Propagation of optical pulses in a resonantly absorbing medium: Observation of negative velocity in Rb vapor

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Propagation of optical pulses in a resonantly absorbing medium is studied. Propagation time of nanosecond pulses was measured for the Rb D_1 transition. At the center of two absorption lines, delay of the pulse peak which is about ten times as large as the pulse width was observed, where zero delay is defined for the propagation with the light velocity in vacuum. On the other hand, at the peak of an absorption line, negative delay was observed for large absorption, where the advance time is as large as 25% of the pulse width. Simulation including the effect of absorption and phase shift reproduced well the experimental results.

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I. INTRODUCTION

Since Sommerfeld and Brillouin [1,2] treated the problem of propagation of light in dispersive media early in the 20th century, the study of the group velocity in regions of anomalous dispersion has attracted a lot of interest [3–11]. In recent years, various intresting phenomena [12–20] with regard to the propagation of light pulses, such as slowing the velocity down to the velocity of sound [12], stopping the light [13–16], and realizing negative velocity [17,18], have been reported. These phenomena are made possible by using an artificial processing, the electromagnetically induced transparency (EIT) [21,22], where narrow transparent window and steep inclination of refractive index are produced in an absorbing region. The EIT resulting from the quantum interference effect is a nonlinear phenomenon. Confining our attention to the light propagation for the produced absorbing structure, however, the propagation is a linear phenomenon. Even in natural resonantly absorbing media propagation of light pulses is not necessarily understood completely.

Garret and McCumber [3] and Crisp [4] showed theoretically that negative pulse velocity can be possible when the incident frequency lies in the anomalous dispersion region, but it never violates the causality. Experimental obseration of the negative pulse velocity was reported by Chu and Wong [6] for a thin sample of GaP:N. Grischkowsky [5] observed slow pulse velocities in the Rb vapor in agreement with the conventional group velocity. Superluminal pulse propagation without significant distortion or broadening in amplifying media near gain resonance were theoretically suggested by Chiao *et al.* [8,7]. Talukder *et al.* [11] verified experimentally the differences from the conventional group velocity in dye solutions which were theoretically proposed by Tanaka *et al.* [9] and by Peatross *et al.* [10]. Although many theoretical and experimental studies on the light propagation have been reported, the propagation of superluminal pulses in natural atomic transitions have not been observed so far.

The amplitude of a wave packet propagating to the z direction is expressed by a Fourier integral

$$u(z,t)=\int_{-\infty}^{\infty}A(\omega)\exp\left[i\omega\left(\frac{n(\omega)}{c}z-t\right)\right]d\omega, \tag{1}$$

where $n(\omega)=kc/\omega$ is the refractive index and $A(\omega)$ is the spectral density of the wave packet. We assume that $\Delta\omega \ll \omega_c$, where $\Delta\omega$ and ω_c are the spectral width and center frequency of the wave packet, respectively. If change of $n(\omega)$ is linear around ω_c then the following approximation is valid:

$$n(\omega)\omega \simeq n(\omega_c)\omega_c+\left(\frac{d(n\omega)}{d\omega}\right)_{\omega_c}(\omega-\omega_c). \tag{2}$$

Applying Eq. (2) to Eq. (1), the amplitude is given by

$$u(z,t)=u_0(z,t)\exp\left[i\omega_c\left(\frac{n(\omega_c)}{c}z-t\right)\right], \tag{3}$$

where $u_0(z,t)$ represents the envelope of the wave packet and is expressed as

$$u_0(z,t)=\int_{-\infty}^{\infty}A(\omega)\exp\left[i\frac{\omega-\omega_c}{c}\left\{\left(\frac{d(n\omega)}{d\omega}\right)_{\omega_c}z-ct\right\}\right]d\omega. \tag{4}$$

According to Eq. (4) the velocity of the wave packet is given by

$$v_g=\frac{c}{\dfrac{d(n\omega)}{d\omega}}=\frac{c}{n+\omega\dfrac{dn}{d\omega}}=\frac{d\omega}{dk}. \tag{5}$$

The velocity v_g in Eq. (5) is the well-known group velocity. Steep inclination of refractive index in resonantly absorbing regions can give a large change to the value of v_g through the contribution of the term $\omega(dn/d\omega)$, even if the value of n is not so changed. It should be noted that the meaning of v_g is broken down when the approximation of Eq. (2) is not valid.

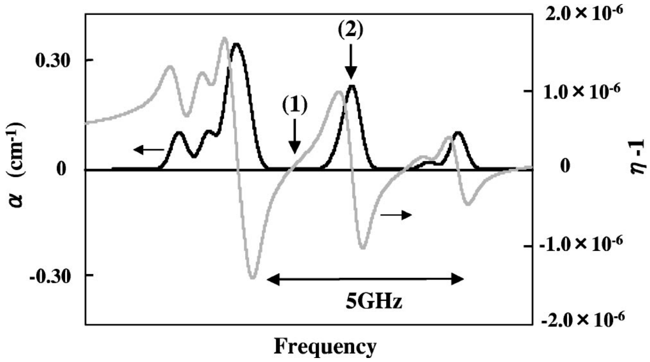

FIG. 1. Absorption coefficient α and change $\eta-1$ of the refractive index of the Rb D_1 transition for the atomic density of 6.5 $\times 10^{10}$ cm^{-3} (45 °C). Arrows (1) and (2) indicate the frequency of the probe pulse.

Absorption coefficient and refractive index of a Rb cell at 45 °C is shown in Fig. 1. The refractive index $\eta(\omega)$ is obtained from the observed absorption coefficient $\alpha(\omega)$ by applying the Kramers-Kronig relations [23]. The solid line in Fig. 2 shows the expected propagation time of pulses which takes to pass through the Rb vapor of 1 cm thickness, where the propagation time is calculated from the refractive index in Fig. 1 and the group velocity of Eq. (5). The broken line indicates the propagation time for *c*, light velocity in vacuum. The result of the simulation in Fig. 2 shows that the group velocity can exceed the light velocity in vacuum ($v_g > c$) and can be negative ($v_g<0$) where the propagation time is negative. Delay of propagation time is expected in the normal dispersion region and negative delay is expected in the anomalous dispersion region. In the present work we report the negative velocity for the *D* transition of an alkali-metal atom.

We observed propagation of optical pulses in a resonantly absorbing medium. Propagation time of nanosecond pulses was measured for the D_1 transition of Rb vapor. Delay at the

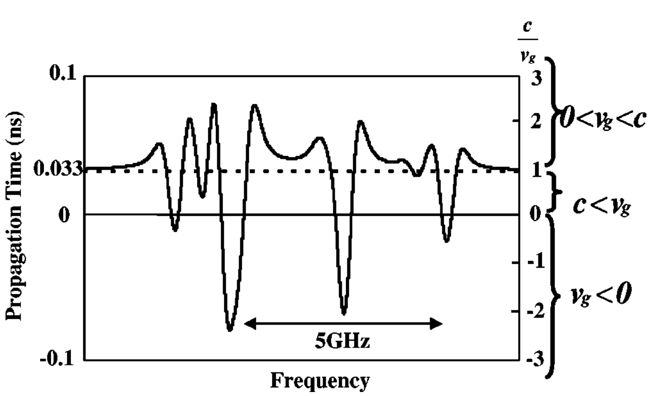

FIG. 2. Simulation of the propagation time for a 1 cm Rb cell of Fig. 1 (45 °C). The broken line indicates the propagation time (33 ps) for *c*, light velocity in vacuum.

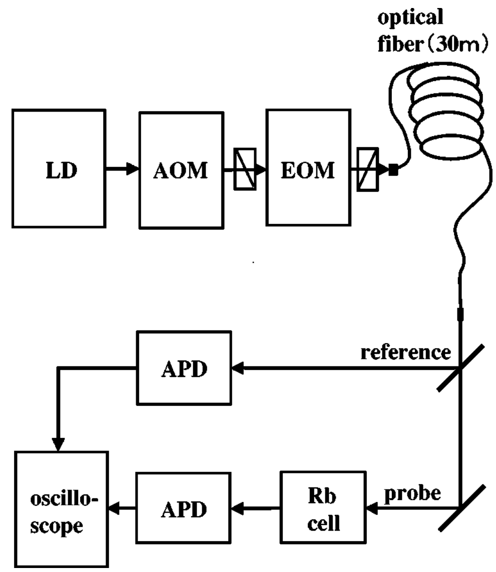

FIG. 3. Experimental setup. LD, laser diode; AOM, acousto-optic modulator; EOM, electro-optic modulator; APD, avalanche photo diode.

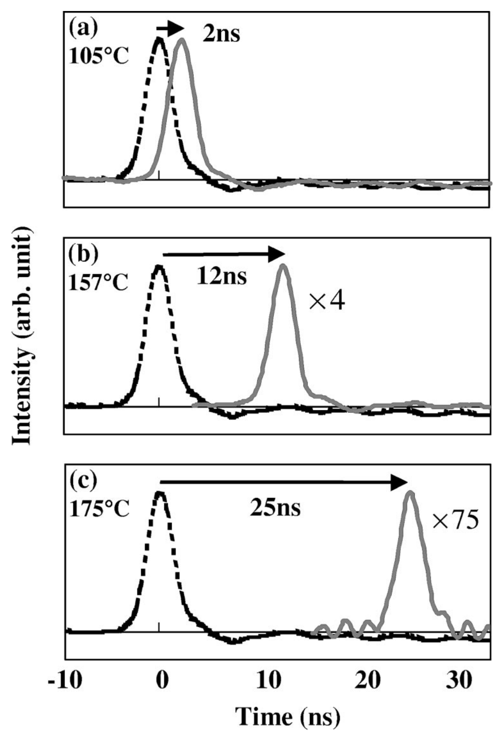

FIG. 4. Delay of the probe pulse (solid lines) observed at the center of two absorption lines indicated by arrow (1) in Fig. 1. The atomic density is (a) 6.5×10^{12} cm^{-3} (105 °C), (b) 9.8 $\times 10^{13}$ cm^{-3} (157 °C), and (c) 2.3×10^{14} cm^{-3} (175 °C). The broken lines show zero delay of the probe pulse observed at an off-resonant frequency, where the laser frequency is detuned by 20 GHz. The intensity of the delayed pulses in (b) and (c) is enlarged by 4 and 75 times as large as that for zero delay, respectively.

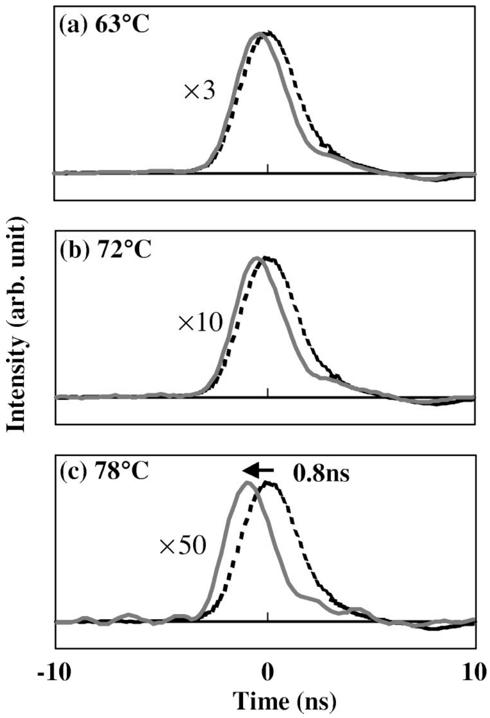

FIG. 5. Negative delay of the probe pulse (solid lines) observed at the peak of an absorption line indicated by arrow (2) in Fig. 1. The atomic density is (a) 3.0×10^{11} cm^{-3} (63 °C), (b) 6.0×10^{11} cm^{-3} (72 °C), (c) 9.3×10^{11} cm^{-3} (78 °C). The broken lines show zero delay of the probe pulse observed at an off-resonant frequency where the laser frequency is detuned by 20 GHz. The intensity of the solid lines in (a), (b), and (c) is enlarged by 3, 10, and 50 times as large as that for zero delay, respectively.

center of two absorption lines and negative delay at the peak of an absorption line for large absorption are observed without using EIT. The observed negative delay time is as large as 25% of the pulse width, while the previously observed ratio by using EIT is less than 2% [17]. The experimental results are simulated by taking the effect of absorption and phase shift in consideration.

II. EXPERIMENT AND RESULT

We used Rb vapor as a resonantly absorbing medium. The experimental setup is shown in Fig. 3. Optical pulses, whose pulse width is 3.2 ns, are generated by an electro-optic modulator (EOM) from the continuous output of a laser diode tuned to the Rb D_1 transition 794.8 nm. An acousto-optic modulator, which produces 50 ns pulses is used to improve the extinction ratio of the laser light and to prevent the hyperfine pumping. The pulses are divided into probe and reference pulses by a beam splitter. The arrival time of the reference pulse is detected by an avalanche photo diode (APD). The delay of the arrival time of the probe pulse after passing through a Rb cell whose thickness is 1 cm is detected by another APD. The density of the Rb vapor is varied by changing the temperature of the cell. An optical fiber is used to give a time delay ($\sim$150 ns) and to avoid the effect of electronic noise from the EOM at the arrival time of the pulses to the Rb cell.

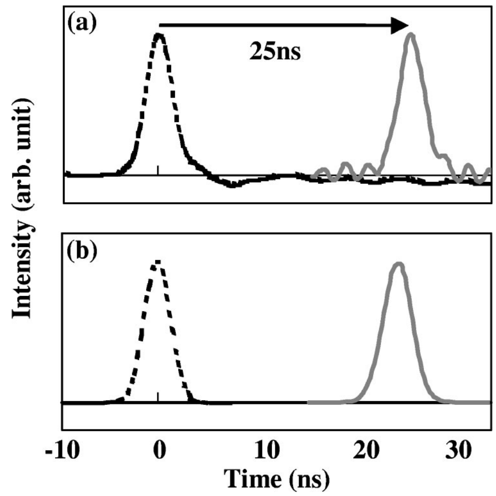

FIG. 6. (a) Observed delay of the probe pulse (solid line) in the normal dispersion region for the atomic density of 2.3×10^{14} cm^{-3} [175 °C, Fig. 4(c)]. The broken line is zero delay for off-resonance. (b) Simulation of (a) calculated from Eq. (9).

The solid lines in Fig. 4 show delay of the probe pulse,

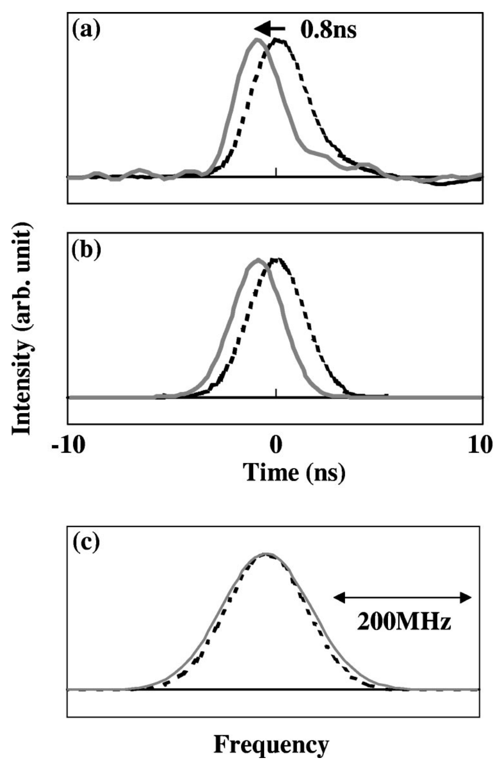

FIG. 7. (a) Observed negative delay of the probe pulse (solid line) in the anomalous dispersion region for the atomic density of 9.3×10^{11} cm^{-3} [78 °C, Fig. 5(c)]. The broken line is zero delay for off-resonance. (b) Simulation of (a) calculated from Eq. (9). (c) Simulation of incident (broken line) and transmitted (solid line) spectra.

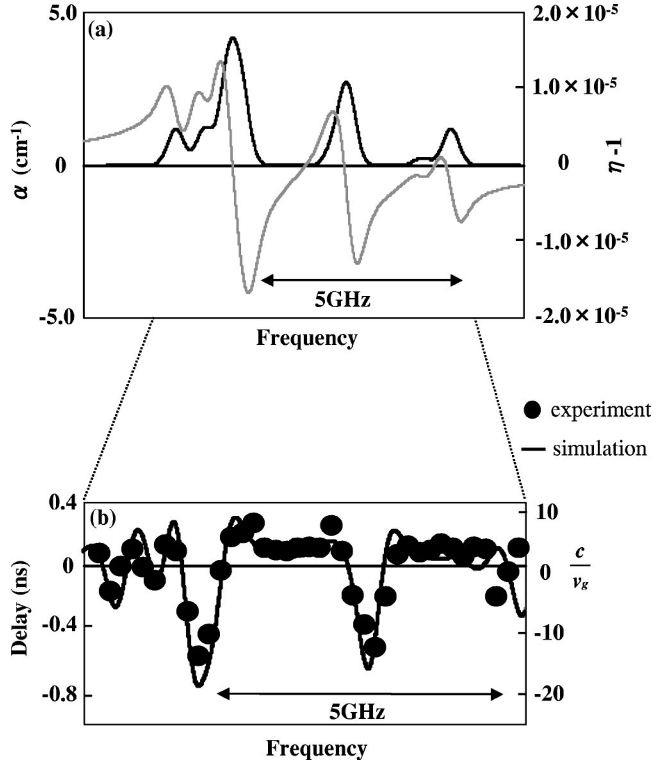

FIG. 8. (a) Absorption coefficient α and change $\eta-1$ of the refractive index for the atomic density of 4.5×10^{11} cm^{-3} (65 °C). (b) Frequency dependence of the delay time of the probe pulse (solid circles). The solid line is a theoretical curve calculated by applying $d\omega/dk$ of Eq. (5).

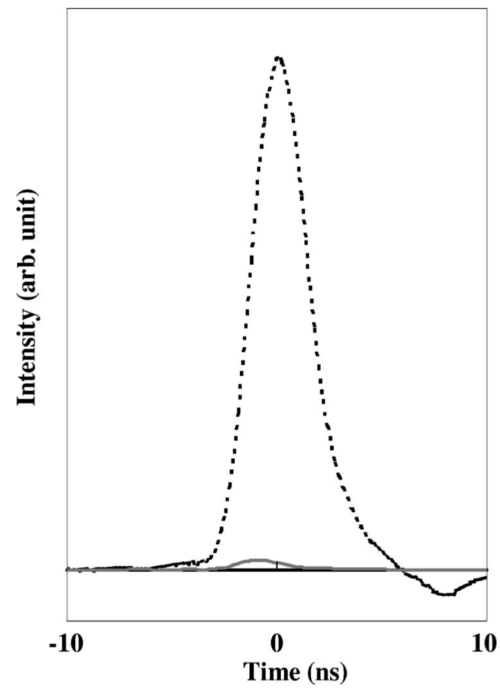

FIG. 9. Observed intensities of the probe pulse for the case of the negative delay in Fig. 5(c) drawn on the same vertical scale. The solid and broken lines show the transmitted pulse at the absorption peak and the zero-delay pulse at an off-resonant frequency. The leading edge of the transmitted pulse never precedes that of the incident pulse. The negative intensity of the zero-delay pulse is due to the back shooting in the detector circuit.

where the frequency of the probe pulse is tuned to the center of two absorption lines as shown by arrow (1) in Fig. 1. The broken lines show zero delay of the probe pulse observed at an off-resonant frequency, where the laser frequency is detuned by 20 GHz and the probe pulse propagates with the light velocity in vacuum. As the atomic density is increased, the delay time becomes larger. In case of Fig. 4(c) the delay time is 25 ns, and this means that the propagation velocity of the probe pulse is reduced to about 10^{-3} of c.

The solid lines in Fig. 5 show negative delay of the probe pulse, where the frequency of the probe pulse is tuned to the peak of an absorption line as shown by arrow (2) in Fig. 1. The broken lines show zero delay of the probe pulse observed at an off-resonant frequency. As the atomic density is increased, the advance time becomes larger. In case of Fig. 5(c) the advance time is 0.8 ns, and this means that the propagation velocity of the probe pulse is negative and its value is about -4×10^{-2} of c.

The observed advance is quite clear, and the advance time corresponds to 25% of the pulse width. This is in contrast with the previously observed advance ratio which was 1.7% of the pulse width [17], although the advance time was larger than ours by using EIT.

The observed delay and advance times were independent of the incident probe intensity and the observed propagation is a linear phenomenon without any nonlinear effect. It is noted that the intensity of the transmitted probe has large reduction due to the strong absorption.

III. SIMULATION AND DISCUSSION

We give a simulation of the pulse propagation in a resonantly absorbing medium. The effect of absorption, which is not included in Eq. (3) has to be included and the approximation of Eq. (2) cannot be applied for general cases. We consider a pulse which incidents into an absorbing medium at $z=0$ from the vacuum. The thickness of the medium is z_0. The amplitude $u_{in}(z,t)$ of the incident pulse is given by Eq. (1) with $n(\omega)=1$:

$$u_{in}(z,t)=\int_{-\infty}^{\infty}A(\omega)\exp\left[i\omega\left(\frac{z}{c}-t\right)\right]d\omega. \tag{6}$$

In the absorbing medium the complex refractive index $n(\omega)$ is given by

$$n(\omega)=\eta(\omega)+i\kappa(\omega), \tag{7}$$

$$\eta(\omega)=1+\Delta\eta(\omega).$$

The refractive index $\eta(\omega)$ and the extinction coefficient $\kappa(\omega)$ contribute to phase shift and absorption, respectively. Considering the effect of phase shift and absorption for each Fourier component, the amplitude $u_{out}(z_0,t)$ of the transmitted pulse at z_0 is given by

$$u_{out}(z_0,t)=\int_{-\infty}^{\infty}A(\omega)\exp\left[i\omega\left\{\frac{n(\omega)}{c}z_0-t\right\}\right]d\omega$$
$$=\int_{-\infty}^{\infty}A(\omega)\exp\left[-\frac{\omega\kappa(\omega)}{c}z_0\right]\exp\left[i\frac{\omega\Delta\eta(\omega)}{c}z_0\right]\exp\left[i\omega\left(\frac{z_0}{c}-t\right)\right]d\omega$$
$$=f(z_0,t)\exp\left[i\omega_c\left(\frac{z_0}{c}-t\right)\right], \tag{8}$$

$$f(z_0,t)=\int_{-\infty}^{\infty}A(\omega)\exp\left[-\frac{\omega\kappa(\omega)}{c}z_0\right]\exp\left[i\left\{\frac{\omega-\omega_c+\omega\Delta\eta(\omega)}{c}z_0-(\omega-\omega_c)t\right\}\right]d\omega. \tag{9}$$

The intensity of the transmitted pulse can be obtained from $|u_{out}(z_0,t)|^2=|f(z_0,t)|^2$.

Simulation for the delay of the probe pulse in Fig. 4(c) is shown in Fig. 6(b), which is calculated from Eq. (9) by assuming a Gaussian spectrum $A(\omega)$. Figure 6(a) is the observed delay. Simulation for the negative delay of the probe pulse in Fig. 5(c) is shown in Fig. 7(b). Figure 7(a) shows the observed probe pulse. Simulation of spectra is also shown in Fig. 7(c), where the solid and broken lines are normalized spectra for the transmitted and incident pulses, respectively. The simulations both for the delay and negative delay reproduce well the experimental results. A slight narrowing of the pulse width observed in the negative-delay experiment is also reproduced by the simulation. This result is well understood by the broadening of the transmitted spectrum in Fig. 7(c).

Frequency dependence of the delay time of the probe pulse is shown in Fig. 8. The solid circles in Fig. 8(b) are the observed delay time and the solid line is the theoretical curve calculated by applying $d\omega/dk$ of Eq. (5). Agreement between the experimental result and the calculation is good.

We observed delay and negative delay for the D_1 transition of the Rb vapor, and the observed delay and negative delay in our experiment are explained well by the conventional group velocity $d\omega/dk$. However, the intensity of the transmitted pulse is reduced by the strong absorption, and the observed velocity is not the velocity of energy flow. The observed negative delay does not conflict with the casuality or the theory of relativity. The observed intensities of the probe pulse for the case of the negative delay in Fig. 5(c) drawn on the same vertical scale are shown in Fig. 9. Since the intensity of the transmitted pulse is much smaller than the incident pulse, the leading edge of the former never precedes to that of the latter [19,20].

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