# Transverse Fresnel-Fizeau drag effects in strongly dispersive media

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A light beam normally incident upon an uniformly moving dielectric medium is, in general, subject to bendings due to a transverse Fresnel-Fizeau light drag effect. In most familiar dielectrics, the magnitude of this bending effect is very small and hard to detect. Yet, the effect can be dramatically enhanced in strongly dispersive media where slow group velocities in the m/s range have been recently observed taking advantage of the electromagnetically induced transparency effect. In addition to the usual downstream drag that takes place for positive group velocities, we discuss a significant anomalous upstream drag which is expected to occur for negative group velocities. Furthermore, for sufficiently fast speeds of the medium, higher-order dispersion terms are found to play an important role and to be responsible for light propagation along curved paths or the restoration of the time and space coherence of an incident noisy beam. The physics underlying this class of slow-light effects is thoroughly discussed.

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## I. INTRODUCTION

A constant effort has always been devoted to the search for new effects and materials to control the propagation of light waves. Over the past few years, in particular, the use of quantum interference has led to an astonishing control of light waves propagating through specific classes of atomic and solid-state media. These materials exhibit superior properties that cannot be found in conventional ones. *Slow light* propagation at group velocities in the m/s range, e.g., has been observed in Bose-Einstein condensates of sodium atoms [1,2], in hot rubidium vapors [3,4] as well as in a solid doped Pr:$Y_2SiO_5$ crystal [5], in Ruby [6], and in alexandrite crystals [7]. Reversible *stopping* of a laser pulse [8] in ultracold and hot alkali vapors [9,10] as well as in Pr:$Y_2SiO_5$ has also been observed [5].

Light stopping and slow-light propagation effects typically originate from electromagnetically induced transparency (EIT). Such a phenomenon arises from quantum interference and is characterized by a strong enhancement of the refractive index dispersion within a narrow frequency window around the medium resonance where absorption turns out to be largely quenched. The study of such a phenomenon goes back to the late 1970s when nonabsorbing resonances in atomic sodium have first been observed in Gozzini's group and later interpreted in terms of coherent population trapping [11–13].

The interest for such a phenomenon has however revived [14] over the past decade and has now become a rather topical area of research [15,16]. Much of this work deals with fundamental issues such as high nonlinear coupling between weak fields and quantum entanglement of slow photons [17–19], entanglement of atomic ensembles [20], quantum memories [21], and enhanced acousto-optical effects [22], just to mention a few. In particular, a strict analogy between slow-light in moving media and light propagating in curved space times has been unveiled and some of its consequences have been recently discussed [23–29].

Recently it has been also anticipated in Refs. [30,31] that EIT media are ideal candidates for the observation of extremely low and negative group velocities and apparently superluminal behavior. In such media, the group velocity can in fact be readily tuned over a wide range of negative values directly by varying the coupling and probe detunings in a standard three-level $\Lambda$ configuration.

Although most fast- and slow-light experiments have dealt with the basic problem of a light pulse advance or delay during its propagation across a steady dispersive medium, ultraslow positive or negative group velocities can have interesting consequences in other scenario. In this paper we present a thorough investigation of slow-light propagation through a moving medium. Specifically, we examine a configuration in which the electromagnetically induced transparency (EIT) medium uniformly moves along a direction orthogonal to an incident light beam of finite spatial extent [33]. The same geometry, adopted in the early 1970s by Jones in his pioneering work on the *transverse* Fresnel-Fizeau light drag effect [34,35] led to the observation of a very small downstream bending, i.e., in the direction of motion, of a light ray. The use of a strongly dispersive medium supporting slow light, rather than the nondispersive glassy material used by Jones, will not only allow for a remarkable enhancement of the drag effect but also for qualitatively new features.

The possibility of having light propagating with small negative group velocities across the dragging medium is predicted to yield large *upstream* light bendings, i.e., in the direction opposite to that of the medium. Such an anomalous light drag effect, which has been the subject of an old controversy during the late 1970s [36,37], is discussed here in detail unwinding some of its controversial aspects.

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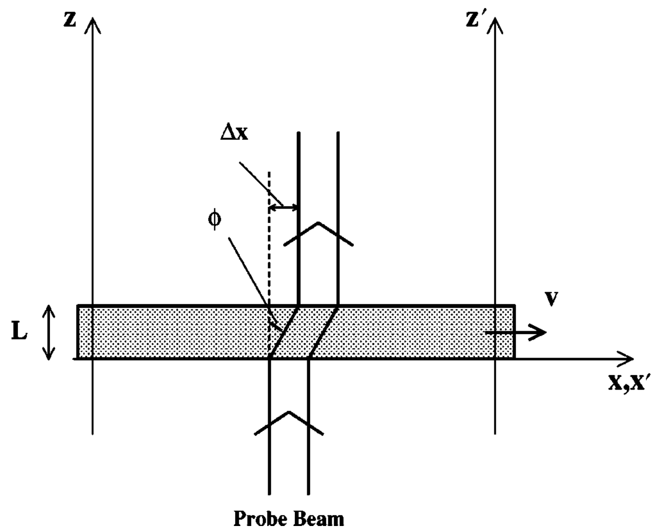


FIG. 1. Scheme of the experimental setup under consideration.

Furthermore, for sufficiently fast dragging speeds, the correct description of the slow-light transverse dragging effect requires that absorption dispersion and group-velocity dispersion be taken into account. It turns out that these higher–order dispersion terms introduce peculiar features, such as propagation along curved light paths and restoration of time and space coherence of a noisy beam. Although all our predictions have been made using realistic parameters taken from slow-light experiments in ultracold atomic clouds [1], our results are not restricted to cold atoms and generally hold also for slow-light solid media that have now become available even at room temperature [6,7].

The paper is organized as follows. In Sec. II we introduce the physical system and we present our model. The general theory of the transverse Fresnel-Fizeau light drag effect is presented in Sec. III. These general results are specialized to the case of a strongly dispersive dressed medium driven into a $\Lambda$ configuration by a resonant and nonresonant coupling beam, in Sec. IV and in Sec. V respectively: in the former case, the group velocity is small and positive, so the transverse drag occurs in the usual downstream direction; in the latter case, the negative group velocity is shown to give an anomalous upstream drag. In Sec. VI we proceed to discuss two effects which arise from the inclusion of higher-order dispersion terms. A summary of the work is given in Sec. VII where conclusions are also drawn. Two alternative derivations of the Fresnel drag effect are discussed in the Appendix.

## II. THE MODEL AND GENERAL THEORY

We consider a monochromatic *probe* light beam of frequency $\omega_0$ propagating along the $z$ axis and normally incident upon a homogeneous dielectric medium uniformly moving along the $x$ direction as shown in Fig. 1. Here we denote the thickness of the slab with $L$ and its velocity with $v$ ($v \ll c$). The probe beam has a Gaussian profile centered in $(x_0,y_0)$ so that at $z=0$ one has

$$E_0(x,y)=E_0\, e^{-[(x-x_0)^2+(y-y_0)^2]/2\sigma_0^2}, \tag{1}$$

where $\sigma_0$ is the beam waist. The corresponding Fourier transform $\tilde{E}_0(k_x,k_y)$ of $E_0(x,y)$ is then a Gaussian of width $\sigma_0^{-1}$.

In the following we shall always restrict our attention to the case of a weak probe beam, which allows us to apply linear-response theory and describe the polarization of the medium by means of a dielectric function $\epsilon$. Assuming for simplicity a nonmagnetic ($\mu=1$) and isotropic $\epsilon_{ij}=\epsilon\,\delta_{ij}$ medium, the scalar dielectric function $\epsilon$ completely characterizes the linear polarization of the medium in its rest frame $\Sigma'$. Spatial locality of the dielectric polarization will also be assumed, i.e., $\epsilon$ will be taken to depend on the frequency $\omega'$ but not on the wave vector $\mathbf{k}'$. Primed quantities will refer to the medium rest frame $\Sigma'$, while the nonprimed ones will refer to the laboratory frame $\Sigma$. The linearity of the optical response and the translational invariance of the system on the $(x,y)$ plane allow us to propagate each transverse Fourier component independently from the others, and to reconstruct the transmitted beam profile by using an inverse Fourier transform.

In the $\Sigma'$ frame, the dispersion law inside the medium has the usual form

$$\epsilon(\omega')\,\omega'^2=c^2(k_x'^2+k_y'^2+k_z'^2). \tag{2}$$

Since $v\ll c$, the linearized form of the Lorentz transformations can be used, namely,

$$\omega'=\omega-k_x v, \tag{3}$$

$$k_x'=k_x-\frac{\omega}{c^2}v, \tag{4}$$

$$k_{y,z}'=k_{y,z}. \tag{5}$$

By inserting the transformations (3)–(5) into the dispersion law (2), one obtains the following expression for the dispersion law in $\Sigma$:

$$\frac{(\omega-k_x v)^2}{c^2}\epsilon(\omega-k_x v)=\left(k_x-\frac{\omega v}{c^2}\right)^2+k_y^2+k_z^2. \tag{6}$$

This equation can be used to determine the propagation of the light beam across the slab. From the frequency $\omega_0$ and the transverse components $k_{x,y}$ of the wave vector, we obtain from Eq. (6) the $z$ component $k_z^{(in)}$ of the wave vector inside the medium:

$$k_z^{(in)}(k_x,k_y)=\sqrt{\left(\frac{\omega_0-k_x v}{c}\right)^2\epsilon(\omega_0-k_x v)-\left(k_x-\frac{\omega_0 v}{c^2}\right)^2-k_y^2} \tag{7}$$

as well as in the space outside the slab:

$$k_z^{(out)}(k_x,k_y)=\sqrt{\frac{\omega_0^2}{c^2}-k_x^2-k_y^2}. \tag{8}$$

Since the energy flux must be along the positive $z$ axis, only roots with positive real part $\mathrm{Re}[k_z^{(in,out)}]>0$ have to be taken [38].

For each component, the amplitude at positions $0\leqslant z\leqslant L$ inside the slab is given by

$$\tilde{E}(k_x,k_y;z)=e^{i\Phi(k_x,k_y;z)}\tilde{E}_0(k_x,k_y) \tag{9}$$

with a phase $\Phi$:

$$\Phi(k_x,k_y;z)=k_z^{(in)}(k_x,k_y)z. \tag{10}$$

Note that the wave vector $k_z^{(in)}(k_x,k_y)$ as well as the phase $\Phi(k_x,k_y;z)$ are generally complex quantities, their imaginary parts vanish only for nonabsorbing, nonamplifying medium. Past the slab, the transmitted amplitude at the position $z>L$ is given by the same Eq. (9) with the phase $\Phi$ now given by

$$\Phi(k_x,k_y;z)=k_z^{(in)}(k_x,k_y)\,L+k_z^{(out)}(k_x,k_y)\,(z-L). \tag{11}$$

The spatial profile of the transmitted beam at any point $z$ can then be obtained taking the inverse Fourier transform of Eq. (9),

$$E(x,y,z)=\int\frac{dk_x\,dk_y}{2\pi}\,e^{i(k_xx+k_yy)}\,e^{i\Phi(k_x,k_y;z)}\,\tilde{E}_0(k_x,k_y). \tag{12}$$

## III. THE TRANSVERSE FRESNEL-FIZEAU DRAG EFFECT

Provided that the incident beam waist $\sigma_0$ is wide enough, only a very small window of wave vectors $(k_x,k_y)$ around $k_{x,y}=0$ is effectively relevant to the propagation dynamics and one can safely expand the phase (11) in powers of $k_{x,y}$ so that to the lowest order one has

$$\Phi(k_x,k_y;z)=\frac{\omega_0}{c}\sqrt{\epsilon(\omega_0)}\left[1-\frac{k_xv}{\omega_0}\left(1-\frac{1}{\epsilon(\omega_0)}\right)-\frac{k_x v}{2\,\epsilon(\omega_0)}\,\frac{d\epsilon}{d\omega}\right]L+\frac{\omega_0}{c}(z-L). \tag{13}$$

The position of the center $(x_c,y_c)$ of the emerging wave packet at a given $z$ can be obtained inserting the expansion (13) into (12) and then invoke the so-called *stationary-phase* principle. This states that the integral in Eq. (12) has its maximum value at those points $(x_c,y_c)$ for which constructive interference between the different Fourier components occurs, that is at those points where the phase of the integrand is stationary [39]:

$$\frac{\partial}{\partial k_{x,y}}\{\mathrm{Re}[\Phi(k_x,k_y;z)]+k_xx_c+k_yy_c\}\bigg|_{k_x=k_y=0}=0. \tag{14}$$

Inserting the phase (13) into (14) and assuming the imaginary part $\epsilon_i$ of $\epsilon=\epsilon_r+i\,\epsilon_i$ to be negligible, one finds that in the laboratory frame $\Sigma$ the beam propagates inside the moving slab at a nonvanishing angle $\theta$ with respect to the $z$ direction given by

$$\tan\theta=\frac{v}{c}\left[\sqrt{\epsilon_r(\omega_0)}+\frac{\omega_0}{2\sqrt{\epsilon_r(\omega_0)}}\,\frac{d\epsilon_r}{d\omega}-\frac{1}{\sqrt{\epsilon_r(\omega_0)}}\right]=\frac{v}{c}\left[\frac{c}{v'_{\mathrm{gr}}}-\frac{v'_{\mathrm{ph}}}{c}\right]. \tag{15}$$

Here $v'_{\mathrm{gr}}$ and $v'_{\mathrm{ph}}$ denote the group and phase velocities, respectively, in the medium rest frame $\Sigma'$:

$$v'_{\mathrm{ph}}=\frac{c}{\sqrt{\epsilon_r(\omega_0)}}, \tag{16}$$

$$v'_{\mathrm{gr}}=\frac{c}{\sqrt{\epsilon_r(\omega_0)}+\frac{\omega_0}{2\sqrt{\epsilon_r(\omega_0)}}\,\frac{d\epsilon_r}{d\omega}}. \tag{17}$$

This deflection of the light beam can be interpreted as a *transverse* Fresnel-Fizeau drag effect, in which the beam of light is dragged by the transverse motion of the medium. After exiting the slab, the beam again propagates along the normal direction. Its center, however, turns out to be laterally shifted by an amount

$$\Delta x=Lv\left[\frac{1}{v'_{\mathrm{gr}}}-\frac{v'_{\mathrm{ph}}}{c^2}\right] \tag{18}$$

along a direction parallel to the medium velocity. This expression is in agreement with the one derived by Player and by Rogers [40,41]. Two alternative derivations of this lateral shift are discussed in the Appendix.

As one can see from the expression (18) the magnitude of the transverse drag $\Delta x$ is largest for a strongly dispersive medium in which $\epsilon$ has a rapid frequency dependence and the group velocity $v'_{\mathrm{gr}}$ results much slower than the vacuum speed of light $v'_{\mathrm{gr}}\ll c$. In this case, Galilean velocity composition law can be safely applied and Eq. (18) simplifies to

$$\Delta x=\frac{Lv}{v'_{\mathrm{gr}}}, \tag{19}$$

which yields a very intuitive interpretation for $\Delta x$ as the displacement of the medium during the time $\Delta t=L/v'_{\mathrm{gr}}$ it takes the light to cross.

## IV. THE TRANSVERSE FRESNEL-FIZEAU DRAG EFFECT IN AN EIT MEDIUM

The first experimental observation of the transverse Fresnel-Fizeau drag effect was performed in the mid 1970s by Jones [34] using a rotating glass disk as moving dielectric medium. In such a nondispersive medium the phase and group velocities were of the order of $c$ ($c/v'_{\mathrm{gr}}\approx c/v'_{\mathrm{ph}}\approx 1.5$)

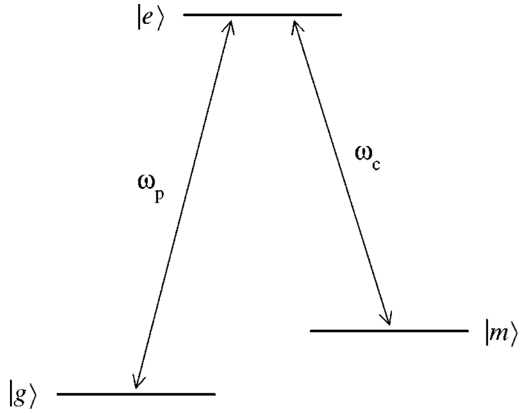


FIG. 2. Scheme of the energy levels involved in the optical transitions.

and the limited rotation velocity of the disk at the beam spot ($v \simeq 2\times 10^4$ cm/s) limited the lateral displacement to a distance of the order of a few nanometers. However, a clever optical alignment technique allowed not only to observe the effect, but also to discriminate the validity of the result (15) from other possible expressions for the displacement [35].

As remarked in the preceding section, the use of a strongly dispersive medium allows for a significant enhancement of the drag effect. Very slow group velocities can be obtained by optically dressing a resonant transition with a coherent *coupling* laser beam as shown in the $\Lambda$-level scheme of Fig. 2.

Under the assumption of a weak probe beam, linear-response theory holds and the resulting dielectric constant is to be interpreted as describing the linear response of the optically driven medium to a weak probe. In the simplest case of nondegenerate levels, the rest frame dielectric constant of our optically driven $\Lambda$ configuration acquires the following well-known form [12–15]:

$$\epsilon(\omega)=\epsilon_\infty+\frac{4\pi f}{\omega_e-\omega-i\frac{\gamma_e}{2}-\frac{|\Omega_c|^2}{\omega_m+\omega_c-\omega-i\frac{\gamma_m}{2}}}, \quad (20)$$

where $\omega_{e(m)}$ and $\gamma_{e(m)}$ are the frequency and the linewidth of the excited $e$ and metastable $m$ states, respectively, where we have set $\omega_g=0$. Here $\omega_c$ is the frequency of the $e\leftrightarrow m$ coherent coupling beam [42] and $\Omega_c$ its Rabi frequency. The linewidth $\gamma_m$ of the metastable state is much smaller than the linewidth $\gamma_e$ of the excited state. The $f$ parameter is proportional to the oscillator strength of the optical transition: for an ultracold atomic gas [1] at atomic densities $n \approx 10^{12}$ cm$^{-3}$, $f$ is of the order of a few $10^{-3}\gamma_e$. The background dielectric constant $\epsilon_\infty$ takes into account the effect of all the other nonresonant transitions, but for an atomic gas $\epsilon_\infty \simeq 1$ to a very good approximation.

For a resonant coupling beam, i.e., $\delta_c=\omega_c-(\omega_e-\omega_m)=0$, the dielectric function (20) in the neighborhood of the resonance $|\omega-\omega_e|\ll\gamma_e$ can be rewritten as

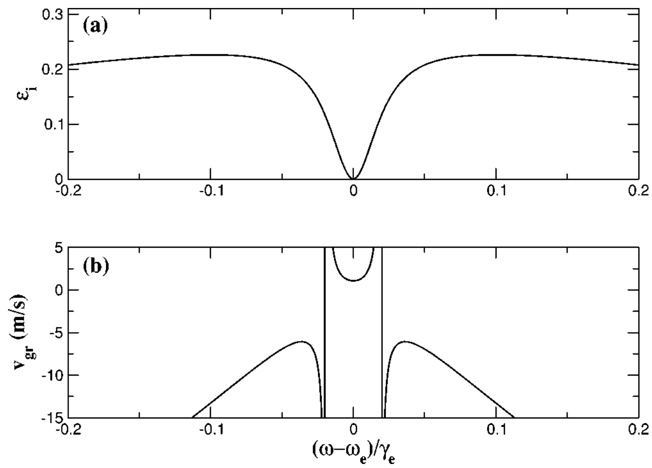


FIG. 3. Resonantly dressed EIT medium at rest: plot of the imaginary part $\epsilon_i(\omega)$ of the dielectric function (a) and of the group velocity $v_{\rm gr}$ (b). Medium parameters correspond to the case of an ultracold $^{23}$Na gas: $\gamma_e \simeq 2\pi\times 10$ MHz, $\lambda_e=589$ nm, $f=0.009\gamma_e$, $\epsilon_\infty=1$, $\gamma_m=10^{-4}\gamma_e$. The Rabi frequency of the resonant ($\delta_c=0$) coupling beam is $\Omega_c=0.1\gamma_e$. For this choice of parameters, the minimum (positive) group velocity is $v_{\rm gr}\approx 1$ m/s.

$$\epsilon(\omega)=\epsilon_\infty+\frac{8\pi f i}{\gamma_e}\left[1+\frac{2i\Omega_c^2/\gamma_e}{\omega_e-\omega-\frac{i}{2}\left(\gamma_m+\frac{4\Omega_c^2}{\gamma_e}\right)}\right]. \quad (21)$$

The imaginary part of Eq. (21) shown in Fig. 3(a) shows a narrow dip around $\omega=\omega_e$ in the otherwise wide absorption profile of the $g\rightarrow e$ transition. Provided $\Omega_c^2/\gamma_e \gg \gamma_m$, absorption at the center of the dip is strongly suppressed, yielding nearly perfect transparency for a resonant probe beam. This is the so-called EIT effect [11]. The linewidth of the dip is $\Gamma \simeq 4\Omega_c^2/\gamma_e$ and becomes strongly subnatural ($\Gamma \ll \gamma_e$) for $\Omega_c \ll \gamma_e$. The assumed inequality $\gamma_m \ll \gamma_e$ guarantees that a good level of transparency can be obtained simultaneously with a subnatural linewidth of the dip.

In the dip region, the real part of the dielectric function (20) shows an extremely steep dispersion yielding the group velocity

$$\frac{v_{\rm gr}}{c}\simeq\frac{|\Omega_c|^2}{2\pi f\omega_e}, \quad (22)$$

which can be orders of magnitude smaller than $c$ [Fig. 3(b)]. This suggests that the importance of the transverse drag effect (18) should be strongly enhanced in an EIT medium.

To verify this prediction, a complete calculation of the profile of the transmitted probe beam can be numerically performed by inserting the explicit expression of the dielectric function (20) into the dispersion law (6) and then performing the inverse Fourier transform (12). For a probe exactly on resonance with the $g\rightarrow e$ transition ($\omega_0=\omega_e$), the numerical result plotted in Fig. 4(a) is in perfect agreement with the analytic prediction (18) when the value (22) for the group velocity is used. The nearly total absence of absorption

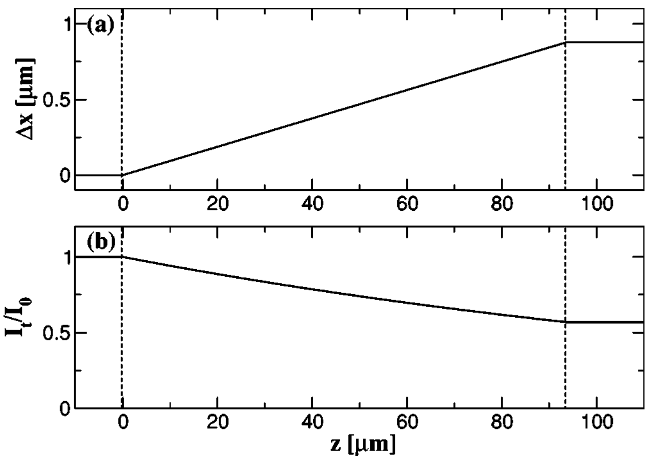


FIG. 4. Propagation of a light beam through a slowly moving ($v=0.01$ m/s), resonantly dressed EIT medium. The optical parameters of the medium are the same as in Fig. 3, the vertical dashed lines correspond to the surfaces of the medium, whose thickness is taken as $L=93$ $\mu$m. The incident beam is resonant $\omega_0=\omega_e$ and its waist is $\sigma_0=20$ $\mu$m. *Downstream* transverse drag (a) and corresponding weak absorption of the beam (b). The $x$ axis is oriented in the downstream direction.

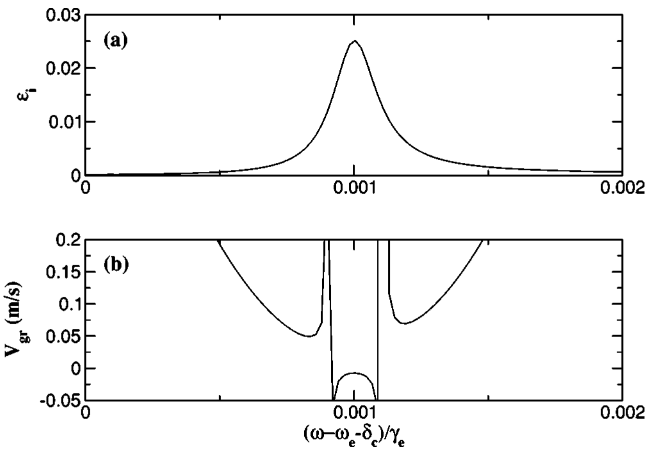


FIG. 5. Nonresonantly ($\delta_c=10\ \gamma_e$) dressed EIT medium at rest: plot of the imaginary part $\epsilon_i(\omega)$ of the dielectric function (a) and of the group velocity $v_{\rm gr}(\omega)$ (b). The other parameters are the same as in Fig. 3.

at the center of the dip guarantees that only a small fraction of the incident probe intensity is absorbed by the medium [Fig. 4(b)].

Group velocities as low as 1 m/s have been observed in ultracold atomic gases [2]. For $v_{\rm gr}=1$ m/s a medium that is 100 $\mu$m thick and moves with a velocity of the order of $v=0.01$ m/s gives a lateral shift $\Delta x\approx 1$ $\mu$m as shown in Fig. 4. This is orders of magnitude larger than the one originally observed by Jones [34,35]. Even larger lateral shifts should be attained by using solid EIT materials [5,43,44] as dragging medium. Group velocities as slow as 45 m/s have in fact been recently observed in a Pr-doped $Y_2SiO_5$ [5] kept at very low temperatures. Furthermore, the mechanical rigidity and the possibility of working at higher temperatures should allow one to study dragging effects on a thicker sample moving at a higher speed $v$. The new generation of slow-light solid media where velocities between 60 and 90 m/s can be achieved at room temperature [6,7] are certainly the best candidates for the observation of a substantial transverse drag effect.

## V. ANOMALOUS TRANSVERSE FRESNEL-FIZEAU DRAG EFFECT

The discussion of the preceding sections has focused on the most common case of media with *normal* dispersion. Because for all frequency at which the medium is transparent and nonamplifying, the Kramers-Kronig causality relations [45] ensure that

$$\frac{c}{v'_{\rm gr}}-\frac{v'_{\rm ph}}{c}>0, \tag{23}$$

the transverse drag (18) turns out to be directed in the *downstream* direction, as if the light were to be dragged by the moving medium.

On the other hand, several papers during 1970s [36,37] have discussed the possibility of having an *upstream* transverse drag in the presence of *anomalous* dispersion, i.e., in the presence of negative group velocity. As negative group velocities in nonmagnetic media [46] are forbidden by Kramers-Kronig relations [45] in all frequency regions where the medium is transparent and nonamplifying, negative group velocities are possible only in the presence of substantial absorption [48–50] or in amplifying media [51].

In most experiments performed up to now, negative group velocities have been demonstrated by observing that the pulse advances in time with respect to the same wave packet propagating in vacuum. This kind of apparent superluminal behavior refers to the peak of the wave packet only, and not to the propagation velocity of information. As discussed in Ref. [52], the velocity of the front of a step function signal can never exceed $c$.

As we have recently anticipated in Ref. [53], a significant upstream drag effect should be observable in nonresonantly dressed EIT media, when the coupling beam is not exactly on resonance with the $m\rightarrow e$ transition, but has a finite detuning $|\delta_c|=|\omega_m+\omega_c-\omega_e|\geqslant\gamma_e$. This kind of media have in fact been shown to be good candidates for the observation of ultraslow and negative group velocities [30]. In the case $|\delta_c|\geqslant\gamma_e$, the dielectric function (20) in the neighborhood of $\omega=\omega_m+\omega_c$ (Fig. 5) can be written in the following Lorentzian form:

$$\epsilon(\omega)=\epsilon_\infty-\frac{4\pi f}{\delta_c}+\frac{4\pi f_2}{\omega_2-\omega-i\frac{\gamma_2}{2}}. \tag{24}$$

Largest absorptions occur at the Raman resonance with the two-photon transition from the ground $g$ state to the metastable $m$ state via the excited $e$ state, i.e., at the two-photon resonance frequency

$$\omega_2=\omega_m+\omega_c+\frac{\Omega_c^2}{\delta_c}. \tag{25}$$

The shift $\Omega_c^2/\delta_c$ from the bare resonant frequency is due to the optical Stark effect induced by the coupling beam. The linewidth $\gamma_2$ is the sum of the bare linewidth of the metastable $m$ level plus a contribution which takes into account its decay via the excited $e$ state:

$$\gamma_2=\gamma_m+\frac{\Omega_c^2}{\delta_c^2}\gamma_e. \tag{26}$$

For large coupling beam detunings $|\delta_c|\gg\Omega_c$, the linewidth $\gamma_2$ is much smaller than $\gamma_e$, while the oscillator strength $f_2$ of the transition,

$$f_2=\frac{\Omega_c^2}{\delta_c^2}f, \tag{27}$$

is itself weakened. The peak absorption (proportional to $f_2/\gamma_2$) does not vary for increasing values of $|\delta_c|/\Omega_c$, at least as far as $\gamma_2\gg\gamma_m$. On the other hand, the anomalous dispersion at resonance $\omega=\omega_2$ is under the same conditions enhanced due to the narrower linewidth $\gamma_2$. The corresponding group velocity can be written as

$$\frac{v_{\mathrm{gr}}}{c}=-\frac{\gamma_2^2}{8\pi f_2\,\omega_e}, \tag{28}$$

and, in the limit $\gamma_2\gg\gamma_m$, this is a factor $\gamma_e^2/4\delta_c^2$ smaller in magnitude than the one obtained in the resonant $\delta_c=0$ case considered in the preceding section. The use of a detuned coupling with $|\delta_c|>\gamma_e$ should then enable one to observe significant upstream drags over an optical thickness $L$ of the order of the absorption length, still leaving the transmitted probe intensity to be an appreciable fraction of the incident one.

In order to verify this expectation, we have again inserted the explicit expression of the dielectric function (20) into the propagation Eq. (6) and we have numerically performed the inverse Fourier transform so as to obtain the profile of the transmitted beam [Fig. 6(a)]. As expected, light bendings are found to occur in the upstream direction and the magnitude of the effect is in good agreement with the approximated analytical expression (18) in which the imaginary part of $\epsilon$ was neglected. We have also verified in Fig. 6(b) that the beam is not completely absorbed in the medium. Note that for the same set of material parameters and the same experimental configuration as in the preceding section, except for the coupling beam detuning $\delta_c$, the group velocity $|v_{\mathrm{gr}}|$ is now of the order of 0.01 m/s, i.e., a factor 100 slower than in the case of a resonant coupling. This explains the bigger deflection angle $\theta$.

Nonresonantly dressed EIT media are therefore good candidates for the experimental observation of significant upstream deflections and consequently large anomalous transverse drags. Such an observation should finally resolve the longstanding controversy about the possibility of observing

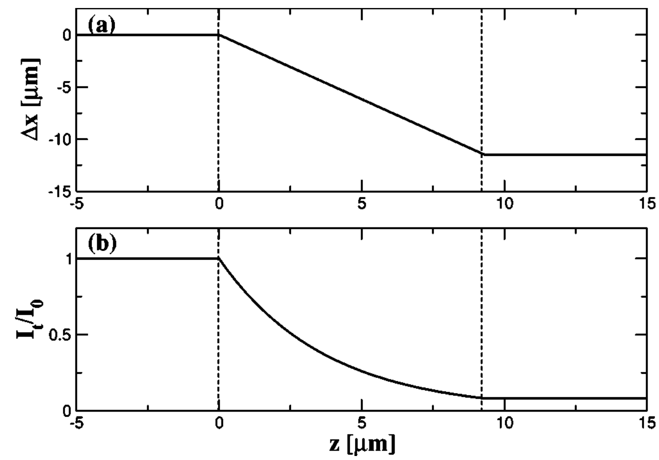


FIG. 6. Propagation of a light beam through a slowly moving ($v=0.01$ m/s), nonresonantly dressed ($\delta_c=10\,\gamma_e$) EIT medium; probe frequency on resonance with the two-photon transition ($\omega_0=\omega_2$). The optical parameters of the medium are the same as in Fig. 5; the vertical dashed lines correspond to the surfaces of the medium, whose thickness is taken as $L=1.9$ $\mu$m. The incident beam waist is $\sigma_0=20$ $\mu$m. Anomalous *upstream* transverse drag (a) and corresponding absorption of the beam (b). The $x$ axis is oriented in the downstream direction.

such an effect [36,37]. Conversely, the anomalous upstream drag could provide an interesting way of probing negative group velocities, in place of the usual [48–51] measurement of the negative temporal delay of the pulse after its propagation across the medium.

## VI. HIGHER-ORDER DISPERSION EFFECTS

The effects discussed in the preceding two sections entirely rely on the strongly reduced value of the group velocity of light in EIT media. For the slow medium velocity range addressed above, higher-order dispersion effects such as the dispersion of absorption or the group-velocity dispersion are very small and hardly contribute to the results plotted in Figs. 4 and 6. Yet, these terms may be no longer negligible for larger values of $v$, regime in which the light propagation can exhibit qualitatively new features [54]. In what follows we shall discuss in detail two of such features.

### A. Light propagation along a curved path

In the present section, we shall discuss the effect of nonrectilinear light propagation due to simultaneously large dispersions of both absorption [55] (proportional to $d\epsilon_i/d\omega$) and group velocity (proportional to $d^2\epsilon_r/d\omega^2$).

For large enough values of the absorption dispersion $d\epsilon_i/d\omega$, the absorption coefficient can have significant variations across the range of transverse $k_x$ vectors present in the incident beam. In this case, the center of mass $k_x^{\mathrm{cm}}(z)$ of the $k_x$ wave vector distribution is shifted from its initial value $k_x^{\mathrm{cm}}(z=0)=0$ as the beam propagates through the medium.

In the slow group-velocity regime ($v'_{\mathrm{gr}}\ll c$), the dependence of the propagation wave vector (7) on the transverse

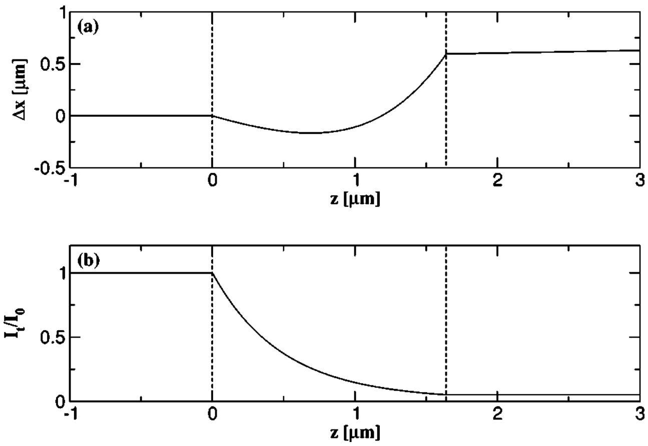


FIG. 7. Nonrectilinear light beam propagation through a moving ($v=4$ m/s) EIT medium for a resonant coupling ($\delta_c=0$) and a slightly detuned probe ($\omega_0-\omega_e=0.04\,\gamma_e$); the incident beam waist is $\sigma_0=3\ \mu$m. Panel (a) curved beam path across the moving medium. Panel (b) intensity of the light beam at different depths in the medium. The optical parameters of the medium are the same as in Fig. 3; the vertical dashed lines correspond to the surfaces of the medium, whose thickness is taken as $L=1.65\ \mu$m.

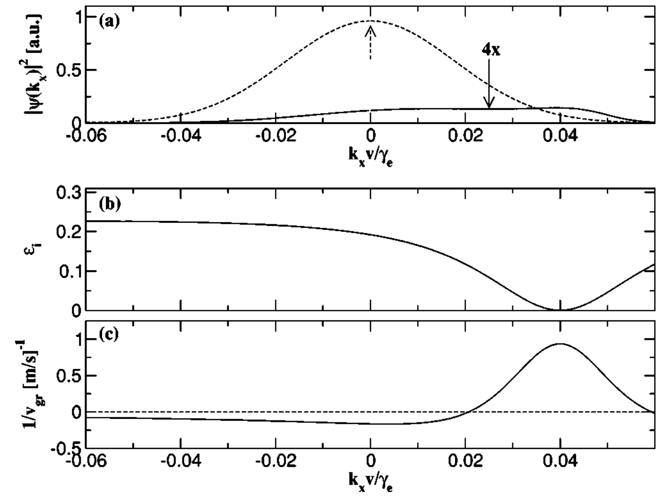


FIG. 8. Physical interpretation of the nonrectilinear light propagation of Fig. 7. Panel (a) spatial Fourier transform of the incident (dashed) and transmitted (solid) beam profile; for each spectrum, the arrow indicates the position of the spectral center of mass. For the sake of clarity, the transmitted spectrum has been multiplied by 4. Panels (b) and (c) absorption and inverse group velocity $1/v_{\rm gr}$ spectra as a function of the transverse wave vector $k_x$. The corresponding Doppler-shifted frequency in the rest frame $\Sigma'$ is $\omega' =\omega_0-k_x v$. Since $\omega_0-\omega_e=0.04\gamma_e$, the resonance is found at $k_x v=0.04\gamma_e$.

wave vector $k_{x,y}$ mainly comes from the Doppler effect combined with the strong frequency dispersion of the dielectric function, so that transmitted phase reads

$$\Phi(k_x,k_y;z)\simeq\frac{\omega_0 z}{c}\sqrt{\epsilon(\omega_0-k_x v)}. \qquad (29)$$

By applying the same stationary-phase arguments used in Sec. III to the finite $k_x^{\rm cm}(z)$ case, we find that after propagation for a distance $z$ the spatial center of mass of the beam is located at

$$\Delta x(z)=z\frac{v}{v'_{\rm gr}(\omega_D)}. \qquad (30)$$

In physical terms, as the spectral center of mass $k_x^{\rm cm}$ of the beam varies with $z$ because of the filtering action of the absorption, the transverse drag at a given position $z$ has to be evaluated using the group velocity $v'_{\rm gr}$ at the Doppler shifted frequency $\omega_D(z)=\omega_0-v\,k_x^{\rm cm}(z)$. This implies that the light beam is no longer rectilinear, but acquires a finite curvature. In particular, the deflection angle $\theta(z)$ given by

$$\tan\theta(z)=\frac{\partial\Delta x(z)}{\partial z}=\frac{v}{v'_{\rm gr}(\omega_D)}+\frac{v^2 z}{[v'_{\rm gr}(\omega_D)]^2}\frac{dv'_{\rm gr}}{d\omega}\frac{dk_x^{\rm cm}}{dz} \qquad (31)$$

now contains a term explicitly depending on the spectral center of mass shift $dk_x^{\rm cm}/dz$ which was not present in Eq. (15) [56].

Figures 7 and 8, report the result of numerical calculations for the specific case of a moving EIT medium when the coupling beam is resonant ($\delta_c=0$) and the probe beam is slightly blue detuned ($\omega_0-\omega_e=0.04\,\gamma_e$). As one can verify by comparing the spectrum in Fig. 8(a) with the group-velocity spectrum in Fig. 8(c), most of the incident beam spectrum lies in the negative group-velocity region ($k_x v<0.02\,\gamma_e$), so the beam is initially dragged in the upstream direction. As absorption is weaker for the $k_x>0$ components for which the Doppler-shifted frequency $\omega'=\omega_0-k_x v$ is closer to resonance [see Fig. 8(b)], the negative $k_x<0$ components are more rapidly quenched than the positive $k_x>0$ components and the center of mass $k_x^{\rm (cm)}$ of the spectral distribution moves towards the positive $k_x>0$ values. As one can see in Fig. 8(c), $(\partial/\partial k_x)(1/v'_{\rm gr})>0$ in the region $0<k_x v<0.04\,\gamma_e$ of interest and therefore the curvature of the beam will be towards the downstream direction. After the first 1 $\mu$m of propagation, most of the $k_x$ spectrum is found in the positive $v'_{\rm gr}$ region ($k_x v>0.02\gamma_e$), so that the transverse drag is from now on in the downstream direction [Fig. 7(a)].

As a consequence of the shift of the spectral center of mass $k_x^{\rm cm}$, the beams exits from the rear face of the medium at a small but finite angle with respect to the normal towards the downstream direction. Although this effect is hardly visible on the scale of Fig. 8(a), this small bending of the beam direction may have a significative effect on the subsequent rectilinear propagation of the beam in the free space.

Unfortunately, this effect of nonrectilinear propagation is associated with a rather severe absorption of the beam. For the parameters used in Fig. 7, transmission is about 5%.

It is also worth noticing that the curvature effect described in the present section follows from a reshaping of the beam in momentum space and hence is physically different from the ones discussed in [23,24], which instead originate from a nonuniform velocity field of the slow-light medium.

### B. Temporal and spatial coherence restoration

If both probe and coupling beams are exactly on resonance ($\omega_0-\omega_e=\delta_c=0$), both absorption $d\epsilon_i/d\omega'$ and group velocity $d^2\epsilon_r/d\omega'^2$ dispersion vanish, so that the effects described in Sec. VI A do not take place. In the present section, we shall show how one can rather take advantage of the large value of $d^2\epsilon_i/d\omega'^2$ to improve the coherence level of a noisy incident probe beam. In the following dection, the case of *temporal* coherence restoration in a stationary EIT medium will be adressed, while in Sec. VI B 2 we shall show how the same concepts can be applied in the case of a moving EIT medium to improve the level of *spatial* coherence.

#### 1. Temporal coherence restoration

Consider an incident probe beam of carrier frequency $\omega_0$ whose complex amplitude $E_0(t)$ is assumed to fluctuate in time over a characteristic time scale $\tau_c$:

$$E(t)=E_0(t)\,e^{-i\omega_0 t}. \tag{32}$$

Following a standard model [58], the decay in time of the first-order coherence function $g^{(1)}(\tau)$ is taken to be Gaussian:

$$g^{(1)}(\tau)=\frac{\langle E_0^*(\tau)E_0(0)\rangle}{\langle E_0^*(0)E_0(0)\rangle}=\exp(-\tau^2/2\tau_c^2). \tag{33}$$

The frequency spectrum $|\tilde{E}(\omega)|^2$ of the beam, being proportional to the Fourier transform of the coherence function $g^{(1)}(\tau)$, is also Gaussian with linewidth $\sigma_c=1/\tau_c$.

In a one-dimensional geometry, the propagation of a pulse through a stationary dielectric medium is described by the usual Fresnel equation

$$k_z^2(\omega)=\epsilon(\omega)\,\frac{\omega^2}{c^2}. \tag{34}$$

In the neighborhood of the resonance at $\omega_e$, the dielectric function of a slow-light EIT medium can be approximately written as

$$\epsilon(\omega)\simeq 1+\frac{2c}{\omega_e v_{\rm gr}^{(0)}}(\omega-\omega_e)+i\,\frac{\alpha}{2}(\omega-\omega_e)^2, \tag{35}$$

where $v_{\rm gr}^{(0)}$ is the group velocity at resonance and the real quantity $\alpha$ is given by

$$\alpha=\left.\frac{d^2\epsilon_i(\omega)}{d\omega^2}\right|_{\omega=\omega_e}=\frac{4\pi f\gamma_e}{\Omega_c^4}. \tag{36}$$

To the lowest order in $\omega-\omega_e$, the real and imaginary parts of the wave vector $k_z(\omega)$ can then be written as

$$k_z(\omega)|_r\simeq\frac{\omega_e}{c}\left[1+\frac{c}{\omega_e v_{\rm gr}^{(0)}}(\omega-\omega_e)\right], \tag{37}$$

$$k_z(\omega)|_i\simeq\frac{\alpha\omega_e}{4c}(\omega-\omega_e)^2. \tag{38}$$

Since after propagation over a distance $L$ the amplitude of each frequency component is multiplied by a factor $\exp[ik_z(\omega)L]$, the frequency spectrum after propagation keeps its Gaussian shape, but the frequency linewidth is reduced to

$$\sigma_c(L)=\frac{\sigma_c}{\sqrt{1+\dfrac{\alpha\sigma_c^2\omega_e}{c}L}} \tag{39}$$

and the coherence time correspondingly increased to

$$\tau_c(z)=\tau_c\sqrt{1+\frac{\alpha\omega_e}{c\,\tau_c^2}L}. \tag{40}$$

The absorption associated with this filtering technique may still be fairly high; the beam intensity reduction over a distance $L$ during the line-narrowing process may be estimated to be

$$I(L)\simeq\frac{\sigma_c(L)}{\sigma_c(0)}I(0). \tag{41}$$

#### 2. Spatial coherence restoration

For a moving EIT medium and a strictly monochromatic light beam at $\omega_0$, a similar effect occurs in the $k_x$ space. As discussed in full detail in Sec. II, the propagation of light is described in this case by Eq. (7). For resonant probe and coupling beams, the imaginary part of $\mathrm{Im}[k_z^{(in)}]$ corresponding to absorption is proportional to $v^2k_x^2$:

$$\mathrm{Im}[k_z^{(in)}]\simeq\frac{\alpha\omega_0}{4c}v^2k_x^2. \tag{42}$$

As the amplitude of the transverse spatial fluctuations is proportional to the amplitude of nonvanishing $k_x$ components, the spatial profile of the beam flattens as the beam propagates through the moving EIT medium. The faster the speed $v$ of the medium, the more efficient the spatial coherence restoration process.

Results of a numerical calculation for a spatially noisy incident beam propagating through a moving EIT medium are presented in Fig. 9. Note that how the fluctuation amplitude is strongly suppressed during propagation. At the end of the process, the losses in the total intensity amount to 60%.

The overall shift of the beam which can be seen in the figure is due to the transverse drag effect discussed in detail in Sec. IV. Since both the probe and the coupling are resonant, the shift is directed in the downstream direction.

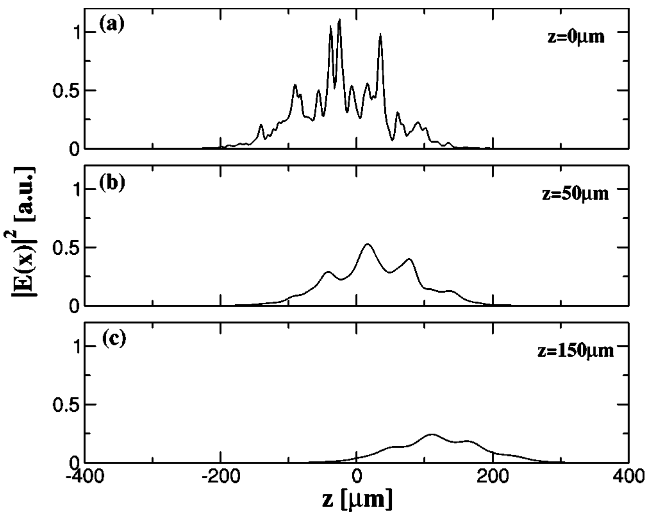


FIG. 9. Transverse spatial coherence restoration during propagation across a moving ($v=1$ m/s) EIT medium in a fully resonant regime ($\delta_c=\omega_0-\omega_e=0,\ \omega_0=\omega_e$). Panel (a) noisy incident beam profile. Panels (b) and (c) beam profile after propagation through the medium. Same medium parameters as in Figs. 3–7.

## VII. CONCLUSIONS

In the present paper we have given a comprehensive analysis of the transverse Fresnel-Fizeau drag effect for light propagating across a uniform slab of moving medium and, in particular, we have dealt with the case of a highly dispersive EIT medium. All calculations have been performed using realistic parameters taken from EIT experiments with ultracold atomic clouds. However, our results are general and thus hold through also for the recently prepared solid-state EIT media. Depending on the detuning of the probe and coupling beams with respect to the medium resonance, different regimes have been identified.

In the presence of a slow and positive group velocity, the magnitude of the downstream drag effect is predicted to be significantly enhanced with respect to previous drag experiments. In the regime of negative and small group velocities, a sizable anomalous upstream drag has been predicted to occur. Not only would this help in solving a long-standing controversy on the observability of such an effect, but it could also provide an interesting alternative way for experimentally probing negative group velocities.

For larger values of the velocity of the moving medium, higher-order dispersion terms such as the dispersion of absorption and the dispersion of group velocity have been shown to play an important role. Depending on the specific choice of probe and coupling detuning, light propagation along curved paths can be observed, as well as the restoration of temporal and spatial coherence of a noisy beam.

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## APPENDIX: TWO ALTERNATIVE PICTURES OF THE TRANSVERSE FRESNEL DRAG

In the present Appendix we shall give two alternative derivations of the transverse Fresnel-Fizeau drag which should help the reader to get a different insight of the underlying physics.

Even though the medium does not exhibit any spatial dispersion in its rest frame $\Sigma'$, spatial dispersion however arises in the laboratory frame $\Sigma$. This stems from the $k_x$ dependence of $\omega'$ in the transformation law (3). Physically, this can be easily understood in the following way: in its rest frame $\Sigma'$, the dielectric polarization of a medium with only frequency dispersion depends on the retarded values of the electric field at the same spatial position. As seen from the laboratory frame $\Sigma$, the polarization at a given point turns out to depend on the electric field at different spatial positions. This means that a moving medium can show spatial dispersion in $\Sigma$ even if it is only temporally dispersive in the rest frame $\Sigma'$.

The most remarkable consequence of this induced spatial dispersion is the nonparallelism of the group and phase velocities even if the medium is isotropic in the rest frame $\Sigma'$. For a light beam normally incident on the slab, the transverse $x$ and $y$ components of the phase velocity are vanishing. On the other hand, the group velocity $\mathbf{v}_{gr}=\boldsymbol{\nabla}_{\mathbf{k}}\omega(\mathbf{k})$ at $k_{x,y}=0$ obtained from the dispersion law (6) has a finite component along $x$:

$$v_{\mathrm{gr},x}=v\left(1-\frac{1}{\epsilon+\dfrac{\omega}{2}\dfrac{d\epsilon}{d\omega}}\right)=v\left(1-\frac{v'_{\mathrm{gr}}v'_{\mathrm{ph}}}{c^2}\right), \tag{A1}$$

$$v_{\mathrm{gr},y}=0, \tag{A2}$$

$$v_{\mathrm{gr},z}=\frac{c}{\sqrt{\epsilon}+\dfrac{\omega}{2\sqrt{\epsilon}}\dfrac{d\epsilon}{d\omega}}=v'_{\mathrm{gr}}, \tag{A3}$$

which implies that the beam propagates in the medium at a finite angle $\theta$ with respect to the normal. The value of the angle $\tan\theta=v_{\mathrm{gr},x}/v_{\mathrm{gr},z}$ obtained from Eqs. (A1) and (A3) agrees indeed with the one found in a different way in Eq. (15).

An again different picture of the same drag effect can be obtained by working in the rest frame $\Sigma'$ as discussed in our previous paper [53]. Because of the aberration of light [39], the direction of the incident beam makes an angle $\theta'_{\rm inc} = -v/c$ with the normal. In $\Sigma'$, the group and phase velocities in the medium are parallel and make an angle $\theta'_{\rm refr} = -v/(c\sqrt{\epsilon})$ with the normal. The magnitude of the group velocity in $\Sigma'$ is given by Eq. (17). By Lorentz-transforming back the group velocity to the laboratory frame $\Sigma$, it is easy to check that the same deflection angle as in Eq. (15) is obtained.


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