Slow light in Doppler-broadened two-level systems

G. S. Agarwal and Tarak Nath Dey
Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India
(Received 25 August 2003; published 16 December 2003)

We show that the propagation of light in a Doppler-broadened medium can be slowed down considerably even though such medium exhibits very flat dispersion. The slowing down is achieved by the application of a saturating counter propagating beam that produces a hole in the inhomogeneous line shape. In atomic vapors, we calculate group indices of the order of 10^3. The calculations include all coherence effects.

DOI: 10.1103/PhysRevA.68.063816 PACS number(s): 42.50.Gy, 32.80.−t

It is now well understood that slow light can be produced by using the electromagnetically induced transparency (EIT) [1,2]. Many experiments have been reported in a variety of atomic and condensed media [3–8]. Such experiments reveal that the group velocity of the light pulses depends on the parameters of the control field, which produces EIT. Various applications of slow light have been proposed and realized [9–14]. Recently, Bigelow *et al.* [15] showed that one can produce slow light in systems such as Ruby, without the need for applying a control field. They made a hole in homogeneous line in systems, where the transverse and longitudinal relaxation times are of very different order.

In this paper, we consider the possibility of producing slow light in a Doppler-broadened system. This is somewhat counterintuitive as one would think that Doppler broadening would make the dispersion, or more precisely, the derivative of the susceptibility, rather negligible. We, however, suggest the use of the method of saturation absorption spectroscopy [16–20] to produce a hole of the order of the homogeneous width in the Doppler-broadened line. The application of a counter propagating saturated beam can result in considerable reduction in absorption, and adequate normal dispersion to produce slow light. We calculate group index of the order of 10^3 . We illustrate our results using the case of the atomic vapors. However, similar or even more remarkable results on slowing of light can be obtained for inhomogeneously broadened solid-state systems, where the densities are large.

Consider the geometry as shown in Fig. 1. Here a modulated pulse of light propagates in the direction $\hat{z}$ in a medium of two-level atoms. For simplicity we consider the incident pulse of the form

$$\vec{E}(t) \equiv \vec{\mathcal{E}}(1 + m \cos \nu t) e^{i(kz-\omega t)} + \text{c.c.}, \quad k = \frac{\omega}{c}. \tag{1}$$

Here m and ν are the modulation index and frequency, respectively. A counterpropagating pump cw field $\vec{E}_c(t)$ is used for producing saturation

$$\vec{E}_c(t) \equiv \vec{\mathcal{E}}_c e^{i(kz-\omega_c t)} + \text{c.c.} \tag{2}$$

The effective linear susceptibility $\chi(\omega)$ of the two-level atomic systems which is interacting with the field $\vec{\mathcal{E}} e^{i(kz-\omega t)}$ and $\vec{E}_c(t)$, can be calculated to all orders in the counterpropagating field (2). The effective susceptibility $\chi(\omega)$ is well known from the work of Mollow [21],

$$\chi = -\frac{N|d|^2}{\hbar} \frac{1+\Delta^2 T_2^2}{(1+\Delta^2 T_2^2 + 4|G|^2 T_1 T_2)(\Delta + \delta + i/T_2)} \left[1 - \frac{2|G|^2 (\Delta - i/T_2)^{-1} (\delta + 2i/T_2)(\delta - \Delta + i/T_2)}{(\delta + i/T_1)(\delta + \Delta + i/T_2)(\delta - \Delta + i/T_2) - 4|G|^2 (\delta + i/T_2)} \right], \tag{3}$$

where $\Delta = \omega_c - \omega_{1g}$ and $\delta = \omega - \omega_c$ are the detuning of the pump and probe field, respectively. For an atom moving with velocity $\vec{v}$, we replace ω_c by $(\omega_c + kv)$, and ω by $(\omega - kv)$. The Rabi frequency of the pump is given in terms of the dipole moment matrix element $\vec{d}_{1g}$ by

$$2G = \frac{2\vec{d}_{1g} \cdot \vec{\mathcal{E}}_c}{\hbar}. \tag{4}$$

The T_1 and T_2 are, the longitudinal and transverse relaxation times, respectively, and N is the density of atoms. The susceptibility (3) is to be averaged over the Doppler distribution of velocities

$$P(kv)d(kv) = \frac{1}{\sqrt{2\pi D^2}} e^{[-(kv)^2/2D^2]} d(kv), \tag{5}$$

where D is the Doppler width defined by

$$D = \sqrt{K_B T \omega^2 / M c^2}. \tag{6}$$

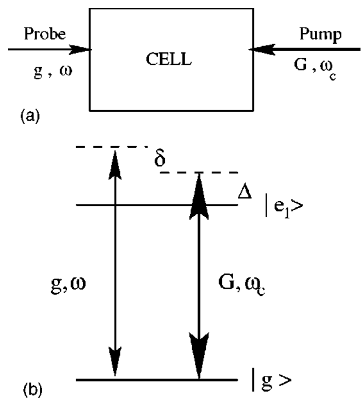

FIG. 1. (a) A block diagram where the pump (ω_c) and probe (ω) fields are counterpropagating inside the medium. (b) Schematic representation of a two level atomic system with ground state $|g\rangle$ and excited state $|e_1\rangle$.

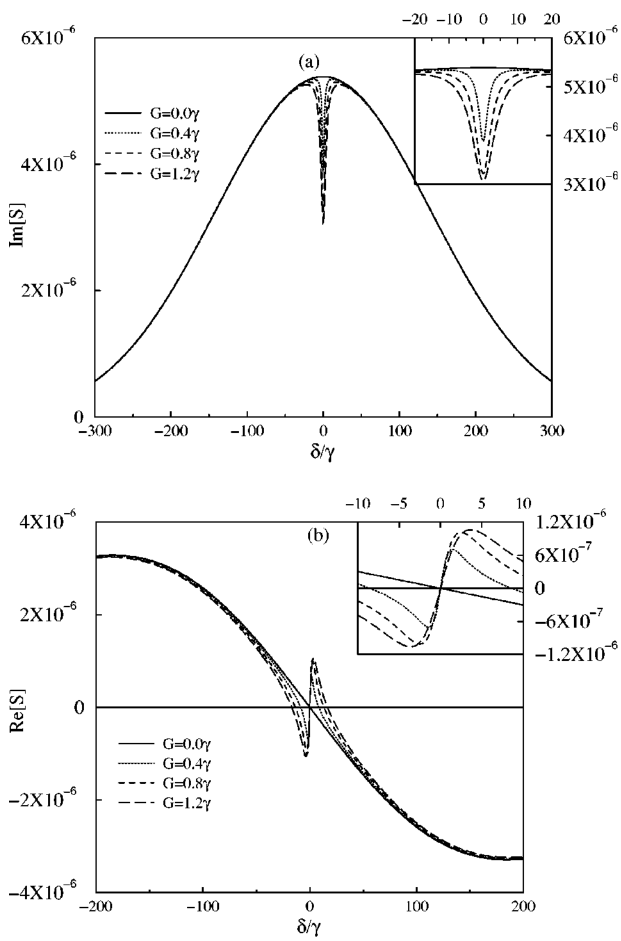

FIG. 2. (a) and (b) The imaginary and real parts of susceptibility $S(\omega)$ at the probe frequency ω in the presence of pump field G. Here we considered the pump field in resonance. The inset shows a magnified part of the same. The common parameters of the above four curves for [87]Rb vapor are chosen as Doppler width parameter $D=1.33\times10^9$ rad/s, density $N=2\times10^{11}$ atoms/cm^3, and $\gamma=3\pi\times10^6$ rad/s.

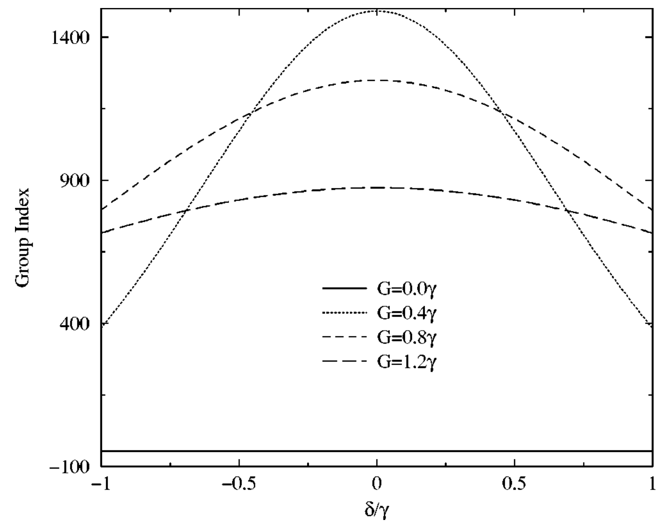

FIG. 3. The variation of group index with the detuning of the probe field. The parameters are chosen as $N=2\times10^{11}$ atoms/cm^3, $D=1.33\times10^9$ rad/s, $\gamma=3\pi\times10^6$ rad/s, and $\Delta=0$.

We denote the average of $\chi(\omega)$ by $\mathcal{S}(\omega)$. For small modulations, we can use the approximation

$$S(\omega\pm\nu)=S(\omega)\pm\nu\frac{\partial S}{\partial\omega}. \tag{7}$$

The probe field in Eq. (1) at the output face $z=l$ of the medium can be expressed as

$$\vec{E}(l,t)=\vec{\mathcal{E}}(1+m\cos[\nu(t+\theta)])e^{i(kl-\omega t)+i\frac{\omega}{c}2\pi lS(\omega)}+\text{c.c.}, \tag{8}$$

where the delay time θ is defined by

$$\theta=2\pi l\frac{\omega}{c}\frac{\partial\text{Re}[S]}{\partial\omega}. \tag{9}$$

Note that θ will be positive if $\partial\text{Re}[S]/\partial\omega>0$, i.e, if the medium exhibits normal dispersion. Note further the relation of the parameter θ to the group velocity and the group index

$$v_g=\frac{c}{n_g}=\frac{c}{\left(1+2\pi\text{Re}[S(\omega)]+2\pi\omega\frac{\partial\text{Re}[S]}{\partial\omega}\right)}. \tag{10}$$

Further the imaginary part of S will give the overall attenuation of the pulse.

We present numerical results for the group index by evaluating Eq. (10) for different intensities of the counterpropagating beam. We use typical parameter for [87]Rb transition: $T_1=T_2/2=1/2\gamma$, $\gamma=3\pi\times10^6$ rad/s, $D=1.33\times10^9$ rad/s (at room temperature), and $N=2\times10^{11}$ atoms/cm^3. We show in Fig. 2, the behavior of real and imaginary parts of the susceptibility $S(\omega)$ assuming that the counterpropagating pump is in resonance with atomic transition, i.e, $\omega_c=\omega_{1g}$. The imaginary part of $S(\omega)$ shows the typical Lamb dip [22] which, becomes deeper with the increase in the intensity of the saturating beam. The real part of

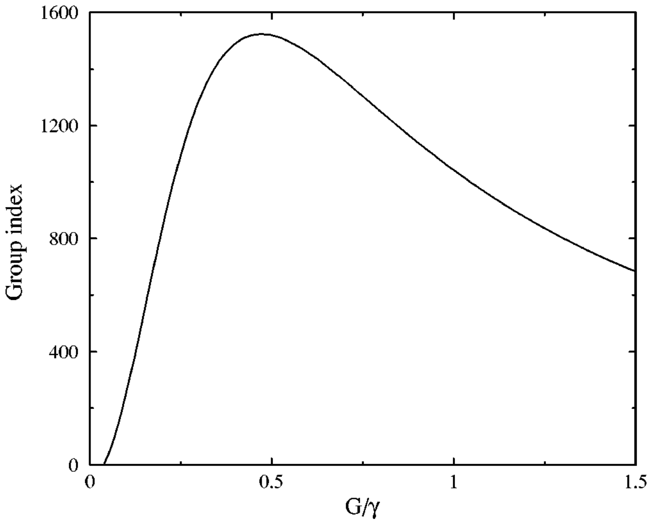

FIG. 4. Group index variation with the Rabi frequency of the saturating field. The parameters are chosen as $N = 2\times 10^{11}$ atoms/cm^3, $D = 1.33\times 10^9$ rad/s, $\gamma = 3\pi\times 10^6$ rad/s, $\Delta = 0$, and $\delta = 0$.

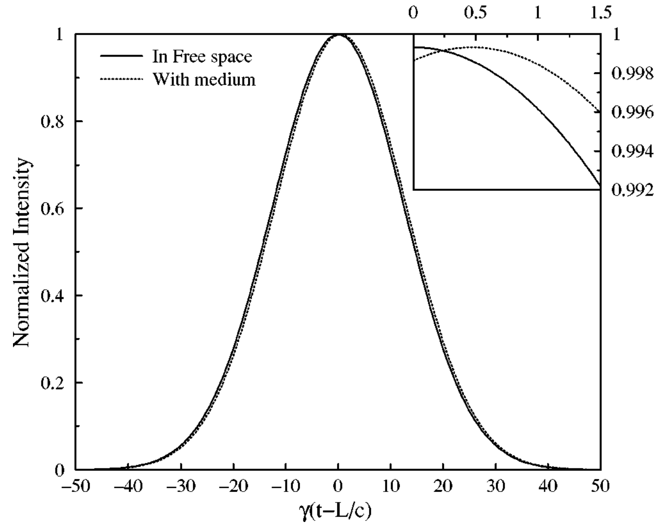

FIG. 5. The solid curve shows light pulse propagating at speed c through 1 cm of vacuum. The dotted curve shows same light pulse propagation through a medium of length 1 cm with time delay of 0.05 μs in the presence of saturating pump with Rabi frequency $G = 0.4\gamma$. The common parameters of the above graph for ^{87}Rb vapor are chosen as $N = 2\times 10^{11}$ atoms/cm^3, $D = 1.33\times 10^9$ rad/s, $\gamma = 3\pi\times 10^6$ rad/s, $\Delta = 0$, and $\delta = 0$. The transmission intensity is 2.1%. The inset shows the close-up of the Gaussian pulse with a spectral width, 120 kHz.

$S(\omega)$ exhibits normal dispersion, which in fact, is very pronounced. It is this sharp dispersion which can produce slow light. The calculated group index n_g as a function of the detuning of the probe from the atomic transition is shown in Fig. 3. We show the behavior in the region of Lamb dip. Clearly the group index increases with the intensity of the saturating pump. One can calculate n_g as a function of G, for $\delta = 0$, and the result is shown in Fig. 4. To confirm these results, we also studied the propagation of a Gaussian pulse with an envelope given by

$$\mathcal{E}(t - L/c) = \frac{\mathcal{E}_0}{2\pi}\exp[-(t - L/c)^2/\tau^2],$$

$$\mathcal{E}(\omega) = \frac{\mathcal{E}_0}{\sqrt{\pi\Gamma^2}}\exp[-(\omega - \omega_0)^2/\Gamma^2], \qquad (11)$$

where $\Gamma\tau$ is equal to 2 and L is the length of the medium. We use $\Gamma = 120$ kHz for our numerical simulation. The pulse delay of 0.05 μs due to the medium is seen in Fig. 5. The group velocity of the pulse, calculated from the relative delay between the reference pulse and the output pulse, is in good agreement, with the value of group index $[(c/v_g) = 1500]$. We get the transmission of Gaussian pulse of the order of 2.1% [23]. This value of transmission can be understood by evaluating $\mathrm{Im}[4\pi l\omega S(\omega)/c]$ [cf. Eq.(8)], which is found to be 3.84. This implies a transmission $e^{-3.84} \sim 2.1\%$. The condition for distortionless pulse propagation is that spectral width of the Gaussian pulse to be well contained within the region of Lamb dip of the medium. If the pulse spectrum becomes too broad relative to the width of the Lamb dip then a simple expression like Eq. (10) does not hold. One can, however, still calculate numerically the output pulse.

In conclusion, we have shown how Lamb dip and saturated absorption spectroscopy can be used to produce slow light with group indices of the order of 10^3 in a Doppler broadened medium, which otherwise has very flat dispersion.

[1] S.E. Harris, J.E. Field, and A. Imamoglu, Phys. Rev. Lett. **64**, 1107 (1990); S.E. Harris, J.E. Field, and A. Kasapi, Phys. Rev. A **46**, R29 (1992).
[2] S.E. Harris, Phys. Today **50**(7), 36 (1997).
[3] A. Kasapi, M. Jain, G.Y. Yin, and S.E. Harris, Phys. Rev. Lett. **74**, 2447 (1995).
[4] L.V. Hau, S.E. Harris, Z. Dutton, and C.H. Behroozi, Nature (London) **397**, 594 (1999).
[5] M.M. Kash, V.A. Sautenkov, A.S. Zibrov, L. Hollberg, G.R. Welch, M.D. Lukin, Y. Rostovtsev, E.S. Fry, and M.O. Scully, Phys. Rev. Lett. **82**, 5229 (1999); A.B. Mastko, O. Kocharovskaya, Y. Rostovtsev, A.S. Zibrov, and M.O. Scully, Adv. At., Mol., Opt. Phys. **46**, 191 (2001).
[6] D. Budker, D.F. Kimball, S.M. Rochester, and V.V. Yashchuk, Phys. Rev. Lett. **83**, 1767 (1999).
[7] O. Schmidt, R. Wynands, Z. Hussein, and D. Meschede, Phys. Rev. A **53**, R27 (1996).
[8] A.V. Turukhin, V.S. Sudarshanam, M.S. Shahriar, J.A. Musser, B.S. Ham, and P.R. Hemmer, Phys. Rev. Lett. **88**, 023602 (2002).
[9] C. Liu, Z. Dutton, C.H. Behroozi, and L.V. Hau, Nature (London) **409**, 490 (2001).
[10] M. Fleischhauer and M.D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000); M. Fleischhauer and M.D. Lukin, Phys. Rev. A **65**,

022314 (2002); D.F. Phillips, A. Fleischhauer, A. Mair, R.L. Walsworth, and M.D. Lukin, Phys. Rev. Lett. **86**, 783 (2001).

[11] A.S. Zibrov, A.B. Matsko, O. Kocharovskaya, Y.V. Rostovtsev, G.R. Welch, and M.O. Scully, Phys. Rev. Lett. **88**, 103601 (2002).

[12] A.B. Matsko, Y.V. Rostovtsev, O. Kocharovskaya, A.S. Zibrov, and M.O. Scully, Phys. Rev. A **64**, 043809 (2001).

[13] C. Mewes and M. Fleischhauer, Phys. Rev. A **66**, 033820 (2002).

[14] T.N. Dey and G.S. Agarwal, Phys. Rev. A **67**, 033813 (2003).

[15] M.S. Bigelow, N.N. Lepeshkin, and R.W. Boyd, Phys. Rev. Lett. **90**, 113903 (2003).

[16] T.W. Hansch, M.D. Levenson, and A.L. Schawlow, Phys. Rev. Lett. **26**, 946 (1971).

[17] S. Haroche and F. Hartmann, Phys. Rev. A **6**, 1280 (1972).

[18] B. Couillaud and A. Ducasse, Phys. Rev. Lett. **35**, 1276 (1975).

[19] M. Sargent III, Phys. Rep. **43**, 223 (1978).

[20] P.G. Pappas, M.M. Burns, D.D. Hinshelwood, M.S. Feld, and D.E. Murnick, Phys. Rev. A **21**, 1955 (1980).

[21] B.R. Mollow, Phys. Rev. A **5**, 2217 (1972).

[22] W. Demtroder, *Laser Spectroscopy Basic Concepts and Instrumentation*, 3rd ed. (Springer-Verlag, Berlin, 1981), p. 450.

[23] The transmitted intensity is almost same as in any saturation absorption experiment [cf. S. Svanberg, G.-Y. Yan, T.P. Duffey, and A.L. Schawlow, Opt. Lett. **11**, 138 (1986)].