic plane wave. A survey of this early work on laser-assisted Møller scattering can be found in the review by Fedorov [15]. The possibility of the formation of bound states in the electron-electron system at very high laser powers was investigated by Kazantsev and Sokolov [16,17]. The problem of laser-assisted Møller scattering at high radiation powers, in the resonant and nonresonant regions, was reinvestigated more recently in several papers by Roshchupkin [18,19] and Denisenko and Roshchupkin [20,21]. A survey of the most recent work on the present topic and related processes can be found in the paper by Roshchupkin *et al.* [22]. In the above investigations, comparatively little numerical work was done to manage in general the evaluation of the generalized Bessel functions that appear in the matrix elements of all these laser-dressed fundamental processes of quantum electrodynamics.

After having successfully dealt with Compton scattering and Mott scattering in the presence of a monochromatic, elliptically polarized electromagnetic plane wave field of relativistic radiation power such that the ponderomotive energy of a free electron becomes of the order of magnitude or even larger than the electron's rest energy, i.e., $U_p \gtrsim mc^2$ [23–26], we shall reinvestigate in the present work Møller scattering under similar conditions, since presently available laser sources are able to produce such high radiation field intensities. We shall derive in the next section the general cross section formula of Møller scattering in an elliptically polarized laser field and perform in Sec. III a detailed analysis of the scattering resonance conditions. On the basis of this analysis we shall present in Sec. IV some characteristic numerical examples of cross section data in the vicinity of these resonances at moderately high radiation powers. In Sec. V we shall summarize our findings and present some concluding remarks. We shall set $\hbar = 1$ throughout this work.

## II. LASER-DRESSED MØLLER CROSS SECTIONS

The $S$-matrix element of Møller scattering in a laser field can be easily written down by using, for example, the matrix element for Møller scattering, presented in the book of Bjorken and Drell [3], and by replacing the free particle wave functions by the corresponding Volkov solutions [27] of electrons in an electromagnetic plane wave field. For two bare ingoing electrons of four momenta $(p_i, q_i)$ and corresponding outgoing electrons of momenta $(p_f, q_f)$ we thus find

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$$S_{fi}=S(p_f,q_f;p_i,q_i)$$
$$=-\frac{i}{c}\int dxdy[\overline{\Psi}_{p_f}(x)e\gamma^{\mu}\Psi_{p_i}(x)]D_{\mu\nu}(x-y)$$
$$\times[\overline{\Psi}_{q_f}(y)e\gamma^{\mu}\Psi_{q_i}(y)]+\frac{i}{c}\int dxdy\cdots p_f\leftrightarrow q_f\cdots \tag{1}$$

in which the photon propagator $D_{\mu\nu}(x-y)$ is given by

$$D_{\mu\nu}(x-y)=\int\frac{d\mathcal{K}}{(2\pi)^4}\tilde{D}_{\mu\nu}(\mathcal{K})e^{-i\mathcal{K}\cdot(x-y)},$$
$$\tilde{D}_{\mu\nu}(\mathcal{K})=\frac{1}{\mathcal{K}^2}\left(g_{\mu\nu}-\kappa\frac{\mathcal{K}_\mu\mathcal{K}_\nu}{\mathcal{K}^2}\right), \tag{2}$$

and $\kappa$ is the gauge-fixing constant. The electron wave functions $\Psi_{p_{i(f)}}(x)$ and $\Psi_{q_{i(f)}}(y)$ are Volkov solution. Thus we can take over certain results from our calculations of Compton scattering and Mott scattering [23,25]. For a plane electromagnetic wave, $A_\mu(k\cdot x)$, with $k^2=0$, the normalized Volkov solution for an electron of four momentum $p$ reads

$$\Psi_p(x)=\sqrt{\frac{m}{VE_p}}f(x;p)\left(1-\frac{e\gamma^\nu A_\nu(k\cdot x)\gamma^\mu k_\mu}{2k\cdot p}\right)u_p,$$
$$f(x;p)=\exp\left(-ip\cdot x-i\int_{-\infty}^{k\cdot x}\left[\frac{eA(\phi)\cdot p}{p\cdot k}-\frac{e^2A^2(\phi)}{2p\cdot k}\right]d\phi\right), \tag{3}$$

where $u_p$ is a free particle Dirac spinor, obeying

$$(\gamma^\mu p_\mu-mc)u_p=0. \tag{4}$$

We consider a monochromatic, elliptically polarized plane wave with two unit vectors $\vec{\epsilon}_1$, $\vec{\epsilon}_2$ and an angle $\delta$, describing the ellipticity. Then its vector potential reads

$$A_\mu(k\cdot x)=A_0[\epsilon_{1\mu}\cos(\delta)\cos(k\cdot x)+\epsilon_{2\mu}\sin(\delta)\sin(k\cdot x)],$$
$$\epsilon_1=(0,\vec{\epsilon}_1),\quad \epsilon_2=(0,\vec{\epsilon}_2),$$
$$\epsilon_1^2=\epsilon_2^2=-\vec{\epsilon}_1^{\,2}=-\vec{\epsilon}_2^{\,2}=-1,\quad \epsilon_1\cdot\epsilon_2=\vec{\epsilon}_1\cdot\vec{\epsilon}_2=0. \tag{5}$$

In that case (neglecting an unknown constant phase factor) the function $f(x;p)$ in the Volkov wave Eq. (3) can be written in the form

$$f(x;p)=\exp[-i(\overline{p}\cdot x+Q(k\cdot x;p))] \tag{6}$$

in which we introduced the renormalized four momenta $\overline{p}$, defined by

$$\overline{p}=p+dn,\quad n=\frac{k}{|k_0|},$$
$$d=\frac{(mc)^2\mu^2}{4n\cdot p},\quad \mu=\frac{|eA_0|}{mc},$$
$$\overline{p}\cdot\overline{p}=\overline{m}^2c^2,\quad \overline{m}^2=m^2\left(1+\frac{\mu^2}{2}\right) \tag{7}$$

and the function $Q(k\cdot x;p)$ stands for the expression

$$Q(k\cdot x;p)=eA_0\frac{p\cdot\epsilon_1}{p\cdot k}\cos(\delta)\sin(k\cdot x)$$
$$-eA_0\frac{p\cdot\epsilon_2}{p\cdot k}\sin(\delta)\cos(k\cdot x)$$
$$+\frac{e^2A_0^2}{8}\frac{1}{k\cdot p}\cos(2\delta)\sin 2(k\cdot x). \tag{8}$$

Therefore the Volkov wave equation (3) can be written in the form

$$\Psi_p(x)=\sqrt{\frac{m}{VE_p}}e^{-i[\overline{p}\cdot x+Q(k\cdot x;p)]}$$
$$\times\left(1-\frac{e\gamma^\nu A_\nu(k\cdot x)\gamma^\mu k_\mu}{2k\cdot p}\right)u_p. \tag{9}$$

With such Volkov solutions for the ingoing electrons of four momenta $p_i$ and $q_i$ and outgoing electrons of momenta $p_f$ and $q_f$ the matrix element, Eq. (1), using Eq. (2), has to be evaluated explicitly. We find

$$S(p_f,q_f;p_i,q_i)=-i4\pi\alpha\int dxdy\frac{d\mathcal{K}}{(2\pi)^4}$$
$$\times\{\tilde{D}_{\mu\nu}(\mathcal{K})e^{-i\mathcal{K}\cdot(x-y)}$$
$$\times[\overline{\Psi}_{p_f}(x)\gamma^\mu\Psi_{p_i}(x)][\overline{\Psi}_{q_f}(y)\gamma^\nu\Psi_{q_i}(y)]$$
$$-\cdots p_f\leftrightarrow q_f\cdots\}. \tag{10}$$

Since the Volkov waves in Eq. (10) are periodic functions in $k\cdot x$ and $k\cdot y$, respectively, we can easily decompose the $S$ matrix into its Fourier components and perform the integrations over $x$ and $y$. This yields

$$S(p_f,q_f;p_i,q_i)=-i4\pi\alpha(2\pi)^4\sum_N\delta(\overline{p_i}+\overline{q_i}-\overline{p_f}-\overline{q_f}$$
$$+Nk)\frac{m^2}{V^2\sqrt{E_{p_f}E_{p_i}E_{q_f}E_{q_i}}}t_N(p_f,q_f,p_i,q_i), \tag{11}$$

where the matrix elements, $t_N$, corresponding to the absorption or emission of $N$ laser photons are given by

$$t_N(p_f,q_f,p_i,q_i)=\sum_M \{\tilde{D}_{\mu\nu}(\overline{q_i}-\overline{q_f}+Mk)\Phi^{\mu}_{N-M}(p_f,p_i)\times\Phi^{\nu}_M(q_f,q_i)-\tilde{D}_{\mu\nu}(\overline{q_i}-\overline{p_f}+Mk)\times\Phi^{\mu}_{N-M}(q_f,p_i)\Phi^{\nu}_M(p_f,q_i)\}. \tag{12}$$

In Eq. (10) we have introduced the fine structure constant $\alpha=e^2/4\pi c$ and in Eq. (12) the Fourier coefficients $\Phi^{\lambda}_N(p',p)$ are conveniently defined by the following relation:

$$\overline{\Psi}_{p'}(x)\gamma^{\lambda}\Psi_p(x)=\frac{m}{V\sqrt{E_{p'}E_p}}\sum_N e^{i(\overline{p'}-\overline{p}-Nk)\cdot x}\Phi^{\lambda}_N(p',p) \tag{13}$$

in which these coefficients are explicitly given by

$$\Phi^{\lambda}_N(p',p)=\overline{u}_{p'}\Bigg[\gamma^{\lambda}+\left(\frac{eA_0}{2k\cdot p'}\gamma^{\nu}\epsilon_{1\nu}\gamma^{\mu}k_{\mu}\gamma^{\lambda}-\frac{eA_0}{2k\cdot p}\gamma^{\lambda}\gamma^{\varrho}\epsilon_{1\varrho}\gamma^{\sigma}k_{\sigma}\right)\cos(\delta)\frac{1}{2}(B_{N+1}+B_{N-1})+\left(\frac{eA_0}{2k\cdot p'}\gamma^{\nu}\epsilon_{2\nu}\gamma^{\mu}k_{\mu}\gamma^{\lambda}-\frac{eA_0}{2k\cdot p}\gamma^{\lambda}\gamma^{\varrho}\epsilon_{2\varrho}\gamma^{\sigma}k_{\sigma}\right)\sin(\delta)\frac{1}{2i}(B_{N+1}-B_{N-1})-\frac{e^2A_0^2}{4(k\cdot p')(k\cdot p)}\Bigg[\gamma^{\nu}\epsilon_{1\nu}\gamma^{\mu}k_{\mu}\gamma^{\lambda}\gamma^{\varrho}\epsilon_{1\varrho}\gamma^{\sigma}k_{\sigma}\cos^2(\delta)\left(\frac{1}{2}B_N+\frac{1}{4}(B_{N+2}+B_{N-2})\right)+\frac{1}{4}(\gamma^{\nu}\epsilon_{1\nu}\gamma^{\mu}k_{\mu}\gamma^{\lambda}\gamma^{\varrho}\epsilon_{2\varrho}\gamma^{\sigma}k_{\sigma}+\gamma^{\nu}\epsilon_{2\nu}\gamma^{\mu}k_{\mu}\gamma^{\lambda}\gamma^{\varrho}\epsilon_{1\varrho}\gamma^{\sigma}k_{\sigma})\sin(2\delta)\frac{1}{2i}(B_{N+2}-B_{N-2})+\gamma^{\nu}\epsilon_{2\nu}\gamma^{\mu}k_{\mu}\gamma^{\lambda}\gamma^{\varrho}\epsilon_{2\varrho}\gamma^{\sigma}k_{\sigma}\sin^2(\delta)\left(\frac{1}{2}B_N-\frac{1}{4}(B_{N+2}+B_{N-2})\right)\Bigg]\Bigg]u_p, \tag{14}$$

where the generalized Bessel functions $B_N$ are defined by the Fourier expansion of the following generating function:

$$e^{i[Q(k\cdot x;p')-Q(k\cdot x;p)]}=\sum_N e^{iNk\cdot x}B_N(a,b;\alpha). \tag{15}$$

The arguments $a,b,\alpha$ of the functions $B_N$ follow from Eq. (8), according to which the generating function in Eq. (15) for the $B_N$ can be explicitly expressed in the form $\exp\{i[a\sin(k\cdot x-\alpha)+b\sin 2(k\cdot x)]\}$. While the Fourier decompositions of the $S$-matrix element and the decompositions into generalized Bessel functions is straightforward and was done analytically, the algebraic evaluations of the matrix elements in Eq. (14), containing a considerable number of $\gamma$ matrices, was done by means of computer programs for that purpose.

With the above results we can now evaluate the nonlinear differential cross sections $d\sigma_N$ of laser-assisted Mϕller scattering. We find, with reference to our definitions and notations for renormalized, laser-dressed four-momenta (7),

$$d\sigma_N=\frac{4m^4c^6\alpha^2}{\sqrt{(p_i\cdot q_i)^2-(mc)^4}}\int\frac{d^3p_f}{E_{p_f}}\frac{d^3q_f}{E_{q_f}}\delta(\overline{p_i}+\overline{q_i}-\overline{p_f}-\overline{q_f}+Nk)|t_N(p_f,q_f,p_i,q_i)|^2. \tag{16}$$

In order to evaluate the integrals in Eq. (16) we first have to find the Jacobian $|\partial\vec{q}_f/\partial\vec{\overline{q}}_f|$, which can be evaluated to yield

$$\left|\frac{\partial\vec{q}_f}{\partial\vec{\overline{q}}_f}\right|=1+\frac{(mc)^2\mu^2}{4(\overline{q}_f\cdot n)^2}\left[\frac{\vec{\overline{q}}_f\cdot\vec{n}}{\overline{q}_{f0}}-1\right] \tag{17}$$

by means of which the integration over $d^3q_f$ in Eq. (16) can be performed and we obtain

$$d\sigma_N=\frac{4m^4c^6\alpha^2}{\sqrt{(p_i\cdot q_i)^2-(mc)^4}}\frac{J(\overline{q}_f,q_f)}{E_{p_f}E_{q_f}}\int d^3p_f\delta(\overline{p}_{i0}+\overline{q}_{i0}-\overline{p}_{f0}-\overline{q}_{f0}+Nk_0)|t_N(p_f,q_f,p_i,q_i)|^2, \tag{18}$$

where we have introduced for the Jacobian the abbreviation $|\partial\vec{q}_f/\partial\vec{\overline{q}}_f|=J(\overline{q}_f,q_f)$. Finally, we are left with the integration over the remaining one-dimensional $\delta$ function. To this end, we first rewrite $d^3p_f$ into the following expression

$$d^3p_f=d\Omega_{p_f}\vec{p}_f^{\,2}dp_f=d\Omega_{p_f}|\vec{p}_f|p_{f0}dp_{f0} \tag{19}$$

and we can then perform the final integration in Eq. (18) over $dp_{f0}$ to obtain the final differential cross section formula

$$\frac{d\sigma_N}{d\Omega_{\vec{p}_f}}=\frac{4m^4c^4\alpha^2}{\sqrt{(p_i\cdot q_i)^2-(mc)^4}}\frac{J(\overline{q}_f,q_f)}{|D|}\frac{|\vec{p}_f|}{q_{f0}}\times|t_N(p_f,q_f,p_i,q_i)|^2, \tag{20}$$

where in the denominator the coefficient $D$ is the derivative of the argument of the $\delta$ function and it is given by the expression

$$D=1-\frac{mU_p\left(\frac{\omega}{c}-\frac{p_{f0}}{\vec{p}_f^2}\vec{k}\cdot\vec{p}_f\right)\frac{\omega}{c}}{(k\cdot p_f)^2}-\frac{p_{f0}}{\bar{q}_{f0}}\frac{\vec{\bar{q}}_f\cdot\vec{p}_f}{\vec{p}_f^2}+\frac{mU_p\left(\frac{\omega}{c}-\frac{p_{f0}}{\vec{p}_f^2}\vec{k}\cdot\vec{p}_f\right)}{(k\cdot p_f)^2}\vec{\bar{q}}_f\cdot\vec{k}. \tag{21}$$

In the formula (20) there appear the outgoing four-momenta $p_f$ and $q_f$, and also the renormalized ones, $\bar{p}_f$ and $\bar{q}_f$, defined by Eqs. (7) [or by Eqs. (33) and (34) below]. However, due to the energy-momentum conservation equation

$$\bar{p}_i+\bar{q}_i-\bar{p}_f-\bar{q}_f+Nk=0 \tag{22}$$

and the on-mass-shell conditions,

$$p_f^2=q_f^2=m^2c^2, \tag{23}$$

the outgoing four-momenta can be expressed in terms of the incoming four-momenta $p_i$ and $q_i$, and the scattering angles $\theta_{p_f}$ and $\varphi_{p_f}$ of one of the outgoing momenta $\vec{p}_f$. Due to the nonlinear character of the Eqs. (22), (23), and (7), this solution does not exist in a closed form and, for this reason, we have not substituted it into Eq. (20). Nevertheless, this solution can be determined numerically. In this sense, $p_f$, $q_f$, $\bar{p}_f$, and $\bar{q}_f$, appearing in Eq. (20), are known provided that $p_i,q_i$ and the scattering angles $\theta_{p_f}$ and $\varphi_{p_f}$ are given.

### III. ANALYSIS OF SCATTERING RESONANCES

In the early work on laser-assisted Møller scattering [9], the appearance of laser-induced resonances was discussed. It is the purpose of the present section, to derive the necessary kinematical conditions for the observation of these resonances in order to define a convenient set of parameters which permit us to analyze them numerically. These resonances are defined by poles of the photon propagators in Eq. (12). Namely, among all the possible values of $M$ there could exist such integers $M_2$ for which

$$(\bar{q}_i-\bar{q}_f+M_2k)^2=0 \tag{24}$$

or

$$(\bar{q}_i-\bar{p}_f+M_2k)^2=0. \tag{25}$$

Since our further analysis is similar for both of these two equations, we shall therefore concentrate on the first of them. Hence, the resonance conditions are defined by the five scalar equations

$$\bar{p}_i+\bar{q}_i-\bar{p}_f-\bar{q}_f+Nk=0, \tag{26}$$

$$k'^2=0, \tag{27}$$

where

$$k'=\bar{q}_i-\bar{q}_f+M_2k=\bar{p}_f-\bar{p}_i-M_1k \tag{28}$$

and

$$N=M_1+M_2. \tag{29}$$

In our discussion below, the intensity of the laser field will be determined by the ponderomotive energy

$$U_p=\frac{e^2A_0^2}{4m}=\frac{mc^2\mu^2}{4}, \tag{30}$$

so that the following relations between the renormalized and bare four-momenta are fulfilled:

$$\bar{p}_i=p_i+\frac{mU_p}{k\cdot p_i}k, \tag{31}$$

$$\bar{q}_i=q_i+\frac{mU_p}{k\cdot q_i}k, \tag{32}$$

$$\bar{p}_f=p_f+\frac{mU_p}{k\cdot p_f}k, \tag{33}$$

$$\bar{q}_f=q_f+\frac{mU_p}{k\cdot q_f}k. \tag{34}$$

The above equations are invariant under boosts and rotations. Consequently, there is no need to retain a particular symmetry and we can choose one specific reference frame, defined as follows:

$$\vec{p}_i=-\vec{q}_i, \tag{35}$$

$$p_x=0, \tag{36}$$

$$p_y=0, \tag{37}$$

$$k_y=0. \tag{38}$$

Using these relations, let us parametrize the solution of Eqs. (26) and (27) as follows:

$$k=\frac{\omega}{c}(1,\sin\theta,0,\cos\theta), \tag{39}$$

$$p_i=\left(\frac{E}{c},0,0,p\right), \tag{40}$$

$$q_i=\left(\frac{E}{c},0,0,-p\right), \tag{41}$$

$$k'=\frac{\omega'}{c}(1,\xi,\pm\sqrt{1-\xi^2-\zeta^2},\zeta), \tag{42}$$

where

$$p=\sqrt{\frac{E^2}{c^2}-m^2c^2} \tag{43}$$

and $\xi$ and $\zeta$ define the null four-vector $k'$. In order to make sure that the components of this vector are real numbers, the parameters $\xi$ and $\zeta$ have to fulfill the condition

$$\xi^2+\zeta^2\leqslant 1, \tag{44}$$

which appears to be crucial for determining the resonances. The energies, belonging to the final four-momenta $p_f$ and $q_f$ should be positive. No further conditions need to be imposed. The case $\omega'=0$ satisfies by itself the condition $k'^2=0$ and therefore does not need to be excluded. This particular case will be briefly discussed later.

Let us now analyze the on-mass-shell equations for the final momenta $p_f$ and $q_f$

$$p_f^2=m^2c^2, \tag{45}$$

$$q_f^2=m^2c^2. \tag{46}$$

After some algebraic manipulations they lead to

$$M_1p_i\cdot k+\left(p_i+\frac{mU_p}{p_i\cdot k}k\right)\cdot k'+M_1k\cdot k'=0 \tag{47}$$

and

$$-M_2q_i\cdot k+\left(q_i+\frac{mU_p}{q_i\cdot k}k\right)\cdot k'+M_2k\cdot k'=0. \tag{48}$$

The four-vector products in the last two equations can be expressed in terms of our parameters. We find that

$$p_i\cdot k=\frac{\omega}{c}p_-, \tag{49}$$

$$q_i\cdot k=\frac{\omega}{c}p_+, \tag{50}$$

$$p_i\cdot k'=\frac{\omega'}{c}\left[\frac{E}{c}-p\zeta\right], \tag{51}$$

$$q_i\cdot k'=\frac{\omega'}{c}\left[\frac{E}{c}+p\zeta\right], \tag{52}$$

$$k\cdot k'=\frac{\omega\omega'}{c^2}[1-\xi\sin\theta-\zeta\cos\theta], \tag{53}$$

where

$$p_\pm=\frac{E}{c}\pm p\cos\theta. \tag{54}$$

In this way, Eqs. (45) and (46) can be transformed into

$$-M_1\frac{\omega}{c}p_-=\frac{\omega'}{c}\left[\left(\frac{E}{c}-p\zeta\right)+\frac{mU_p}{p_-}[1-\xi\sin\theta-\zeta\cos\theta]+M_1\frac{\omega}{c}[1-\xi\sin\theta-\zeta\cos\theta]\right], \tag{55}$$

$$M_2\frac{\omega}{c}p_+=\frac{\omega'}{c}\left[\left(\frac{E}{c}+p\zeta\right)+\frac{mU_p}{p_+}[1-\xi\sin\theta-\zeta\cos\theta]+M_2\frac{\omega}{c}[1-\xi\sin\theta-\zeta\cos\theta]\right]. \tag{56}$$

As we can see, for the case $\omega'=0$, this set of equations can be solved only if $M_1=M_2=0$. Then the equation $k'=0$, which follows from the definition (42), leads to the result $p_i=p_f$ and this relates to the case of forward elastic scattering. We shall not consider this case any further. Next we eliminate $\omega'$ from the foregoing two equations and find

$$M_2p_+\left(\frac{E}{c}-p\zeta\right)+M_2mU_p\frac{p_+}{p_-}[1-\xi\sin\theta-\zeta\cos\theta]+M_1p_-\left(\frac{E}{c}+p\zeta\right)+M_1mU_p\frac{p_-}{p_+}[1-\xi\sin\theta-\zeta\cos\theta]+2M_1M_2\frac{E}{c}\frac{\omega}{c}[1-\xi\sin\theta-\zeta\cos\theta]=0. \tag{57}$$

This is evidently a linear equation in $\xi$, $\zeta$ of the general form $A\xi+B\zeta+C=0$ with the coefficients $A$, $B$, and $C$ being explicitly given by

$$A=-\frac{p_-}{p_+}mU_pM_1\sin\theta-\frac{p_+}{p_-}mU_pM_2\sin\theta-2\frac{E\omega}{c^2}M_1M_2\sin\theta,$$

$$B=-pp_+M_2-\frac{p_+}{p_-}mU_pM_2\cos\theta+pp_-M_1-\frac{p_-}{p_+}mU_pM_1\cos\theta-2\frac{E\omega}{c^2}M_1M_2\cos\theta,$$

$$C=\frac{E}{c}p_+M_2+\frac{p_+}{p_-}mU_pM_2+\frac{E}{c}p_-M_1+\frac{p_-}{p_+}mU_pM_1+2\frac{E\omega}{c^2}M_1M_2. \tag{58}$$

It is well known from analytic geometry that the distance of a straight line

$$A\xi+B\zeta+C=0 \tag{59}$$

from the origin of the Cartesian coordinate system is equal to

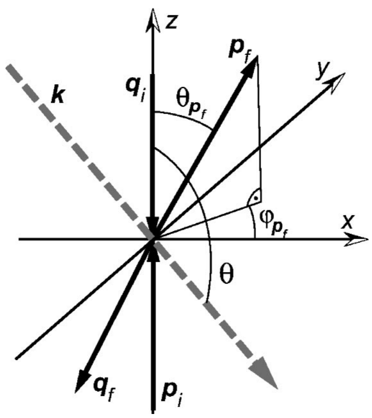


FIG. 1. Shown is the scattering geometry. In the center of mass system the electrons collide with the momenta $\vec{p}_i$ and $\vec{q}_i$ along the $z$ axis while the laser beam with wave vector $\vec{k}$ propagates in the $(x,y)$ plane. $\theta$ is the angle between $\vec{k}$ and the $z$ axis. $\vec{p}_f$ and $\vec{q}_f$ are the momenta of the outgoing electrons and the direction of $\vec{p}_f$ is specified by the angles $\theta_{\vec{p}_f}$ and $\varphi_{\vec{p}_f}$.

$$D=\frac{|C|}{\sqrt{A^2+B^2}}. \tag{60}$$

On the other hand, we know that the parameters $\xi$ and $\zeta$ must fulfill the inequality (44). This means that $D\leqslant 1$ and we therefore obtain the condition

$$f(M_1,M_2)=C^2-A^2-B^2\leqslant 0. \tag{61}$$

This inequality, defined for a set of initial scattering parameters $E$, $U_p$, $\theta$ and $\omega$, and for the physical conditions that the energies of the final electrons should be larger than $mc^2$, is permitting us to determine all possible integers $M_1$ and $M_2$ [hence also the total number of laser photons absorbed or emitted during the scattering process, $N=M_1+M_2$, as well as $\omega'$ from Eq. (55) or (56)] for which resonance scattering takes place. By choosing one of these possible sets of integers $(M_1,M_2)$, we can determine the range of permitted parameter values for $\xi$ and $\zeta$ by analyzing Eqs. (59) and (44). It appears that for all these allowed parameter values the three-dimensional unit vector $\vec{n}_{k'}=\vec{k}'/|\vec{k}'|$ draws a closed trajectory on the unit sphere. This circle can be parametrized by means of the angle $0\leqslant\beta\leqslant 2\pi$ such that

$$\xi=-\frac{B\sqrt{A^2+B^2-C^2}}{A^2+B^2}\cos\beta+\frac{AC}{A^2+B^2}, \tag{62}$$

$$\zeta=+\frac{A\sqrt{A^2+B^2-C^2}}{A^2+B^2}\cos\beta+\frac{BC}{A^2+B^2}, \tag{63}$$

$$\pm\sqrt{1-\xi^2-\zeta^2}=\frac{\sqrt{A^2+B^2-C^2}}{\sqrt{A^2+B^2}}\sin\beta. \tag{64}$$

This trajectory determines all possible values of the final momenta $p_f$ and $q_f$ in the following way: Any angle $\beta$ defines the four-vector $k'$ from which we calculate the four-momentum $\bar{p}_f$, applying Eq. (28), whereas the energy-momentum conservation condition (26) determines the four-momentum $\bar{q}_f$. Finally, by applying the equations

$$p_f=\bar{p}_f-\frac{mU_p}{k\cdot\bar{p}_f}\bar{k}, \tag{65}$$

$$q_f=\bar{q}_f-\frac{mU_p}{k\cdot\bar{q}_f}\bar{k}, \tag{66}$$

we can determine the momenta of the scattered electrons.

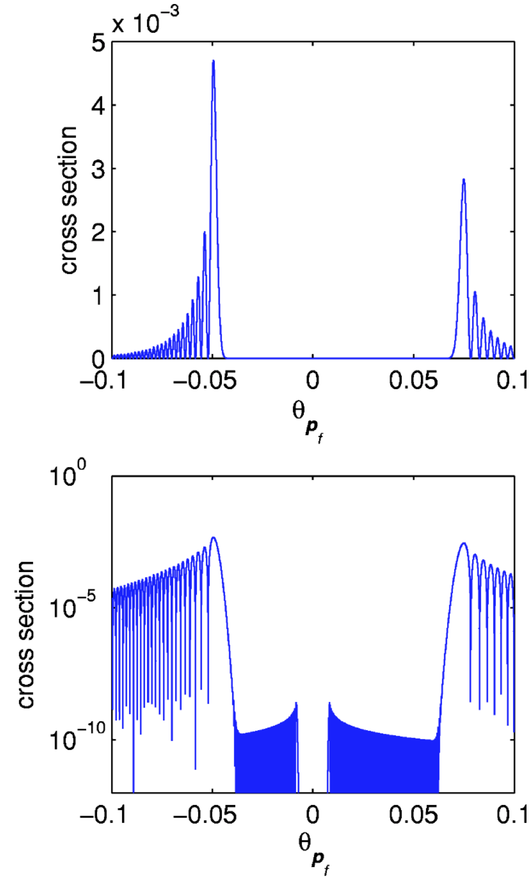


FIG. 2. For the laser and electron parameters $\omega=1.54$ eV, $I=10^{17}$ W cm$^{-2}$ and $E_i=10^4$ eV cross section data are presented for the nonresonant region of pairs $(M_1,M_2)$. The parameters are $\theta=130°$, $\varphi_{\vec{p}_f}=30°$, and $N=-100$, i.e., 100 photons of the linearly polarized laser field are emitted during the scattering process. The polarization vector $\vec{\varepsilon}_1$ of this field is located in the $(x,y)$ plane and $\theta_{\vec{p}_f}$ is measured in radians. The data in the upper and lower panels are the same, except that they are presented on different scales.

## IV. NUMERICAL EXAMPLES

In the present work, we are particularly interested in the conditions of resonances occurring in laser-assisted Møller scattering and in the corresponding cross section values for particular values $N$ of absorbed or emitted laser photons $\omega$. For our analysis, we have chosen the following parameter values. The initial kinetic energy of the colliding electrons in

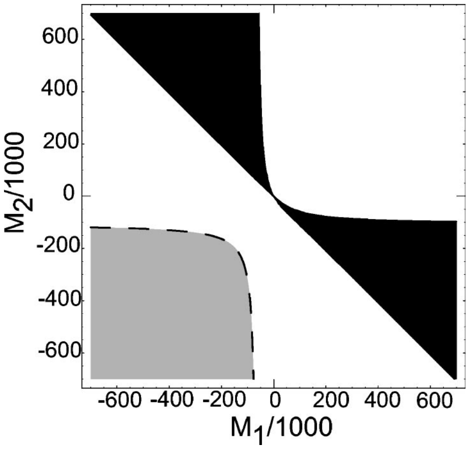


FIG. 3. For the same electron and laser parameters as in Fig. 2 all pairs $(M_1,M_2)$, permitted by the condition (61), are presented for $\theta=130°$.

the center of mass system is $E_i=10^4$ eV and the laser field of frequency $\omega=1.54$ eV has the moderate field intensity of $I=10^{17}\ \mathrm{W\,cm^{-2}}$.

In Fig. 1 we show the envisaged geometry of the scattering process. Along the $z$ axis the electrons collide in the center of mass system with the momenta $\vec{p}_i$ and $\vec{q}_i$ in the presence of the laser beam with the wave vector $\vec{k}$ propagating in the $(x,z)$ plane. The direction of propagation is determined by the angle $\theta$ between $\vec{k}$ and the $z$ axis. The momenta of the outgoing electrons are $\vec{p}_f$ and $\vec{q}_f$, respectively, where the direction of propagation of the outgoing electrons of momentum $\vec{p}_f$ is specified by the polar angles $\theta_{\vec{p}_f}$ and $\varphi_{\vec{p}_f}$.

Before we discuss the resonance phenomena in Møller scattering, we first present in Fig. 2 cross section data for this process in the nonresonant domain of points $(M_1,M_2)$. The data were evaluated for the laser and electron parameters quoted above with $N=-100$, while, with reference to Fig. 1, the polar angle $\theta=130°$ and the azimuth $\varphi_{\vec{p}_f}=30°$. The laser field is linearly polarized and the unit vector $\vec{\varepsilon}_1$ is located in the $(x,z)$ plane of Fig. 1. The scattering angle $\theta_{\vec{p}_f}$ in our figure is measured in radians. The cross section data shown in the upper and in the lower panels are the same, but

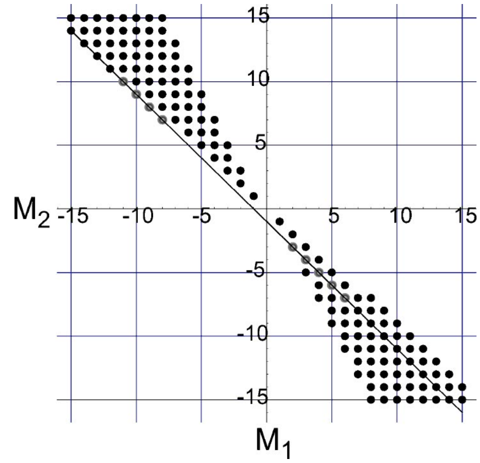


FIG. 4. Shown is an enlarged section of the data $(M_1,M_2)$ depicted in Fig. 3 for which, in particular, the condition $M_1+M_2=-1$ holds. These pairs $(M_1,M_2)$ refer to resonances in inelastic Møller scattering with the emission of one laser photon $\omega$.

they are presented on different scales. In this way we can recognize the enormous differences in the values of the data in the regions of large and small scattering angles. Moreover, we find in the lower panel that at very small scattering angles $\theta_{\vec{p}_f}$ there is a "dark window" into which marginally few electrons are scattered.

In Fig. 3 we present for the above laser and electron parameters in the plane of numbers $M_1$ and $M_2$ all pairs $(M_1,M_2)$ permitted by the inequality (61) for the particular polar angle $\theta=130°$ between $\vec{k}$ and the $z$ axis. The pairs $(M_1,M_2)$ shown as gray region in this figure have to be discarded, since they would correspond to negative kinetic electron energies. The units along the abscissa and ordinate for $M_1$ and $M_2$ are in powers of $10^{-3}$.

In Fig. 4 we have enlarged a certain part of the data of Fig. 3, showing along the straight line those pairs $(M_1,M_2)$ for which the auxiliary condition $M_1+M_2=-1$ is fulfilled. The corresponding points in the $(M_1,M_2)$ plane refer to resonances in inelastic Møller scattering with the emission of one laser photon $\omega$.

For the selected points $(M_1,M_2)$ of Fig. 4, we show in Fig. 5, with reference to Fig. 1, all possible projections of the

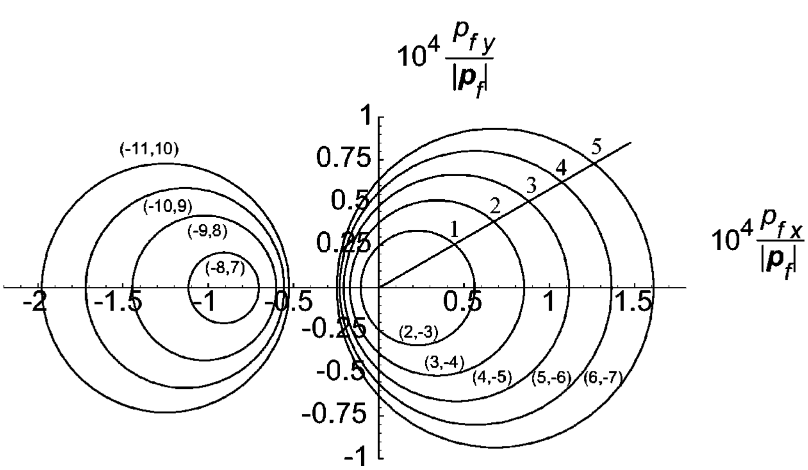


FIG. 5. Presented for the selected pairs $(M_1,M_2)$ with $M_1+M_2=-1$, shown as gray circles in Fig. 3, are the possible projections of $\vec{p}_f$ into the $(x,y)$ plane close to forward direction for which the Møller cross sections are infinite on account of the singularities determined by the condition (24). Apparently, to any permitted pair $(M_1,M_2)$ there corresponds a closed line, almost a circle, for close to forward scattering. The cross sections for $\varphi_{\vec{p}_f}=30°$ and for small $\theta_{\vec{p}_f}$ are forming a straight line. Their crossing points with the above circles indicate the occurrence of resonances at the points, denoted by 1,2, . . . ,5.

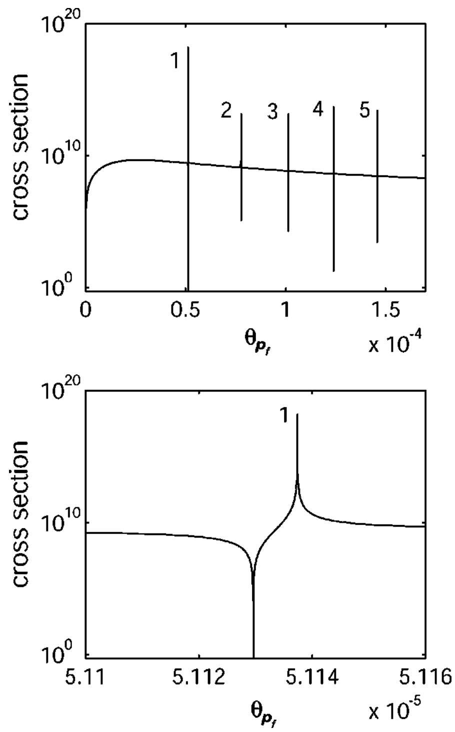


FIG. 6. Shown in the upper panel are the cross sections in the vicinity of the expected resonances of Møller scattering, determined by the crossing points of the straight lines with the circles, presented in Fig. 5. Details of the first resonance in the upper panel are presented in the lower panel in which both the resonant peak as well as the dip can be recognized.

corresponding final momenta $\vec{p}_f$ into the $(x,y)$ plane in the close to forward direction for which the cross sections of Møller scattering become infinite on account of the singularities, determined by the condition (24). The singularities, due to the second condition (25) are not considered in the present discussion. As we can observe, to any allowed pair $(M_1,M_2)$ of integers there corresponds in this figure a closed line, that can be well approximated by a circle in the case of close to forward scattering. By calculating the cross sections of inelastic Møller scattering for a particular value $\varphi_{\vec{p}_f}=30°$ of the azimuth and for small $\theta_{\vec{p}_f}$ (shown as a straight line in this figure), we should expect to observe resonances at the points marked by 1,2, . . . ,5.

This is demonstrated in Fig. 6 where the cross sections of inelastic Møller scattering (in a.u.) are presented for the emission of one laser photon $\omega$ and for $\varphi_{\vec{p}_f}=30°$ and for the same parameter values chosen initially. As indicated before, we really observe in the upper panel rapid changes of the cross section values at the points 1,2, . . . ,5 of Fig. 4 where the resonance condition (24) is fulfilled. For the particular case of the crossing point, denoted by 1 in Fig. 4, we can recognize in the lower panel that the resonance, at which the cross sections become infinite, is accompanied by a dip.

In order to explain why the appearance of an infinity at the laser-induced resonance is accompanied by a dip of the cross sections of Møller scattering, we shall approximate the scattering amplitude very close to the resonance by two terms, namely its singular part, leading to the infinity, and a constant contribution of the remaining background

$$f(\theta)=\frac{a}{\theta-\theta_r}+b, \qquad (67)$$

where $\theta_r$ is the polar angle of the direction of $\vec{p}_f$ at which the resonance appears and $a$ and $b$ are constants. Hence, near the resonance the differential cross section has the structure

$$|f(\theta)|^2=\frac{|b|^2}{(\theta-\theta_r)^2}\left(\theta-\theta_r+\frac{a}{b}\right)\left(\theta-\theta_r+\frac{a^*}{b^*}\right) \qquad (68)$$

and we see that there are in general two complex angles $\theta_r - a/b$ and $\theta_r - a^*/b^*$ for which the cross section vanishes. The fact that the dip shows up very close to the resonance indicates that $|a| \ll |b|$ which is also compatible with our approximation. The expression (68) shows that for $\theta=\theta_r+\mathrm{Re}(a/b)$ we should expect a minimum of the cross section which can have the value zero only if $a/b$ is a real number. Since the numerical work of our present investigation is rather complicated, we were not able to also consider any damping mechanisms that would eliminate the resonance divergences, as considered in the work of Bös *et al.* [11] and Borisov *et al.* [12].

## V. SUMMARY AND CONCLUSIONS

In the present work we reconsidered Møller scattering in a laser field of relativistic radiation power and presented scattering formulas for arbitrary elliptic polarization. Particular attention was devoted to a detailed analysis of the conditions required, in order to observe resonances during the laser-assisted scattering process. For a selected set of initial parameter values we presented characteristic examples of these laser-induced resonances. Since the numerical work of these investigations is rather complicated and time-consuming, we were not able to also consider any damping mechanisms which would prevent the appearance of divergences at the points of resonance. In the nonresonant regions of scattering we found "dark angular windows" at very small scattering angles where the cross sections are extremely small.

## ACKNOWLEDGMENTS

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