# Comment on "Mott scattering in strong laser fields"

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The first Born differential cross section for Mott scattering of a Dirac-Volkov electron is reviewed. The expression (26) derived by Szymanowski *et al.* [Phys. Rev. A **56**, 3846 (1997)] is corrected. In particular, we disagree with the expression of $(d\sigma/d\Omega)$ they obtained and we give the exact coefficients multiplying the various Bessel functions appearing in the scattering differential cross section. Comparison of our numerical calculations with those of Szymanowski *et al.* shows qualitative and quantitative differences when the incoming total electron energy and the electric-field strength are increased particularly in the direction of the laser propagation.

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## I. INTRODUCTION

In a pioneering paper, Szymanowski *et al.* [1] have studied the Mott scattering process in a strong laser field. The main purpose was to show that the modifications of the Mott scattering differential cross section for the scattering of an electron by the Coulomb potential of a nucleus in the presence of a strong laser field can yield interesting physical insights concerning the importance and the signatures of the relativistic effects. Their spin dependent relativistic description of Mott scattering permits to distinguish between kinematics and spin-orbit coupling effects. They have compared the results of a calculation of the first Born differential cross section for the Coulomb scattering of the Dirac-Volkov electrons dressed by a circularly polarized laser field to the first Born cross section for the Coulomb scattering of spinless Klein-Gordon particles, and also to the nonrelativistic Schröodinger-Volkov treatment. The aim of this Comment is to provide the correct expression for the first-Born differential cross sections corresponding to the Coulomb scattering of the Dirac-Volkov electrons. On the one hand, we show that the terms proportional to $\sin(2\phi_0)$ are missing in Ref. [1], where $\phi_0$ is the phase stemming from the expression of the circularly polarized electromagnetic field. The claim of Ref. [1] that they vanish is not true. These terms do not depend on the chosen description of the circular polarization in cartesian components. On the other hand we use atomic units $(\hbar=e=m=1)$, where $m$ denotes the electron mass. DCS stands for the differential cross section.

The organization of this comment is as follows : in Sec. II, we establish the expression of the *S*-matrix transition amplitude as well as the formal expression of scattering DCS. We give an account on the various trace calculations and show that indeed there is a missing term proportional to $\sin(2\phi_0)$ that is not equal to zero. This term as well as a term proportional to $\cos(2\phi_0)$ contribute to $d\sigma/d\Omega$ and multiply the product $J_{s+1}(z)J_{s-1}(z)$, where $J_s(z)$ is an ordinary Bessel function of argument $z$ and index $s$. The argument $z=\xi$ appearing in the above mentioned product is defined in Ref. [1]. Then, we carry out the derivation of the correct expression of the scattering DCS associated to the exchange of a given number of laser photons. In Sec. III, we discuss the numerical significance of our corrections and we end by a brief conclusion.

## II. THE *S*-MATRIX ELEMENT AND THE SCATTERING DIFFERENTIAL CROSS SECTION

All the theoretical formalism needed here can be found in Ref. [1]. Therefore, we refer the reader to this work for the notations and conventions. We calculate the transition amplitude. The interaction of the dressed electrons with central Coulomb field

$$A^{\mu}=\left(-\frac{Z}{|\mathbf{x}|},0,0,0\right) \tag{1}$$

is considered as a first-order perturbation. This is well justified if $Z\alpha\ll 1$, where $Z$ is the nuclear charge of the nucleus considered and $\alpha$ is the fine-structure constant. We evaluate the transition matrix element for the transition $(i\rightarrow f)$

$$S_{fi}=\frac{iZ}{c}\int d^4x\,\overline{\psi}_{q_f}(x)\frac{\gamma^0}{|\mathbf{x}|}\psi_{q_i}(x). \tag{2}$$

After some algebraic manipulations, one gets

$$\begin{aligned}&\overline{u}(p_f,s_f)\overline{R}(p_f)\gamma^0R(p_i)u(p_i,s_i)\\&\quad=\overline{u}(p_f,s_f)[C_0+C_1\cos(\phi)+C_2\sin(\phi)]u(p_i,s_i),\end{aligned} \tag{3}$$

where $R(p)=R(q)=1+1/2c(kp)[\not{k}\not{a}_1\cos(\phi)+\not{k}\not{a}_2\sin(\phi)]$ and the three coefficients $C_0$, $C_1$, and $C_2$ are, respectively, given by

$$\begin{aligned}C_0&=\gamma^0-2k_0a^2\not{k}c(p_i)c(p_f),\\C_1&=c(p_i)\gamma^0\not{k}\not{a}_1+c(p_f)\not{a}_1\not{k}\gamma^0,\\C_2&=c(p_i)\gamma^0\not{k}\not{a}_2+c(p_f)\not{a}_2\not{k}\gamma^0,\end{aligned} \tag{4}$$

with $c(p)=1/2c(kp)$ and $k_0=k^0=\omega/c$. Therefore, the transition matrix element becomes

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$$S_{fi}=\frac{iZ}{c}\int d^4x\frac{1}{\sqrt{2Q_iV}}\frac{1}{\sqrt{2Q_fV}}\overline{u}(p_f,s_f)[C_0+C_1\cos(\phi)+C_2\sin(\phi)]u(p_i,s_i)\exp[i(q_f-q_i)x-iz\sin(\phi-\phi_0)]. \tag{5}$$

We now invoke the well-known identities involving ordinary Bessel functions $J_s(z)$

$$\begin{Bmatrix} 1 \\ \cos(\phi) \\ \sin(\phi) \end{Bmatrix} e^{-iz\sin(\phi-\phi_0)}=\sum_{s=-\infty}^{\infty}\begin{Bmatrix} B_s \\ B_{1s} \\ B_{2s} \end{Bmatrix} e^{-is\phi}, \tag{6}$$

with

$$\begin{Bmatrix} B_s \\ B_{1s} \\ B_{2s} \end{Bmatrix}=\begin{Bmatrix} J_s(z)e^{is\phi_0} \\ [J_{s+1}(z)e^{i(s+1)\phi_0}+J_{s-1}(z)e^{i(s-1)\phi_0}]/2 \\ [J_{s+1}(z)e^{i(s+1)\phi_0}-J_{s-1}(z)e^{i(s-1)\phi_0}]/2i \end{Bmatrix}. \tag{7}$$

The calculation is now reduced to the computation of traces of $\gamma$ matrices. This is routinely done using REDUCE [2]. We consider the unpolarized DCS. Therefore, the various polarization states have the same probability and the actually measured DCS is given by summing over the final polarization $s_f$ and averaging over the initial polarization $s_i$. The unpolarized DCS is formally given by

$$\frac{d\overline{\sigma}}{d\Omega_f}=\sum_{s=-\infty}^{\infty}\left.\frac{d\overline{\sigma}^{(s)}}{d\Omega_f}\right|_{Q_f=Q_i+sw}, \tag{8}$$

where

$$\left.\frac{d\overline{\sigma}^{(s)}}{d\Omega_f}\right|_{Q_f=Q_i+sw}=\frac{Z^2}{c^4}\frac{|\mathbf{q}_f|}{|\mathbf{q}_i|}\frac{1}{|\mathbf{q}_f-\mathbf{q}_i-s\mathbf{k}|^4}\frac{1}{2}\times\sum_{s_i}\sum_{s_f}\left.|M_{fi}^{(s)}|^2\right|_{Q_f=Q_i+sw}. \tag{9}$$

Since the controversy is very acute and precise about the results of the sum over the polarization, we shall analyze in detail the calculations of the various traces that intervene in the formal expression of the unpolarized DCS given by Eq. (9). We have to calculate

$$\frac{1}{2}\sum_{s_i}\sum_{s_f}|M_{fi}^{(s)}|^2=\frac{1}{2}\sum_{s_i}\sum_{s_f}|\overline{u}(p_f,s_f)\Lambda^{(s)}u(p_i,s_i)|^2, \tag{10}$$

with

$$\begin{aligned}\Lambda^{(s)}=&[\gamma^0-2k_0a^2\not{k}c(p_i)c(p_f)]B_s\\ &+[c(p_i)\gamma^0\not{k}\not{a}_1+c(p_f)\not{a}_1\not{k}\gamma^0]B_{1s}\\ &+[c(p_i)\gamma^0\not{k}\not{a}_2+c(p_f)\not{a}_2\not{k}\gamma^0]B_{2s}.\end{aligned} \tag{11}$$

Using standard techniques of the $\gamma$ matrix algebra, one has

$$\frac{1}{2}\sum_{s_i}\sum_{s_f}|M_{fi}^{(s)}|^2=\frac{1}{2}\mathrm{Tr}\{(\not{p}_fc+c^2)\Lambda^{(s)}(\not{p}_ic+c^2)\overline{\Lambda}^{(s)}\}, \tag{12}$$

with

$$\begin{aligned}\overline{\Lambda}^{(s)}=\gamma^0\Lambda^{(s)\dagger}\gamma^0=&[\gamma^0-2k_0a^2\not{k}c(p_i)c(p_f)]B_s^*\\ &+[c(p_i)\not{a}_1\not{k}\gamma^0+c(p_f)\gamma^0\not{k}\not{a}_1]B_{1s}^*\\ &+[c(p_i)\not{a}_2\not{k}\gamma^0+c(p_f)\gamma^0\not{k}\not{a}_2]B_{2s}^*.\end{aligned} \tag{13}$$

There are nine main traces to be calculated. We write them explicitly

$$\begin{aligned}\mathcal{M}_1&=\mathrm{Tr}\{(\not{p}_fc+c^2)C_0(\not{p}_ic+c^2)\overline{C_0}\}|B_s|^2,\\ \mathcal{M}_2&=\mathrm{Tr}\{(\not{p}_fc+c^2)C_0(\not{p}_ic+c^2)\overline{C_1}\}B_sB_{1s}^*,\\ \mathcal{M}_3&=\mathrm{Tr}\{(\not{p}_fc+c^2)C_0(\not{p}_ic+c^2)\overline{C_2}\}B_sB_{2s}^*,\\ \mathcal{M}_4&=\mathrm{Tr}\{(\not{p}_fc+c^2)C_1(\not{p}_ic+c^2)\overline{C_0}\}B_s^*B_{1s},\\ \mathcal{M}_5&=\mathrm{Tr}\{(\not{p}_fc+c^2)C_1(\not{p}_ic+c^2)\overline{C_1}\}|B_{1s}|^2,\\ \mathcal{M}_6&=\mathrm{Tr}\{(\not{p}_fc+c^2)C_1(\not{p}_ic+c^2)\overline{C_2}\}B_{1s}B_{2s}^*,\\ \mathcal{M}_7&=\mathrm{Tr}\{(\not{p}_fc+c^2)C_2(\not{p}_ic+c^2)\overline{C_0}\}B_{2s}B_s^*,\\ \mathcal{M}_8&=\mathrm{Tr}\{(\not{p}_fc+c^2)C_2(\not{p}_ic+c^2)\overline{C_1}\}B_{1s}^*B_{2s},\\ \mathcal{M}_9&=\mathrm{Tr}\{(\not{p}_fc+c^2)C_2(\not{p}_ic+c^2)\overline{C_2}\}|B_{2s}|^2.\end{aligned} \tag{14}$$

To simplify the notations, we will drop the argument of the various ordinary Bessel functions that appear. The diagonal terms give rise to $\mathcal{M}_1\propto|B_s|^2,\mathcal{M}_5\propto|B_{1s}|^2$, and $\mathcal{M}_9\propto|B_{2s}|^2$. So, taking into account the fact that the traces multiplying $|B_s|^2$, $|B_{1s}|^2$, and $|B_{2s}|^2$ are not zero, one expects that terms proportional to $J_{s+1}J_{s-1}\cos(2\phi_0)$ will be present in the expression of the scattering DCS. The first controversy between our work and the result of Szymanowski *et al.* [1] concerns the traces $\mathcal{M}_6$ and$\mathcal{M}_8$. Since $\mathcal{M}_6\propto B_{1s}B_{2s}^*$ and $\mathcal{M}_8\propto B_{1s}^*B_{2s}$ and with little familiarity with the $\gamma$ matrix algebra, one can see at once that if the corresponding traces are not zero then the net contribution of $\mathcal{M}_6+\mathcal{M}_8$ will contain a term proportional to $J_{s+1}J_{s-1}\sin(2\phi_0)$. Explicitly, we give the result for $\mathcal{M}_6$ and $\mathcal{M}_8$. One has

$$\begin{aligned}\mathcal{M}_6=\frac{w^2}{c^2}\Bigg\{&2\sin(2\phi_0)\left[\frac{(a_1p_i)}{(kp_i)}\frac{(a_2p_f)}{(kp_f)}+\frac{(a_2p_i)}{(kp_i)}\frac{(a_1p_f)}{(kp_f)}\right]\\ &\times J_{s+1}J_{s-1}+i[-\{(a_1p_i)(a_2p_f)+(a_1p_f)(a_2p_i)\}\\ &\times J_{s-1}^2+\{(a_1p_i)(a_2p_f)+(a_1p_f)(a_2p_i)\}J_{s+1}^2]\Bigg\},\end{aligned} \tag{15}$$

while $\mathcal{M}_8$ is given by

$$\mathcal{M}_8=\frac{w^2}{c^2}\Bigg\{2\sin(2\phi_0)\left[\frac{(a_1p_i)}{(kp_i)}\frac{(a_2p_f)}{(kp_f)}+\frac{(a_2p_i)}{(kp_i)}\frac{(a_1p_f)}{(kp_f)}\right]\times J_{s+1}J_{s-1}-i[-\{(a_1p_i)(a_2p_f)+(a_1p_f)(a_2p_i)\}\times J_{s-1}^2+\{(a_1p_i)(a_2p_f)+(a_1p_f)(a_2p_i)\}J_{s+1}^2]\Bigg\}. \tag{16}$$

The fact that complex numbers appear in the expressions of $\mathcal{M}_6$ and $\mathcal{M}_8$ is not surprising since the former is the complex conjugate of the latter and their real sum is such that

$$\mathcal{M}_6+\mathcal{M}_8=\frac{4w^2}{c^2}\sin(2\phi_0)\left[\frac{(a_1p_i)}{(kp_i)}\frac{(a_2p_f)}{(kp_f)}+\frac{(a_2p_i)}{(kp_i)}\frac{(a_1p_f)}{(kp_f)}\right]J_{s+1}J_{s-1}. \tag{17}$$

So, the first controversy is settled and there is indeed a term containing $\sin(2\phi_0)$ in the expression of the scattering cross section. We have written a REDUCE program that calculates analytically the traces in Eq. (12). Before writing our REDUCE program, we have extensively studied the textbook by A. G. Grozin [3] which is full of worked examples in various fields of physics particularly in QED. We give the final result for the unpolarized DCS for the Mott scattering of a Dirac-Volkov electron

$$\frac{d\overline{\sigma}^{(s)}}{d\Omega_f}=\frac{Z^2}{c^2}\frac{|\mathbf{q}_f|}{|\mathbf{q}_i|}\frac{1}{|\mathbf{q}_f-\mathbf{q}_i-s\mathbf{k}|^4}\frac{2}{c^2}\{J_s^2A+(J_{s+1}^2+J_{s-1}^2)B+(J_{s+1}J_{s-1})C+J_s(J_{s-1}+J_{s+1})D\}, \tag{18}$$

where for notational simplicity we have dropped the argument $z$ in the various ordinary Bessel functions. The coefficients $A$, $B$, $C$, and $D$ are, respectively given by

$$A=c^4-(q_fq_i)c^2+2Q_fQ_i-\frac{a^2}{2}\left(\frac{(kq_f)}{(kq_i)}+\frac{(kq_i)}{(kq_f)}\right)+\frac{a^2\omega^2}{c^2(kq_f)(kq_i)}[(q_fq_i)-c^2]+\frac{(a^2)^2\omega^2}{c^4(kq_f)(kq_i)}+\frac{a^2\omega}{c^2}(Q_f-Q_i)\left(\frac{1}{(kq_i)}-\frac{1}{(kq_f)}\right), \tag{19}$$

$$B=-\frac{(a^2)^2\omega^2}{2c^4(kq_f)(kq_i)}+\frac{\omega^2}{2c^2}\left(\frac{(a_1q_f)}{(kq_f)}\frac{(a_1q_i)}{(kq_i)}+\frac{(a_2q_f)}{(k.q_f)}\frac{(a_2q_i)}{(kq_i)}\right)-\frac{a^2}{2}+\frac{a^2}{4}\left(\frac{(kq_f)}{(kq_i)}+\frac{(kq_i)}{(kq_f)}\right)-\frac{a^2\omega^2}{2c^2(kq_f)(kq_i)}[(q_fq_i)-c^2]+\frac{a^2\omega}{2c^2}(Q_f-Q_i)\times\left(\frac{1}{(kq_f)}-\frac{1}{(kq_i)}\right), \tag{20}$$

$$C=\frac{\omega^2}{c^2(kq_f)(kq_i)}[\cos(2\phi_0)\{(a_1q_f)(a_1q_i)-(a_2q_f)(a_2q_i)\}+\sin(2\phi_0)\{(a_1q_f)(a_2q_i)+(a_1q_i)(a_2q_f)\}], \tag{21}$$

$$D=\frac{c}{2}[(\text{Å}q_i)+(\text{Å}q_f)]-\frac{c}{2}\left(\frac{(kq_f)}{(kq_i)}(\text{Å}q_i)+\frac{(kq_i)}{(kq_f)}(\text{Å}q_f)\right)+\frac{\omega}{c}\left(\frac{Q_i(\text{Å}q_f)}{(kq_f)}+\frac{Q_f(\text{Å}q_i)}{(kq_i)}\right), \tag{22}$$

where $\text{Å}=a_1\cos(\phi_0)+a_2\sin(\phi_0)$. The argument about the missing term proportional to $\sin(2\phi_0)$ having been given a convincing explanation, we now turn to other remarks along the same lines since there are indeed other differences between our result and the result of Ref. [1]. We discuss now the difference occurring in our expression of the coefficient $A$ and the corresponding one of Ref. [1]. In their expression multiplying the product $2J_n^2(\xi)$, the single term $(a^2)^2w^2/c^6(kq)(kq')$ should come with a coefficient 1/2. We have written a second REDUCE program that allows the comparison between the coefficient $A$ of Ref. [1] and the coefficient $A$ of this work. There are so many differences between our result and the result they found for the coefficient $B$ that we refer the reader to our main REDUCE program [5]. The coefficient $C$ has already been discussed. As for the coefficient $D$, we have found an expression that is linear in the electromagnetic potential. In a third REDUCE program, it is shown explicitly that if we ignore the first term in the coefficient multiplying $J_s(J_{s-1}+J_{s+1})$ given in Ref. [1], one easily gets the result we have obtained. This term does not come from the passage from the variables $(p,\tilde{p})$ to the variable $(q,\tilde{q})$. The introduction of such four-vector $\tilde{q}$ is not useful, makes the calculations rather lengthy and gives rise to complicated expressions. As a supplementary consistency check of our procedure used in writing the main Reduce program, we have reproduced the result of the DCS corresponding to the Compton scattering in an intense electromagnetic field given by Berestetzkii, Lifshitz, and Pitaevskii [4].

### III. DISCUSSION

We would like to make general comments on the figures obtained in Ref. [1] starting with Fig. 3. This figure does not represent the envelope of the controversial generalized equation (26) of that work. Indeed, we have checked that it represents the envelope of the nonrelativistic DCS given by Eq. (34) in Ref. [1]. In Fig. 6 of Ref. [1], there is a difference between the Dirac-Volkov DCS (26) and the spinless particle DCS (30) though the overall behavior is smoothly oscillatory. The results we have obtained show the same oscillatory behavior. The curves for the Dirac-Volkov DCS (26) of Ref. [1] and the Dirac-Volkov DCS (18) of our work are almost identical while the difference between the two relativistic DCSs and the spinless particle DCS given by Eq. (30) of Ref. [1] is less important than in Fig. 6 of Ref. [1]. Figure 7 of Ref. [1] is the only figure we agree with. In Fig. 8 of Ref. [1], we disagree with the behavior of the Dirac-Volkov DCS (26) of Ref. [1] particularly for small angles around $\theta_f = 0°$. When programming Eq.(26) of Ref. [1], we obtained a value for the Dirac-Volkov DCS at $\theta_f = 0°$ of nearly $3.2 \times 10^{-14}$ a.u. instead of the $2.2 \times 10^{-14}$ a.u. indicated in Fig. 8 of Ref. [1]. Moreover, the electric field strength $\varepsilon$ being a key parameter (as well as the incoming electron total energy), we have compared our Dirac-Volkov DCS and the Dirac-Volkov DCS (26) of Ref. [1] and we have come to the following important conclusions. First, for the nonrelativistic and low-intensity field strength regime ($\gamma = 1.0053, \varepsilon = 0.05$ a.u) and for the relativistic regime and increasing field strength ($\gamma = 2.00, \varepsilon = 1.00$ a.u the differences between our results and the results found in Ref. [1] are small but approach 1%. Second, we have a different picture for the relativistic-high intensity regime ($\gamma = 2.00, \varepsilon = 5.89$ a.u) where the missing terms in Ref. [1] lead to values of the Dirac-Volkov DCS (26) of Ref. [1] that overestimate the corresponding DCS (18) of our work. Even in the nonrelativistic regime ($\gamma = 1.0053$) but for increasing field strengths, the difference between our results and the results of Ref. [1] begins to appear clearly. To conclude, we derived the correct expression of the first Born differential cross section for the scattering of a Dirac-Volkov electron by a Coulomb potential of a nucleus in the presence of a strong laser field. We have given the correct relativistic generalization of the Bunkin and Fedorov treatment [6] that is valid for an arbitrary geometry. Comparison of our numerical calculations with those of Szymanowski *et al.* [1] shows qualitative and quantitative differences when the incoming total electron energy and the electric-field strength are increased particularly in the direction of the laser propagation. The difference between our results and those of Ref. [1] can only be traced back to the mistakes and omitted term in Eq. (26) of Ref. [1].


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