Mesic molecule formation in collisional Auger transitions of excited mesic hydrogen

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The formation rate of the excited hydrogen mesic molecule due to Auger process is calculated in a quasiclassic approximation. The resulted bound state may decay via predissociation with a large energy release, which leads to a considerable acceleration of the mesic atom. The calculated rates of the mesic molecule formation are compared with those obtained in a semiclassical approach.

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I. INTRODUCTION

The excited mesic hydrogen atom is formed when a negative muon stops in hydrogen and is captured into atomic orbitals. The most probable state has the principal quantum number $n \sim \sqrt{m/m_e}$, where m and m_e are the muon and electron masses, respectively. The rate of radiative transitions from such a state is small, so deexcitation proceeds mainly by collisions with neighbor atoms and molecules. For $n \lesssim 10$ the main processes which lead to the cascade transitions in the mesic atom are Coulomb deexcitation and external Auger process. In the former process the deexcitation energy is transferred to the relative motion of the nuclei, resulting in the enhanced kinetic energy. In the latter process the energy of the mesic atom transition is taken mainly by the electron of the target molecule, the acceleration thus being not large. The calculations [1–7] showed that the Coulomb deexcitation rates are much smaller than those of Auger deexcitation and thermalization rates in elastic collisions [8]. For this reason the experimentally observed large number of fast mesic atoms in the lower-energy states [9,10] has not been explained for a long time.

In Ref. [6] a mechanism of the mesic atom acceleration in the cascade Auger transitions has been proposed. It was shown that after the external Auger process (n, n' are the principal quantum numbers of the mesic atom)

$$(p\mu)_n + \mathrm{H} \rightarrow (p\mu)_{n'} + p + e^-, \quad n' < n, \tag{1}$$

when the transition energy is taken by the electron, the $p\mu + p$ system may appear in the bound state, decaying then via predissociation to the lower term:

$$(p\mu)_n + \mathrm{H} \rightarrow (p\mu p)_{n'} + e^-, \tag{2}$$

$$(p\mu p)_{n'} \rightarrow (p\mu)_{n''} + p, \quad n'' < n'. \tag{3}$$

As a result of the second step (3) the deexcitation energy is shared by the heavy particles, enhancing considerably the mesic atom energy. The estimates of the reaction rates (2) and (3) presented in Refs. [6,7] appeared to exceed significantly the Coulomb deexcitation rates.

One should note that Menshikov [11] suggested that a mesomolecular complex can be formed in the external Auger process. However, he considered only internal Auger decay of such a complex, which led to a small (up to ~ 1 eV) acceleration of the mesic atoms. His version of such a two-step Auger process was as follows:

$$(p\mu)_n + \mathrm{H}_2 \rightarrow [(p\mu)_{n'}, \mathrm{H}_2^+] + e^-, \tag{4}$$

$$[(p\mu)_{n'}, \mathrm{H}_2^+] \rightarrow (p\mu)_{n''} + p + p + e^-. \tag{5}$$

In Ref. [11] it was suggested that the rates of reactions (4) and (5) do not differ from those calculated in earlier Refs. [4,5] on Auger deexcitation. In Refs. [6,7] the rates of reactions (2) and (3) were recalculated taking into account the electron screening in the input channel. The calculation was performed in a semiclassical approach, in which the motion of the nuclei was treated classically and that of the muon and electron was treated quantum mechanically. In accordance with the approach, it was considered that in the bound state of the complex the energy of the relative motion of the nuclei changes continuously. However, according to the quantum mechanics, the energy of the finite motion is quantized, for which reason the semiclassical estimates could be inadequate. In the present paper we calculate the rates of processes (2) and (3) by means of a simplified version of quantum-mechanical description of the relative motion of the nuclei—quasiclassical approximation.

II. PROBLEM DEFINITION

When hitting the hydrogen target the mesic atom interacts with the hydrogen molecule H_2. Nevertheless, as in all previous papers, beginning from the pioneer paper by Leon and Bethe [4], we shall consider the collision with a hydrogen atom H, regarding, however, the electron binding energy to be equal to $I_e = 15.4$ eV, as in a collision with the molecule. Just to be definite, we shall regard the muon and proton, keeping in mind that the results (with slight variations) are valid for all hydrogen isotopes (p,d,t) and all negative mesons (μ^-, π^-, K^-). As before, we do not take into account the identity of the nuclei in the atom and mesic atom, because it does not affect much the main aim of the paper: to estimate the Auger deexcitation rate via the formation of the bound state.

So process (2) is considered under the assumption that the main contribution to the cross section comes, like in process (1), from the internuclear distances R much greater than the dimension of the excited mesic atom. The calculations are

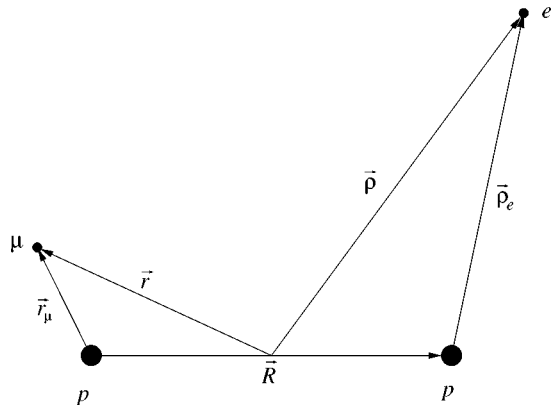

FIG. 1. Coordinate system used in the calculations.

performed in perturbation theory, the perturbation being the interaction V between the mesic atom and target electron, unperturbed wave functions being presented as products of muonic wave functions of the problem of two Coulomb centers $\phi(\vec{r},R)$ and electron single-center wave functions $\psi(\rho)$:

$$V=\frac{1}{|\vec{\rho}-\vec{r}|}-\frac{1}{|\frac{1}{2}\vec{R}+\vec{\rho}|}, \tag{6}$$

$$\Psi_i=F_p(\vec{R})\phi(\vec{r},R)\psi_1(\vec{\rho}), \quad \Psi_f=F_b(\vec{R})\phi'(\vec{r},R)\psi_2(\vec{\rho}). \tag{7}$$

The vectors $\vec{R}$, $\vec{r}$, and $\vec{\rho}$ are shown in Fig. 1. Electron wave functions

$$\psi_1(\vec{\rho})=\psi_{1s}(\vec{\rho}_e), \quad \psi_2(\vec{\rho})=\psi_k(\vec{\rho}_e) \tag{8}$$

are centered on the same nucleus and refer to the initial and ionized states of the hydrogen atom, respectively, k being the electron momentum in the ionized state.

The functions $F_p(\vec{R})$ and $F_b(\vec{R})$ describe the relative motion of nuclei in the input and output channels, respectively. In the input channel the motion is infinite with the asymptotic momentum p, in the output channel it is finite motion corresponding to the bound state of the two nuclei and muon. Since in the initial state muon is localized on one nucleus, and the electron on the other, and we neglect the possibility of muon transfer to another nucleus (the Auger transition proceeds at large distances $R \gg r_\mu$), one should choose the two-center muon function as a linear combination of even g and odd u adiabatic functions, which at $R\to\infty$ represents the wave function of the muonic atom:

$$\phi(\vec{r},R)=\frac{1}{\sqrt{2}}[\phi_g(\vec{r},R)+\phi_u(\vec{r},R)]\to\varphi_{nj}(\vec{r}_\mu), \tag{9}$$

$$\phi'(\vec{r},R)=\frac{1}{\sqrt{2}}[\phi'_g(\vec{r},R)+\phi'_u(\vec{r},R)]\to\varphi_{n'j'}(\vec{r}_\mu), \tag{10}$$

$$j=(n_1n_2m), \quad j'=(n'_1n'_2m'), \tag{11}$$

where j,j' are parabolic quantum numbers. With muonic functions thus chosen, the wave functions of nuclear motion F_p and F_b are obtained by solving the Schrödinger equation with a potential equal to a half sum of the even and odd potentials plus Coulomb repulsion of the nuclei [12,13]. The electron screening effect should also be taken into account in the input channel.

In the first order of perturbation theory the reaction amplitude is

$$M_{fi}=\langle\Psi_f|V|\Psi_i\rangle. \tag{12}$$

Because of the orthogonality of the muonic functions ϕ and ϕ', as well as ψ_{1s} and ψ_k, the only nonzero contribution to matrix element (12) comes from the first term of Eq. (6), which represents the Coulomb interaction between the muon and electron. Then

$$M_{fi}=\int d\vec{R}d\vec{r}d\vec{\rho}F_b^*(\vec{R})\phi'(\vec{r},R)\psi_2^*(\vec{\rho}) \times\frac{1}{|\vec{\rho}-\vec{r}|}\psi_1(\vec{\rho})\phi(\vec{r},R)F_p(\vec{R}). \tag{13}$$

Taking into account that the Auger transition proceeds mainly at large R, let us replace the functions ϕ and ϕ' by their asymptotic values (9) and (10), as well as the coordinates $\vec{r}$ and $\vec{\rho}$ counted from the middle of the internuclear axis, by the coordinates $\vec{r}_\mu$ and $\vec{\rho}_e$, counted from the nuclei at which the corresponding particles are localized. Then

$$M_{fi}\simeq\int d\vec{R}d\vec{r}_\mu d\vec{\rho}_e F_b^*(\vec{R})\varphi_{n'j'}(\vec{r}_\mu)\psi_k^*(\vec{\rho}_e) \times\frac{1}{|\vec{R}+\vec{\rho}_e-\vec{r}_\mu|}\psi_{1s}(\vec{\rho}_e)\varphi_{nj}(\vec{r}_\mu)F_p(\vec{R}). \tag{14}$$

Using the operator identity

$$\frac{1}{|\vec{R}+\vec{\rho}_e-\vec{r}_\mu|}=\frac{1}{2\pi^2}\int\frac{d\vec{q}}{q^2}e^{i\vec{q}(\vec{R}+\vec{\rho}_e-\vec{r}_\mu)}, \tag{15}$$

one obtains

$$M_{fi}=\int d\vec{R}F_b^*(\vec{R})V_{fi}(\vec{R})F_p(\vec{R}), \tag{16}$$

$$V_{fi}(\vec{R})=\frac{1}{2\pi^2}\int\frac{d\vec{q}}{q^2}e^{i\vec{q}\vec{R}}\int d\vec{r}_\mu e^{-i\vec{q}\vec{r}_\mu}\varphi_{n'j'}(\vec{r}_\mu)\varphi_{nj}(\vec{r}_\mu) \times\int d\vec{\rho}_e e^{i\vec{q}\vec{\rho}_e}\psi_k(\vec{\rho}_e)\psi_{1s}(\vec{\rho}_e). \tag{17}$$

We shall consider the Auger transitions to the levels $n'=n-1$ from the levels with $n=5$ and $n=4$, because at higher levels the mesic atoms are quickly thermalized in elastic collisions [8], while at lower n radiative transitions dominate [14,15], which do not lead to the mesic atom acceleration. The energy of the mesic atom transition $\Delta_{nn'}$ is

taken by the ejected electron, whose energy is $\varepsilon_e \sim \Delta_{nn'} - I_e \sim m_0 n^{-3}$, where m_0 is muon reduced mass [m_0 is defined in Eq. (30) below; in energy units $m_0 \sim 5$ keV for $p\mu$ atom]. For the transition $5 \to 4$ ($4 \to 3$) the energy of the Auger-electron $\varepsilon_e \sim 40$ eV (80 eV). This energy is not high enough (especially in the former case) to use the plane-wave approximation for the ejected electron; however, this approximation, as shown in Ref. [7], is not unreasonably crude for the wave function $\psi_k(\vec{\rho}_e)$ in our problem. So we set $\psi_k(\vec{\rho}_e) = \exp(i\vec{k}\vec{\rho}_e)$. The integral over $\vec{\rho}_e$ can then be easily obtained as

$$\int d\vec{\rho}_e e^{i(\vec{q}-\vec{k})\vec{\rho}_e} \psi_{1s}(\vec{\rho}_e) = N_{1s} \frac{8\pi\eta}{[(\vec{q}-\vec{k})^2+\eta^2]^2}, \quad \eta = m_e,$$

$$N_{1s} = (\eta^3/\pi)^{1/2}, \quad k = \sqrt{2m_e\varepsilon_e}, \quad \hbar = e = 1. \tag{18}$$

When calculating the integral over muon coordinates one should take into account that the mesic atom dimension $r_n \sim n^2 a_0$ ($a_0 = 1/m_0$ is the mesic atom radius in the ground state) and characteristic $q \sim k$ (at $R \sim a_e$ the integral over q converges in the region $q \lesssim k$). Let us estimate the characteristic value of qr_μ,

$$qr_\mu \sim k a_0 n^2 \sim \sqrt{2m_e m_0 n^{-3}} n^2 m_0^{-1} = \sqrt{2\frac{m_e}{m_0} n} \simeq 0.1\sqrt{n} \ll 1 \quad \text{for } n < 10. \tag{19}$$

Expanding the exponent $e^{-i\vec{q}\vec{r}_\mu}$ in the muon integral in a series and keeping only the dipole term one has

$$\int d\vec{r}_\mu e^{-i\vec{q}\vec{r}_\mu} \varphi_{n'j'}(\vec{r}_\mu) \varphi_{nj}(\vec{r}_\mu) \simeq -i\vec{q}\int d\vec{r} \varphi_{n'j'}(\vec{r}) \vec{r} \varphi_{nj}(\vec{r}) = -i\vec{q}\vec{D}. \tag{20}$$

Inserting Eqs. (18) and (20) into Eq. (17) and separating the main contribution to the integral over $\vec{q}$ one obtains

$$V_{fi}(\vec{R}) \simeq -4\pi i N_{1s} \frac{\vec{k}\vec{D}}{k^2} e^{i\vec{k}\vec{R} - \eta R}. \tag{21}$$

The larger is the k value the more accurate is equation (21).

When calculating the integral over $\vec{R}$ in Eq. (16) we use the integrand expansion in spherical harmonics, as well as a quasiclassical approximation to describe the radial wave functions of the relative motion of the nuclei (the adiabatic potentials corresponding to the excited states of the muonic atom satisfy the validity condition for the quasiclassical radial motion [16]).

The function $F_p(\vec{R})$ is a solution of a single-channel scattering problem with the asymptotics "plane wave + outgoing spherical wave" with a well-known partial expansion [17]:

$$F_p(\vec{R}) = \frac{1}{2pR} \sum_{\lambda=0}^{\infty} i^\lambda (2\lambda+1) \exp(i\delta_\lambda) \chi_{p\lambda}(R) P_\lambda(\vec{n_p}\vec{n_R}),$$

$$\vec{n_p} = \frac{\vec{p}}{p}, \quad \vec{n_R} = \frac{\vec{R}}{R}, \tag{22}$$

where δ_λ is a phase shift and P_λ is a Legendre polynomial.

The functions $F_b(\vec{R})$ and $\chi_{p\lambda}(R)$ are normalized by the conditions

$$\int F^*_{p'}(\vec{R}) F_p(\vec{R}) d\vec{R} = (2\pi)^3 \delta(\vec{p'} - \vec{p}),$$

$$\int \chi_{p'\lambda}(R) \chi_{p\lambda}(R) dR = 2\pi\delta(p'-p), \tag{23}$$

The function $F_b(\vec{R})$ is a solution of the eigenvalue problem and belongs to a discrete spectrum of the Schrödinger equation. We may take it in the form ($Y_{\Lambda\mu}$ is a spherical harmonic)

$$F_{\nu\Lambda\mu}(\vec{R}) = \frac{1}{R} \chi_{\nu\Lambda}(R) Y_{\Lambda\mu}(\vec{n}_R), \tag{24}$$

which describes the relative motion of the nuclei in the state with vibrational quantum number ν, rotational quantum number Λ, and magnetic number μ. Radial wave function $\chi_{\nu\Lambda}(R)$ is characterized by the energy $\varepsilon_{\nu\Lambda} < 0$ and is normalized by

$$\int \chi^2_{\nu\Lambda}(R) dR = 1. \tag{25}$$

In quasiclassic approximation the radial functions $\chi_{p\lambda}$, $\chi_{\nu\Lambda}$ in a classically approachable region look like

$$\chi_{p\lambda}(R) = \frac{C_1}{\sqrt{P(R)}} \cos\left(\int_a^R P(x) dx - \frac{\pi}{4} \right),$$

$$C_1 = 2\sqrt{P(\infty)}, \quad a < R, \tag{26}$$

$$\chi_{\nu\Lambda}(R) = \frac{C_2}{\sqrt{P'(R)}} \cos\left(\int_{a'}^R P'(x) dx - \frac{\pi}{4} \right),$$

$$C_2 = 2\sqrt{\frac{M\omega}{2\pi}}, \tag{27}$$

$$\omega = 2\pi \left(2M \int_{a'}^b \frac{dx}{P'(x)} \right)^{-1}, \quad a' < R < b. \tag{28}$$

Here ω is a circular frequency of the molecular oscillations, a', b are left and right turning points in the classical motion of the nuclei with energy $\varepsilon_{\nu\Lambda}$, M is reduced mass of the system:

$$M^{-1} = (M_1 + M_3)^{-1} + M_2^{-1}, \tag{29}$$

M_1, M_2 are the masses of the nuclei, M_3 is a meson mass (in our case $M_1 = M_2 = M_p$, $M_3 = M_\mu$, the electron mass is

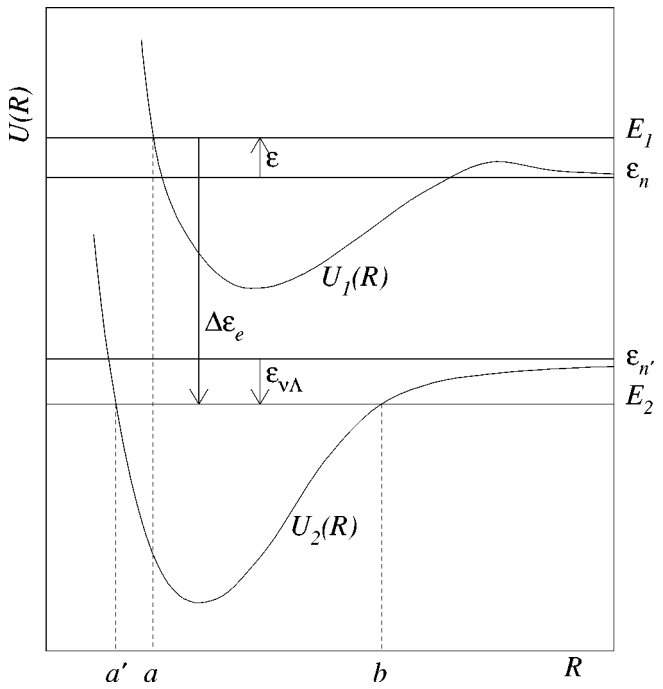

FIG. 2. Energy scheme of the molecule formation during mesic atom deexcitation.

neglected). Hereafter we shall use the reduced mass of the muon m_0 as a unit mass, defined as follows:

$$m_0^{-1}=M_1^{-1}+M_3^{-1}=M_p^{-1}+M_\mu^{-1}. \tag{30}$$

Radial momenta which enter Eqs. (26)–(28) are given by formulas

$$P(R)=\sqrt{2M[E_1-U_1(R)]}, \quad P'(R)=\sqrt{2M[E_2-U_2(R)]}, \tag{31}$$

where $E_{1,2}$ are total energies of the $p\mu p$ system, $U_{1,2}$ are potential energies with centrifugal term included, in the input and output channels, respectively (see Fig. 2). For the potential energy the following equations hold:

$$U_{1,2}(R)=U_{1,2}(\infty)+u_{1,2}(R), \tag{32}$$

$$U_1(\infty)=\varepsilon_n=-\frac{1}{2n^2}, \quad U_2(\infty)=\varepsilon_{n'}=-\frac{1}{2n'^2},$$

$$u_{1,2}(\infty)=0, \tag{33}$$

$$u_1(R)=[\varepsilon_{nj}(R)-\varepsilon_n+R^{-1}]S(R)+\frac{(\lambda+1/2)^2}{2MR^2}, \tag{34}$$

$$S(R)=e^{-2\beta R}(1+2\beta R+2\beta^2R^2), \quad \beta=m_e/m_0=m_e, \tag{35}$$

$$u_2(R)=\varepsilon_{n'j'}(R)-\varepsilon_{n'}+R^{-1}+\frac{(\Lambda+1/2)^2}{2MR^2}, \tag{36}$$

$$\varepsilon(R)=\frac{1}{2}[\varepsilon_g(R)+\varepsilon_u(R)]. \tag{37}$$

Here $\varepsilon_{g,u}(R)$ are the even and odd terms (term indices omitted), $S(R)$ is the screening correction, which takes into account the effect of the electron cloud of the target atom upon the interatomic potential in the input channel. Introducing the collision energy ε and the energy of the molecular level $\varepsilon_{\nu\Lambda}$ by equations (see Fig. 2)

$$\varepsilon=E_1-\varepsilon_n, \quad \varepsilon_{\nu\Lambda}=E_2-\varepsilon_{n'} \tag{38}$$

one can write the radial momenta as

$$P(R)=\sqrt{2M[\varepsilon-u_1(R)]}, \quad P'(R)=\sqrt{2M[\varepsilon_{\nu\Lambda}-u_2(R)]}. \tag{39}$$

Then, expanding the exponent $e^{i\vec{k}\vec{R}}$ in Eq. (21) into partial waves,

$$e^{i\vec{k}\vec{R}}=4\pi\sum_{\ell m} i^\ell j_\ell(kR)Y^*_{\ell m}(\vec{n}_k)Y_{\ell m}(\vec{n}_R), \quad \vec{n}_k=\vec{k}/k, \tag{40}$$

where $j_\ell(kR)=\sqrt{\pi/2kR}J_{\ell+1/2}(kR)$ is a spherical Bessel function, one can obtain the reaction amplitude in the following form:

$$M_{fi}=-\frac{iN_{1s}}{2pk^2}(4\pi)^2\sum_{\lambda\ell m}\sqrt{4\pi(2\lambda+1)}i^{\ell+\lambda}e^{i\delta_\lambda}Y^*_{\ell m}(\vec{n}_k)$$
$$\times\int dR\chi_{p\lambda}(R)\chi_{\nu\Lambda}(R)j_\ell(kR)e^{-\beta R}$$
$$\times\int d\Omega_R(\vec{k}\vec{D})Y^*_{\Lambda\mu}(\vec{n}_R)Y_{\lambda 0}(\vec{n}_R)Y_{\ell m}(\vec{n}_R). \tag{41}$$

Formula (41) is written in the laboratory system with a z axis directed along the vector $\vec{p}$, the spherical harmonics being taken according to

$$P_\lambda(\vec{n}_p\vec{n}_R)=\sqrt{\frac{4\pi}{2\lambda+1}}Y_{\lambda 0}(\vec{n}_R). \tag{42}$$

The factor $(\vec{k}\vec{D})$ enters the angular integral because the dipole moment $\vec{D}$ depends on $\vec{n}_R$ via the Coulomb parabolic functions φ_{nj}, defined in the molecular coordinate system with the z axis directed along the internuclear axis, defined by the vector $\vec{n}_R(\theta,\phi)$. When calculating the angular integral in Eq. (41) one should express $\vec{D}$ via spherical basis vectors in the laboratory system [18]:

$$\vec{D}=\sum_{i=x,y,z}D_i\vec{e'}_i=\sum_{\alpha=0,\pm1}d_\alpha\vec{e^\alpha}. \tag{43}$$

Here $\vec{e'}_i$ are Cartesian basis vectors of the molecular coordinate system, $\vec{e'}_z=\vec{n}_R$, while $\vec{e^\alpha}$ are spherical basis vectors of the laboratory system, $\vec{e}^0=\vec{n}_p$,

$$d_0 = -D_x \sin\theta + D_z \cos\theta, \tag{44}$$

$$d_{\pm 1} = \mp \frac{e^{\pm i\phi}}{\sqrt{2}} [D_x(1+\cos\theta) + D_z \sin\theta]. \tag{45}$$

Cartesian components of the dipole moment D_x, D_y, D_z are calculated as matrix elements of the radius vector $\vec{r} \equiv (x,y,z)$ in the molecular coordinate system for a transition between given parabolic states (n_1,n_2,m) and (n_1',n_2',m'), e.g.,

$$D_x = \int d\vec{r} \varphi_{n'j'} x \varphi_{nj}. \tag{46}$$

Their values can be found by formulas given in Ref. [19]. The calculations showed that the matrix elements D_x, D_z are real, and $D_y = iD_x$ is purely imaginary. This property was used when deriving Eqs. (44) and (45).

III. REACTION CROSS SECTION

The differential cross section of the reaction is given by a well-known formula [17]

$$d\sigma = \frac{2\pi}{v} |M_{fi}|^2 \delta(\mathcal{E}_f - \mathcal{E}_i) \frac{d\vec{k}}{(2\pi)^3} = \frac{m_e k}{v} |M_{fi}|^2 \frac{d\Omega_k}{(2\pi)^2}. \tag{47}$$

The energy $\mathcal{E}$ of the system comprising atom and mesic atom is related to the energy E of the $(p\mu p)$ complex and electron energy ε_e as follows:

$$\mathcal{E}_i = E_1 - I_e, \quad \mathcal{E}_f = E_2 + \varepsilon_e, \tag{48}$$

where $-I_e$ and ε_e are the energies of the bound and free electron, respectively.

Since the molecular state is degenerate over the angular-momentum projection, the cross section for the transition to the state ν, Λ is given by a formula

$$d\sigma = \frac{m_e k}{4\pi^2 v} \sum_{\mu=-\Lambda}^{\Lambda} |M_{fi}^{\mu}|^2 d\Omega_k. \tag{49}$$

Electron momentum k is obtained from the energy conservation $\mathcal{E}_i = \mathcal{E}_f$ and the energy scheme (Fig. 2):

$$\Delta\varepsilon_e = \varepsilon_e + I_e = E_1 - E_2 = \varepsilon_n + \varepsilon - \varepsilon_{n'} - \varepsilon_{\nu\Lambda}. \tag{50}$$

Taking into account that $\varepsilon_{\nu\Lambda} < 0$ and introducing $\Delta_{nn'} = \varepsilon_n - \varepsilon_{n'}$ one has

$$\varepsilon_e = \Delta_{nn'} - I_e + \varepsilon + |\varepsilon_{\nu\Lambda}|, \quad k = \sqrt{2m_e\varepsilon_e}. \tag{51}$$

For slow collisions ($\varepsilon < 1$ eV) and transitions to the molecular states with $\nu \gg 1$ (such transitions are the most probable) the electron momentum k is practically independent of ε and $\varepsilon_{\nu\Lambda}$ so one may write

$$\varepsilon_e \simeq \Delta_{nn'} - I_e. \tag{52}$$

The angular integration over $\Omega(\vec{n}_R)$ and $\Omega(\vec{n}_k)$ is performed with standard technique [18]. Omitting the cumbersome intermediate formulas, one can obtain the final expression for the rate of the reaction in question reduced to the liquid-hydrogen density as

$$\lambda_{\nu\Lambda} = \tau_e^{-1} (N_0 a_e^3) \beta^5 \frac{32\omega}{v v_e} (2\Lambda+1) \times \sum_{\mu=-\Lambda}^{\Lambda} \sum_{\ell=0}^{\infty} \left| \sum_{\lambda=0}^{\infty} (2\lambda+1) i^{\lambda} e^{i\delta_\lambda} \times \sum_{\ell'=\ell\pm1} a_{\ell'} G_{\lambda\ell'}^{\Lambda\mu} I_{\lambda\ell'}^{\nu\Lambda}(k,p) \right|^2, \tag{53}$$

where $\tau_e^{-1} = 4.134 \times 10^{16}\ \mathrm{s}^{-1}$, $N_0 a_e^3 = 0.63 \times 10^{-2}$, $N_0 = 4.25 \times 10^{22}\ \mathrm{cm}^{-3}$ is the liquid-hydrogen density; v is the relative velocity of the nuclei in the initial state, v_e is the velocity of the ejected electron, ω is the frequency of the molecular oscillations. The phase shift δ_λ is determined by the formula [17]

$$\delta_\lambda = \lim_{R\to\infty} \left(\int_a^R P(x)dx - \int_{a_1}^R dx \sqrt{p^2 - (\lambda+1/2)^2/x^2} \right), \tag{54}$$

where a_1 is the left turning point for the classical motion of the particle with momentum p and angular momentum λ without the field, the radial momentum $P(x)$ being defined by Eq. (39).

The coefficients $a_{\ell'}$ have nonzero values only for $\ell' = \ell \pm 1$,

$$a_{\ell-1} = \sqrt{\frac{\ell(2\ell-1)}{2\ell+1}}, \quad a_{\ell+1} = \sqrt{\frac{(\ell+1)(2\ell+3)}{2\ell+1}}. \tag{55}$$

The factors $G_{\lambda\ell'}^{\Lambda\mu}$ appear from angular integrations and can be written as follows:

$$G_{\lambda\ell'}^{\Lambda\mu} = \sum_{\sigma} C_{\ell' q 1\sigma}^{\ell_1 m_1} (h_\sigma D_x + f_\sigma D_z),$$

$$m_1 = q + \sigma, \quad q = M - \mu, \quad M = -\sigma, \quad \sigma = 0, \pm 1, \tag{56}$$

where $C_{a\alpha b\beta}^{d\delta}$ are Clebsh-Gordan coefficients,

$$h_{\pm1} = \pm \sum_L B_{L,\mp1} I(L), \quad f_{\pm1} = \frac{2}{3} B_{1,\mp1},$$

$$h_0 = \sum_L B_{L0} I_2(L), \quad f_0 = -\frac{2}{3} B_{10},$$

$$B_{LM} = \sum_{L'} C_{\lambda\ell'}^{L'} C_{L'\Lambda}^{L} C_{\lambda 0 \ell' q}^{L' q} C_{L' q \Lambda\mu}^{LM}, \quad C_{ab}^{d} \equiv C_{a0b0}^{d0},$$

$$I(L) = \alpha_L [I_1(L) - \beta_L I_2(L)],$$

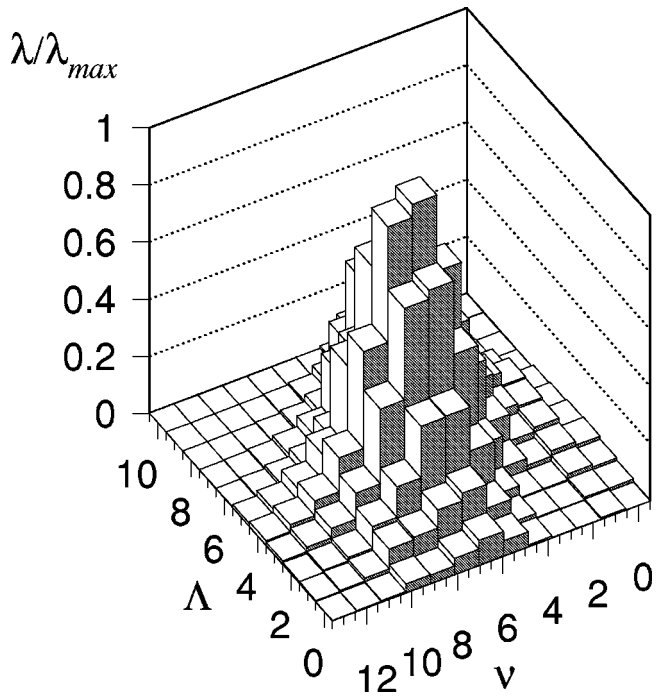

FIG. 3. Typical dependence of $\lambda_{\nu\Lambda}$ on ν and Λ for (022) $\rightarrow$(021) transition at the collision energy of $\epsilon=0.04$ eV.

$$\alpha_L=[2L(L+1)(2L+1)]^{1/2}, \quad \beta_L=L(L+1),$$

$$I_1(L)=\int_{-1}^{+1}\frac{x\,dx}{\sqrt{1-x^2}}P_L(x),$$

$$I_2(L)=\int_{-1}^{+1}dx\sqrt{1-x^2}P_L(x),$$

$I_1(L)$ being zero for even L, $I_2(L)$ being zero for odd L. At last, $I^{\nu\Lambda}_{\lambda\ell'}$ are the radial integrals of the following type:

$$I^{\nu\Lambda}_{\lambda\ell'}(k,p)=\int_a^b dR e^{-\beta R}\tilde{\chi}_{P\lambda}(R)\,\tilde{\chi}_{\nu\Lambda}(R)j_{\ell'}(kR), \quad (57)$$

$$\tilde{\chi}_{P\lambda}(R)=\frac{1}{\sqrt{P(R)}}\cos\left(\int_a^R P(x)dx-\frac{\pi}{4}\right), \quad (58)$$

$$\tilde{\chi}_{\nu\Lambda}(R)=\frac{1}{\sqrt{P'(R)}}\cos\left(\int_{a'}^R P'(x)dx-\frac{\pi}{4}\right). \quad (59)$$

All variables are taken here in mesic atom units ($\hbar=e=m_0=1$).

IV. RESULTS AND DISCUSSIONS

A typical dependence of the $\lambda_{\nu\Lambda}$ on ν and Λ is shown in Fig. 3 for (022)$\rightarrow$(021) transition at the collision energy $\epsilon=0.04$ eV. A distinct maximum in both variables is clearly seen.

The reaction rates are shown in Fig. 4 together with the results of the semiclassical calculations [6] of the Auger-minus process, in which the nuclei in the final state are bound. It is seen that the results of both papers are in a reasonable agreement. Our calculations of the rates of the Coulomb deexcitation for excited muonic hydrogen result in $\lambda^{CD}=1.4\times10^{11}$ s^{-1} and 5.1×10^{10} s^{-1} for $n=5$ and 4, respectively, for a collision energy of 0.04 eV. So, at least for $n=5$ the molecule formation rates are much larger than the Coulomb deexcitation. A considerable drop of the formation rates for $\epsilon>0.1$ eV is naturally explained by the decrease of the probability for the nuclei in the final state to be bound, when the collision energy increases.

The bound state in which the system appeared could decay either via predissociation (Coulomb deexcitation)

$$(p\mu p)_{n'}\rightarrow(p\mu)_{n''}+p, \quad n''=n'-1 \quad (60)$$

or via the external Auger ionization

$$(p\mu p)_{n'}+\mathrm{H}_2\rightarrow(p\mu)_{n''}+\mathrm{H}_2^+ +e. \quad (61)$$

If the binding energy of the quasimolecule $(p\mu p)_{n'}$ is smaller than the thermal energy, the molecule can decay in quasielastic collisions with target molecules,

$$(p\mu p)_{n'}+\mathrm{H}_2\rightarrow(p\mu)_{n'}+p+\mathrm{H}_2. \quad (62)$$

In this case the mesic atoms do not accelerate. As a rule, in the states where the molecule formation rate has a maximum, the average binding energy of the quasimolecule amounts to about several tenths electron volt, so the quasielastic mechanism has low probability.

If the system in the final state represents a quasimolecule

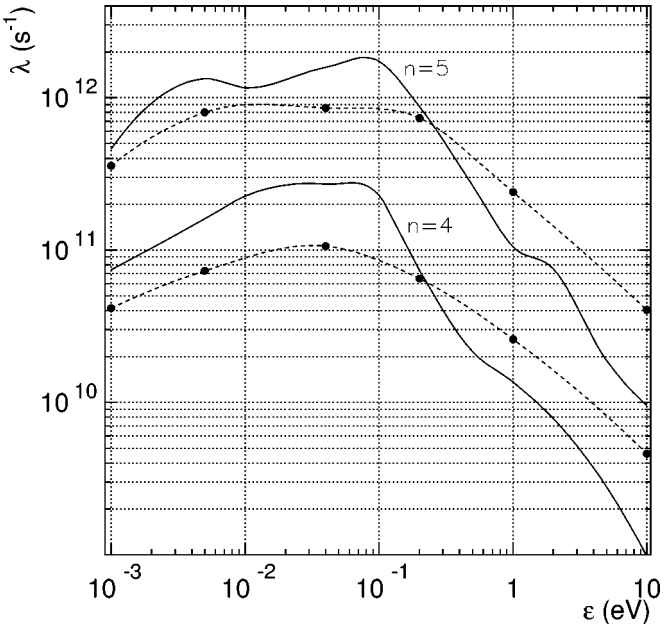

FIG. 4. Energy dependence of the mesic molecule formation rates via Auger collisions for $n=4$ and 5. The solid lines show the rates of the "Auger-minus" process calculated in Ref. [7]. The circles are the results of the present paper, the dashed lines join them just to guide the eye.

comprising the excited mesic atom and hydrogen molecular ion $(p\mu)_{n'}H_2^+$, or $(p\mu p)_{n'}H$, the internal Auger process is also possible. In this case, as it was shown by Menshikov [11], mesic atom acquires some part of the muon energy, accelerating up to energies about 1 eV.

The predissociation process Eq. (60) leads to a two-particle decay, in which the deexcitation energy transforms into the kinetic energy of the fragments because of the Coulomb repulsion of nuclei.

Summing up, one may state that the data obtained corroborate the conclusion made in Ref. [6] about the possibility of the considerable acceleration of the mesic atom during the Auger transition. As a result, the $p\mu$ atoms in the final state acquire the kinetic energy $\epsilon_{p\mu} \sim 1/2n'(n'-1)^2$, which is a half of the transition energy for the transition $n' \rightarrow n'-1$. For $n'=4$, when the Auger process starts from the state $n=5$, $\epsilon_{p\mu} \sim 70$ eV. This effect should be taken into account in the cascade calculations of the mesic atom kinetics.

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