Quantum approach to electromagnetic energy transfer between two dielectric bodies

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(Received 27 June 2003; published 22 October 2003)

The problem of radiative heat transfer between two dielectric bodies is analyzed from the point of view of elementary quantum electrodynamics. The dielectric properties of the bodies are assumed to be linear, but dispersion and losses are allowed. Quantization of the electromagnetic field in inhomogeneous, dispersive, and lossy dielectrics is performed with the help of the Huttner-Barnett procedure. The electromagnetic energy flux is expressed through the expectation value of the Poynting vector. In order to compute the Poynting vector, two techniques suitable for nonequilibrium processes are employed: the Heisenberg equations of motion and the diagrammatic Keldysh procedure. They are shown to give identical final results. These quantum-mechanical calculations provide a solid basis for the further, mainly numerical, development of the theory of thermal scanning microscopy.

DOI: 10.1103/PhysRevA.68.043823 PACS number(s): 42.50.Nn, 44.40.+a, 11.10.Wx

I. INTRODUCTION

In view of the impressive progress recently made in experimental techniques for near-zone thermal microscopy [1–8], the problem of a careful theoretical description of the behavior of thermal electromagnetic fields in the vicinity of surfaces of metals, semiconductors, and dielectrics has acquired some importance. Indeed, in modern experiments with a thermal scanning microscope performed under UHV conditions [8] one can practically exclude any gas-mediated heat transfer between the analyzed body and the tip of the microscope, while keeping under control the transfer due to tunneling electrons. This enables one to concentrate on the radiative heat transfer, that is, on the transfer of electromagnetic energy between bodies with different temperatures. In the past, this problem has been investigated theoretically [9,10] by means of the classical theory of thermal fluctuations of the electromagnetic field [11]. Let us, however, note that the classical theory cannot really be consistent: it suffices to mention that the final formulas contain the Planck constant as an overall factor, as well as the Bose-Einstein function of temperature, which also involves the Planck constant. In Ref. [12] a successful attempt has been made to employ Agarwal's development [13,14] of quantum electrodynamics in the vicinity of dielectrics. That approach is based on the linear-response theory, but, though successful, lacks the explicit quantization of the field. What is more, it heavily relies on the fluctuation-dissipation theorem, which is taken for granted (i.e., no proof is provided). In this paper we go beyond these previous approaches and study the transfer of electromagnetic energy between two dielectric bodies on the grounds of elementary quantum electrodynamics. The dielectric function is then obtained explicitly from the theory of interacting fields.

We assume that the dielectric properties of the bodies are linear. This means that the coupling of matter fields to the electromagnetic field is bilinear in the field operators, so that the equations of motion are linear as well. Still, for the following reasons the solution is not easy to obtain: (i) the system is definitely out of equilibrium. Thus, well-known methods of field theory and condensed-matter theory, such as the Matsubara technique, do not work; we shall, therefore, employ both the Heisenberg equations of motion and the nonequilibrium Keldysh diagrammatic formalism; (ii) the problem of the electromagnetic field quantization in dispersive and lossy dielectrics is far from trivial. Only fairly recently several satisfactory methods to deal with such combined systems (dielectrics plus quantized field) have been proposed. Here, we shall use the Huttner-Barnett quantization scheme as developed in Ref. [15] and simplified in Refs. [16,17]; (iii) even in cases of simplest geometries, the electromagnetic propagators are given by rather complicated Fourier or Fourier-Bessel integrals with a lot of singularities in the integrands. This causes difficulties for their asymptotic evaluation, which is required for developing a clear physical picture of the heat transfer.

We stress that the theory of near-field heat transfer, which underlies thermal scanning microscopy, forces one to take into account dispersion and losses in metallic and dielectric media. Hence, we are led to the interesting and difficult problem of the electromagnetic field quantization in inhomogeneous dispersive media, which has a short but interesting history. So far, the only way to deal with this problem is to neglect (either from the very beginning, or at a certain intermediate stage of the analysis) the grainlike nature of the dielectric or metallic material, and to restrict oneself to the so-called macroscopic electrodynamical description, in which the properties of the media are represented in terms of averaged quantities such as dielectric constants and magnetic permeabilities. This, however, does not preclude the possibility of introducing useful phenomenological models such as the Hopfield model [18]. In the present paper we exploit such a macroscopic electrodynamical description.

To our knowledge, the problem of the quantization of the electromagnetic field in a *homogeneous* but dispersive dielectric has been considered first by Watson and Jauch [19], in a paper which followed their previous works [20,21] on the field quantization in moving dielectrics. These authors quantized the fields under the somewhat restrictive assumption of a one-to-one correspondence between the angular frequency and the wave vector in the medium. A similar approach has been proposed more recently by Drummond [22],

who was also able to take into account nonlinear effects. For the case of *inhomogeneous nondispersive* dielectrics an important theoretical breakthrough has been achieved in Ref. [23]. The authors of that work have canonically quantized the electromagnetic field in a space half filled with the dielectric material. This has been done in such a way that the electromagnetic mode functions included the boundary conditions from the very beginning. These mode functions have been shown in Ref. [23] to be orthogonal; their completeness has then been proven in Ref. [24]. An outline of a general scheme of the electromagnetic field quantization and quantum optics in inhomogeneous nondispersive dielectrics can be found in Ref. [25]. The canonical quantization procedure developed by Carniglia and Mandel [23] has served as a guideline in several further studies of the behavior of quantum electromagnetic fields in more complicated geometrical arrangements, see, e.g., Refs. [26,27], although the tough problem of the completeness of the required mode functions has not been considered.

A new approach to the quantum electrodynamics in the presence of metals and dielectrics has been initiated by Agarwal in a remarkable series of papers [13,14,28–32]. This author was able to include both inhomogeneity and dispersion of media from the very beginning. His approach is based on the properties of electromagnetic correlation functions (Green's functions) of various types. An impressive amount of important features of radiation and of atom-radiation interactions could be calculated with these methods in a fairly elegant manner. Nonetheless, the approach advocated by Agarwal still lacked the explicit quantization of the electromagnetic field. Besides, some important quantum-statistical theorems, such as the fluctuation-dissipation theorem, have been postulated but not derived. This fact stimulated further activity on the subject of quantum electrodynamics in the presence of dispersive and inhomogeneous media and led to considerable theoretical achievements. In particular, in the works of the Jena group [33–36], which heavily rely on the properties of classical retarded Green's functions, the explicit quantization is achieved through the introduction of stochastic currents into the Maxwell equations, while the canonical formalism follows from an ingenious construction of a special Hamiltonian. An elegant equivalent approach [37] has been developed by Tip [38], who used auxiliary variables for obtaining a canonical theory even in the presence of losses. Another quantization scheme in which the (classical) Green's functions play a decisive role is the input-output formalism developed by Savasta and co-workers [39]. A quantization formalism starting from the Maxwell equations with stochastic currents has also been worked out in Refs. [40,41], and the important case of amplifying media could be included [42], but the procedure in these works is quite different from that in Ref. [33]. Finally, we mention that an intriguing technical novelty has been introduced by Bechler [17], who was the first to quantize the field in dielectric media in terms of path integrals, rather than using the canonical quantization scheme. The advantages of path integrals include, but are not reduced to, the trivialization of all calculations on the initial stage. Further, relevant developments are contained in Refs. [43–48,50,51].

In this paper, we resort to the approach worked out by Huttner and Barnett [15]. It is based on the observation, made already by Fano [49], that under rather general conditions the Hamiltonian describing long-wavelength excitations of matter is equivalent to that of a system of independent harmonic oscillators (from which the Hopfield model results). Moreover, it exploits the independent-oscillator model of the reservoir [52], which has a surprisingly wide region of applicability. The reservoir is required to deal with the losses in the dielectric. Technical definitions are given in the following section; here we summarize some arguments in favor of the Huttner-Barnett scheme (while keeping in mind that the choice of the quantization scheme is, to a large extent, a matter of personal understanding, taste, and convenience). First, this approach is very intuitive and simple, especially in its version developed in Refs. [16] and [17]; the dielectric is represented by its own quantum field, and the stochastic currents can be obtained by tracing out the variables representing the dielectric and the reservoir. Second, it is flexible: there is no problem to include magnetic properties and nonlinearities by changing slightly the coupling between the matter fields and the electromagnetic field. Third, within this framework one can, in principle, study excitations in the media (such as polaritons or plasmons) in a natural way, using directly available matter-field correlation functions. Fourth, it is aesthetically appealing: one can initially forget about the existence of the medium, and consider only the interacting quantum fields. The material appears later via some function which can be identified *a posteriori* as the dielectric function. In addition, the coupling between the electromagnetic and matter fields contains—in the case of inhomogeneous media—the coupling ''constant'' which depends on the point in space (and possibly also in time). This is very much in the spirit of an important fundamental development in quantum-field theory which has resulted in (almost) divergence-free QED [53].

The goal of the present paper is to derive fairly general working formulas for the radiative heat transfer between dielectric bodies. That is, we derive expressions from which that heat transfer can be calculated, provided that the Green's functions of the classical wave equation are known for the given geometry. There are only very few geometrical arrangements for which the electromagnetic Green's functions can be written down analytically: one of them is the system of two half-infinite dielectrics with planar surfaces and a vacuum ''layer'' between them, as considered in Refs. [9] and [10]. However, by now, numerical packages are available which allow one to find the required propagators numerically. We are going to study examples of heat transfer in particular geometrical setups in forthcoming publications.

We have employed two very different techniques to derive expressions for the radiative heat transfer. These are (i) an approach based on the Heisenberg equations of motion, and (ii) the Keldysh closed-time-path formalism with its diagrammatic expansion. As we show in this work, both procedures finally yield the same formula for the heat transfer, even though their underlying philosophy is radically different. The equations-of-motion method is much faster and more straightforward, but would become very awkward if we

were to include any nonlinearity. On the other hand, the diagrammatic Keldysh formulation requires the introduction of many different Green's functions, and the calculations are somewhat involved, but this formulation is also suitable for developments aiming beyond macroscopic electrodynamics.

Our paper is organized as follows: in Sec. II we write down the Lagrangian of our system and state the resulting Heisenberg equations of motion, obtained via the Schwinger action principle. In Sec. III we then use these equations of motion to derive the expression for the electromagnetic energy transfer. In Sec. IV we take up the second approach and provide the fundamental Green's functions needed for the application of the Schwinger-Keldysh formalism, and develop an elementary diagrammatic technique. In Sec. V we exploit this technique for computing the expectation value of the Poynting vector. We close the paper with several concluding remarks in Sec. VI.

II. THE MODEL: LAGRANGIAN FORMALISM AND EQUATIONS OF MOTION

For the sake of being at least a little bit unconventional, we start with the Schwinger quantum action principle [54,55], and follow the ideas of Huttner and Barnett [15] to quantize the electromagnetic field in lossy and dispersive dielectrics. Our system comprises the following subsystems: (i) two dielectric bodies, described (in the Hopfield model [18] as applied by Huttner and Barnett) with the help of the polarization vector fields $\mathbf{X}_j(\mathbf{r})$, together with their conjugate momenta $\mathbf{P}_j(\mathbf{r})$ and velocities $\mathbf{V}_j(\mathbf{r})$, where here and in the following the index $j=1,2$ refers to the bodies "1" and "2;" (ii) the electromagnetic field, described by the scalar potential $\Phi(\mathbf{r})$, the vector potential $\mathbf{A}(\mathbf{r})$, the electric field $\mathbf{E}(\mathbf{r})$, and the magnetic induction $\mathbf{B}(\mathbf{r})$, and (iii) the reservoir fields, which are coupled to the polarization fields only. The independent-oscillator model of the reservoir degrees of freedom is employed [52]. The reservoirs are described with the help of the following quantities: the "positions" $\mathbf{Y}_{j\nu}(\mathbf{r})$ of the reservoir oscillators, their momenta $\mathbf{Z}_{j\nu}(\mathbf{r})$, and their velocities $\mathbf{W}_{j\nu}(\mathbf{r})$. The variable ν carries the dimension of a frequency and enumerates the reservoir oscillators coupled to the polarization field at the point $\mathbf{r}$. Thus, there are two collections of reservoirs, corresponding to two dielectrics, and every polarization oscillator has its own reservoir.

Two remarks seem to be appropriate at this point. First, one can model quite complicated dielectric functions of the media using just one polarization field $\mathbf{X}_j(\mathbf{r})$ for each body, one (per medium) collection of the reservoir oscillators $\mathbf{Y}_{j\nu}(\mathbf{r})$, and one coupling function $\rho_{j\nu}$ (introduced below). In particular, one can model a dielectric function with more than one absorption line in this way [15]. On the other hand, it is also possible, and in many cases more convenient, to introduce—for each dielectric body—several polarization fields (and several different reservoir families) with many absorption lines. Here, we develop our theoretical approach employing just one polarization field for each dielectric. The generalization to many such fields and reservoirs does not present, however, any difficulty. Before we start explicit calculations, let us describe some of our notational conventions. We will use Latin indices $j,k,\ldots,$ to enumerate the dielectric bodies which exchange the thermal energy. The sums over $j,k,\ldots$ will be written explicitly. On the other hand, Greek indices $\alpha,\beta,\ldots,\lambda,\mu,\ldots$ are used to denote components of vector fields and Green's functions. The sums over those indices are not written explicitly; the summation convention is used instead.

A. Lagrangian and commutation relations

Our formulation of the action principle closely follows Schwinger [54,55]. Thus, we start with a Lagrangian which still contains both the velocities and the momenta. As is shown below, Schwinger's procedure allows one to write down the dynamical equations and the constraint equations on the same footing.

We assume that every product of noncommuting operators is symmetrized. The total free Lagrangian L_0 reads

$$L_0=L_D+L_{EM}+L_{RD}, \tag{1}$$

where the Lagrangian L_D describing the polarization field of the dielectric is given by

$$L_D=\sum_{j=1}^{2}\int d^3r\left[\mathbf{P}_j(\mathbf{r})\cdot\left(\frac{d\mathbf{X}_j(\mathbf{r})}{dt}-\mathbf{V}_j(\mathbf{r})\right)+\frac{1}{2}\mathcal{M}_j\mathbf{V}_j^2(\mathbf{r})-\frac{1}{2}\mathcal{M}_j\omega_{0j}^2\mathbf{X}_j^2(\mathbf{r})\right]. \tag{2}$$

Within the Hopfield model, the mass parameters $\mathcal{M}_j$ have to be expressed in terms of the static polarizabilities b_j [17,18]:

$$\mathcal{M}_j=(\epsilon_0\omega_{0j}^2 b_j)^{-1}. \tag{3}$$

The degrees of freedom of the electromagnetic field are described with the help of the Lagrangian L_{EM} [54]:

$$L_{EM}=\int d^3r\left[-\epsilon_0\mathbf{E}(\mathbf{r})\cdot\left(\frac{\partial\mathbf{A}(\mathbf{r})}{\partial t}+\boldsymbol{\nabla}\Phi(\mathbf{r})\right)-\frac{1}{\mu_0}\mathbf{B}(\mathbf{r})\cdot[\boldsymbol{\nabla}\times\mathbf{A}(\mathbf{r})]-\frac{1}{2}\epsilon_0\mathbf{E}^2(\mathbf{r})+\frac{1}{2\mu_0}\mathbf{B}^2(\mathbf{r})\right]. \tag{4}$$

Finally, the Lagrangian describing the coupling of the polarization field to the reservoir is postulated to have the form

$$L_{RD}=\sum_{j=1}^{2}\int_0^\infty d\nu\int d^3r\left[\mathbf{Z}_{j\nu}(\mathbf{r})\cdot\left(\frac{d\mathbf{Y}_{j\nu}(\mathbf{r})}{dt}-\mathbf{W}_{j\nu}(\mathbf{r})\right)+\frac{1}{2}\rho_{j\nu}\mathbf{W}_{j\nu}^2(\mathbf{r})-\frac{1}{2}\rho_{j\nu}\nu^2[\mathbf{Y}_{j\nu}(\mathbf{r})-\mathbf{X}_j(\mathbf{r})]^2\right]. \tag{5}$$

The total Lagrangian is then given by the sum $L=L_0+L_I$, with an interaction Lagrangian L_I reading

$$L_I=\sum_{j=1}^{2}\int d^3r\{[\boldsymbol{\nabla}\cdot g_j(\mathbf{r})\mathbf{X}_j(\mathbf{r})]\Phi(\mathbf{r})+g_j(\mathbf{r})\mathbf{V}_j(\mathbf{r})\cdot\mathbf{A}(\mathbf{r})\}. \tag{6}$$

The indicator function $g_j(\mathbf{r})$ is equal to 1 in the region of space occupied by the jth body, and equal to zero outside, so that the polarization field, while occupying the whole space, is coupled to the electromagnetic field only in the regions where the dielectrics are present. We need to impose certain conditions on the coupling functions $\rho_{j\nu}$. They must be real, otherwise, both the Lagrangian and the corresponding Hamiltonian are not Hermitian. What is more, each $\rho_{j\nu}$ should be positive, as it plays the role of a mass density—otherwise there is no stationarity principle for the corresponding action (cf. Ref. [56]). In addition, $\rho_{j\nu}\nu^2$ should have no singularities on the half axis $\nu>0$, for we demand that the coupling of the polarization field with all the reservoir oscillators be finite. Finally, we shall assume that $\rho_{j\nu}$ has an even analytical continuation to real negative values of the variable ν.

Analyzing infinitesimal unitary transformations of the action, one immediately obtains the following equal-time commutation relations for the dynamically independent variables:

$$[X_{j,\alpha}(\mathbf{r},t),P_{k,\beta}(\mathbf{r}',t)]=i\hbar\,\delta_{jk}\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}'), \tag{7}$$

$$[Y_{j\nu,\alpha}(\mathbf{r},t),Z_{k\nu',\beta}(\mathbf{r}',t)]=i\hbar\,\delta_{jk}\delta_{\alpha\beta}\delta(\nu-\nu')\,\delta(\mathbf{r}-\mathbf{r}'), \tag{8}$$

$$[A_\alpha(\mathbf{r},t),E_\beta(\mathbf{r}',t)]=-i\frac{\hbar}{\epsilon_0}\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}'). \tag{9}$$

B. Equations of motion

We can, naturally, eliminate the momenta from the above Lagrangian. To do so, we apply the principle of stationary action under the assumption that the infinitesimal variations commute with the fields, and get

$$\mathbf{E}(\mathbf{r})=-\dot{\mathbf{A}}(\mathbf{r})-\boldsymbol{\nabla}\Phi(\mathbf{r}), \tag{10}$$

$$\dot{\mathbf{E}}(\mathbf{r})=c^2\boldsymbol{\nabla}\times\mathbf{B}(\mathbf{r})-\frac{1}{\epsilon_0}\sum_{j=1}^{2}g_j(\mathbf{r})\mathbf{V}_j(\mathbf{r}), \tag{11}$$

$$\mathbf{B}(\mathbf{r})=\boldsymbol{\nabla}\times\mathbf{A}(\mathbf{r}), \tag{12}$$

$$\boldsymbol{\nabla}\cdot\mathbf{E}(\mathbf{r})=-\frac{1}{\epsilon_0}\sum_{j=1}^{2}\boldsymbol{\nabla}\cdot[g_j(\mathbf{r})\mathbf{X}_j(\mathbf{r})], \tag{13}$$

$$\dot{\mathbf{X}}_j(\mathbf{r})=\mathbf{V}_j(\mathbf{r}), \tag{14}$$

$$\dot{\mathbf{P}}_j(\mathbf{r})=-g_j(\mathbf{r})\boldsymbol{\nabla}\Phi(\mathbf{r})-\int_0^\infty d\nu\,\nu^2[\mathbf{X}_j(\mathbf{r})-\mathbf{Y}_{j\nu}(\mathbf{r})]$$
$$-\mathcal{M}_j\omega_{0j}^2\mathbf{X}_j(\mathbf{r}), \tag{15}$$

$$\mathbf{P}_j(\mathbf{r})=\mathcal{M}_j\mathbf{V}_j(\mathbf{r})-g_j(\mathbf{r})\mathbf{A}(\mathbf{r}), \tag{16}$$

$$\dot{\mathbf{Y}}_{j\nu}(\mathbf{r})=\mathbf{W}_{j\nu}(\mathbf{r}), \tag{17}$$

$$\dot{\mathbf{Z}}_{j\nu}(\mathbf{r})=\rho_{j\nu}\nu^2[\mathbf{X}_j(\mathbf{r})-\mathbf{Y}_{j\nu}(\mathbf{r})], \tag{18}$$

$$\mathbf{Z}_{j\nu}(\mathbf{r})=\rho_{j\nu}\mathbf{W}_{j\nu}(\mathbf{r}). \tag{19}$$

Let us notice that no particular gauge condition has been imposed so far. From the above expressions we conclude that the Maxwell equations take the expected form:

$$\dot{\mathbf{E}}(\mathbf{r})=c^2\boldsymbol{\nabla}\times\mathbf{B}(\mathbf{r})-\frac{1}{\epsilon_0}\sum_{j=1}^{2}g_j(\mathbf{r})\mathbf{V}_j(\mathbf{r}), \tag{20}$$

$$\dot{\mathbf{B}}(\mathbf{r})=-\boldsymbol{\nabla}\times\mathbf{E}(\mathbf{r}), \tag{21}$$

$$\boldsymbol{\nabla}\cdot\mathbf{E}(\mathbf{r})=-\frac{1}{\epsilon_0}\boldsymbol{\nabla}\cdot\sum_{j=1}^{2}[g_j(\mathbf{r})\mathbf{X}_j(\mathbf{r})], \tag{22}$$

$$\boldsymbol{\nabla}\cdot\mathbf{B}(\mathbf{r})=0. \tag{23}$$

Moreover, we find the equations for the polarization degrees of freedom:

$$\dot{\mathbf{X}}_j(\mathbf{r})=\mathbf{V}_j(\mathbf{r}), \tag{24}$$

$$\dot{\mathbf{V}}_j(\mathbf{r})=-\left(\omega_{0j}^2+\frac{1}{\mathcal{M}_j}\int_0^\infty d\nu\rho_{j\nu}\nu^2\right)\mathbf{X}_j(\mathbf{r})+\frac{1}{\mathcal{M}_j}g_j(\mathbf{r})\mathbf{E}(\mathbf{r})$$
$$+\frac{1}{\mathcal{M}_j}\int_0^\infty d\nu\rho_{j\nu}\nu^2\mathbf{Y}_{j\nu}(\mathbf{r}). \tag{25}$$

In the last equation we encounter, in the first term on the right-hand side, the ''frequency renormalization'' of the polarization oscillators caused by their coupling to the reservoir. Finally, the equations of motion for the reservoir degrees of freedom reduce to

$$\dot{\mathbf{Y}}_{j\nu}(\mathbf{r})=\mathbf{W}_{j\nu}(\mathbf{r}), \tag{26}$$

$$\dot{\mathbf{W}}_{j\nu}(\mathbf{r})=\nu^2[\mathbf{X}_j(\mathbf{r})-\mathbf{Y}_{j\nu}(\mathbf{r})]. \tag{27}$$

Thus, within the Huttner-Barnett approach followed here there are initially no dielectrics, but only interacting quantum fields. The dielectrics with their dispersive properties will emerge at a later stage of the development of the theory, via a term which appears in the wave equations satisfied by the retarded electromagnetic propagators. Let us also notice that the factor $g_j(\mathbf{r})$, which in our formalism appears only in the interaction Lagrangian, can also be introduced to multiply the free polarization Lagrangian (see, e.g., Ref. [17]). The difference is that in our approach the polarization field, when uncoupled from the electromagnetic field, oscillates freely; in the other approach, the uncoupled polarization field is ''static'' or ''frozen.'' However, it seems meaningless to state that, e.g., $\mathbf{X}_j(\mathbf{r})=\mathbf{0}$ in a certain region of space.

C. Application of the Laplace transformation

In this and the following section, we consider the problem of computing the fields, and hence, the electromagnetic energy flux, as an initial-value (Cauchy) problem. That is, we assume that we are given the correlation functions at the time $t=0$, and we seek them for later times $t,t',\ldots$ in the dis-

tant future. This is because we are not interested here in transient effects, but rather in sustained energy flow. One efficient method to solve a linear Cauchy problem is to apply the Laplace transformation; then we have the initial values appearing explicitly from the very beginning. Hence, for any operator $f(t)$ we consider its Laplace transform

$$\tilde{f}(s)=\int_0^\infty dt f(t)e^{-st}, \tag{28}$$

and obtain from Eqs. (20)–(27) the following system of algebraic-differential equations:

$$s\tilde{\mathbf{E}}(\mathbf{r},s)=c^2\boldsymbol{\nabla}\times\tilde{\mathbf{B}}(\mathbf{r},s)-\frac{1}{\epsilon_0}\sum_j g_j(\mathbf{r})\tilde{\mathbf{V}}_j(\mathbf{r},s)+\mathbf{E}(\mathbf{r},0), \tag{29}$$

$$s\tilde{\mathbf{B}}(\mathbf{r},s)=-\boldsymbol{\nabla}\times\tilde{\mathbf{E}}(\mathbf{r},s)+\mathbf{B}(\mathbf{r},0), \tag{30}$$

$$s\tilde{\mathbf{X}}_j(\mathbf{r},s)=\tilde{\mathbf{V}}_j(\mathbf{r},s)+\mathbf{X}(\mathbf{r},0), \tag{31}$$

$$s\tilde{\mathbf{V}}_j(\mathbf{r},s)=-\left(\omega_{0j}^2+\frac{1}{\mathcal{M}_j}\int_0^\infty d\nu\rho_{j\nu}\nu^2\right)\tilde{\mathbf{X}}_j(\mathbf{r},s)+\frac{1}{\mathcal{M}_j}g_j(\mathbf{r})\tilde{\mathbf{E}}(\mathbf{r},s)+\frac{1}{\mathcal{M}_j}\int_0^\infty d\nu\rho_{j\nu}\nu^2\tilde{\mathbf{Y}}_{j\nu}(\mathbf{r},s)+\mathbf{V}_j(\mathbf{r},0), \tag{32}$$

$$s\tilde{\mathbf{Y}}_{j\nu}(\mathbf{r},s)=\tilde{\mathbf{W}}_{j\nu}(\mathbf{r},s)+\mathbf{Y}_{j\nu}(\mathbf{r},0), \tag{33}$$

$$s\tilde{\mathbf{W}}_{j\nu}(\mathbf{r},s)=-\nu^2\tilde{\mathbf{Y}}_{j\nu}(\mathbf{r},s)+\nu^2\tilde{\mathbf{X}}_j(\mathbf{r},s)+\mathbf{W}_{j\nu}(\mathbf{r},0). \tag{34}$$

We now adopt the following natural strategy: we first eliminate the polarization and the reservoir variables to obtain inhomogeneous wave equations (in terms of the Laplace variable s) for the electric and magnetic fields, so that their right-hand sides contain the initial values only. Then we solve these equations using Green's functions, form the expectation value of the Poynting vector, and invert the Laplace transform. At the first stage of this procedure, we get

$$\tilde{\mathbf{Y}}_{j\nu}(\mathbf{r},s)=\frac{\nu^2}{s^2+\nu^2}\tilde{\mathbf{X}}_j(\mathbf{r},s)+\frac{1}{s^2+\nu^2}[s\mathbf{Y}_{j\nu}(\mathbf{r},0)+\mathbf{W}_{j\nu}(\mathbf{r},0)], \tag{35}$$

and, moreover,

$$\mathcal{H}_j(s)\tilde{\mathbf{V}}_j(\mathbf{r},s)=-[\mathcal{H}_j(s)-s^2]\mathbf{X}_j(\mathbf{r},0)+s\mathbf{V}_j(\mathbf{r},0)+\frac{g_j(\mathbf{r})}{\mathcal{M}_j}s\tilde{\mathbf{E}}(\mathbf{r},s)+\frac{1}{\mathcal{M}_j}\int_0^\infty d\nu\rho_{j\nu}\frac{\nu^2}{s^2+\nu^2}\times[s^2\mathbf{Y}_{j\nu}(\mathbf{r},0)+s\mathbf{W}_{j\nu}(\mathbf{r},0)], \tag{36}$$

where the resolvents $\mathcal{H}_j(s)$ are given by

$$\mathcal{H}_j(s)=s^2+\omega_{0j}^2+s^2\int_0^\infty d\nu\frac{\rho_{j\nu}}{\mathcal{M}_j}\frac{\nu^2}{s^2+\nu^2}. \tag{37}$$

There are three contributions to the dynamics of the variables $\mathbf{V}_j$. First, there are the dynamics determined by the poles of the resolvent functions $\mathcal{H}_j(s)$ and the initial values $\mathbf{V}_j(\mathbf{r},0)$ and $\mathbf{X}_j(\mathbf{r},0)$. Second, there is the contribution from the electric field $\tilde{\mathbf{E}}(\mathbf{r},s)$, and, third, the contribution from the reservoir, associated with the additional poles at $s=\pm i\nu$. But, as one can prove in a way fully analogous to that in Ref. [17] [p. 910, Eqs. (42)–(44); cf. also the more detailed calculations by Eberlein in Ref. [57], Appendix 2], $\mathcal{H}_j(s)$ does not have any zeros for non-negative values of s. Thus, for sufficiently long time(s), the expectation value and any multilinear correlation function involving $\mathbf{V}_j$ and $\mathbf{X}_j$ will not have any terms associated with the initial conditions for these operators, as these term are damped. In particular, if we make the simplest choice

$$\rho_{j\nu}=\frac{4\gamma_j\mathcal{M}_j}{\pi\nu^2}, \tag{38}$$

so that the product $\rho_{j\nu}\nu^2$ becomes a constant, we find

$$\mathcal{H}_j(s)=s^2+\omega_{0j}^2+2\gamma_j s, \tag{39}$$

with zeros at $-\gamma\pm i\sqrt{\omega_{0j}^2-\gamma_j^2}$, which shows that the initial oscillations (and correlations) of the operators $\mathbf{X}_j$ and $\mathbf{V}_j$ are damped, whatever the relation between ω_{0j} and γ_j might be.

For this reason we write symbolically

$$\tilde{\mathbf{V}}_j(\mathbf{r},s)=\mathcal{H}_j^{-1}(s)\left[\frac{g_j(\mathbf{r})}{\mathcal{M}_j}s\tilde{\mathbf{E}}(\mathbf{r},s)+\frac{1}{\mathcal{M}_j}\int_0^\infty d\nu\rho_{j\nu}\frac{\nu^2}{s^2+\nu^2}\times\{s^2\mathbf{Y}_{j\nu}(\mathbf{r},0)+s\mathbf{W}_{j\nu}(\mathbf{r},0)\}\right], \tag{40}$$

leaving out the initial values of the polarization fields, with the understanding that the Cauchy data for the polarization operators can be omitted only after the expectation values have been formed, and all the time variables have become very large.

After eliminating first the polarization velocity operator and then the magnetic induction operator, we find the following wave equation for the electric field:

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\tilde{\mathbf{E}}(\mathbf{r},s)+\tilde{\boldsymbol{\epsilon}}(\mathbf{r},s)\frac{s^2}{c^2}\tilde{\mathbf{E}}(\mathbf{r},s)=\frac{s}{c^2}\mathbf{E}(\mathbf{r},0)+\boldsymbol{\nabla}\times\mathbf{B}(\mathbf{r},0)-\frac{s^2}{\epsilon_0c^2}\sum_{j=1}^2\frac{g_j(\mathbf{r})}{\mathcal{M}_j\mathcal{H}_j(s)}\times\int_0^\infty d\nu\frac{\rho_{j\nu}\nu^2}{s^2+\nu^2}[s\mathbf{Y}_{j\nu}(\mathbf{r},0)+\mathbf{W}_{j\nu}(\mathbf{r},0)]. \tag{41}$$

Here we have identified the expression

$$\tilde{\epsilon}(\mathbf{r},s)=1+\frac{1}{\epsilon_0}\sum_{j=1}^{2}\frac{g_j^2(\mathbf{r})}{\mathcal{M}_j\mathcal{H}_j(s)}, \tag{42}$$

with the dielectric function, considered as a function of the Laplace variable. Because of the location of the zeros of the resolvent functions $\mathcal{H}_j(s)$, this dielectric function has no poles in the right half of the s plane. In view of the fact that the functions $g_j(\mathbf{r})$ are equal to 1 in the region of space occupied by the jth body, we conclude that the expression

$$\tilde{\epsilon}_j(s)=1+[\epsilon_0\mathcal{M}_j\mathcal{H}_j(s)]^{-1} \tag{43}$$

can be interpreted as the dielectric function of the jth body. Here, the tilde symbol is used to distinguish the dielectric function as expressed in terms of the Laplace variable from the same function expressed in terms of frequency, which we shall denote as $\epsilon_j(\omega)$.

Taking once more the fast damping of the initial polarization correlations into account, the wave equation for the magnetic inductions reads

$$\begin{aligned}\nabla\times\nabla\times\tilde{\mathbf{B}}(\mathbf{r},s)+\tilde{\epsilon}(\mathbf{r},s)\frac{s^2}{c^2}\tilde{\mathbf{B}}(\mathbf{r},s)&\\ =\tilde{\epsilon}(\mathbf{r},s)\frac{s}{c^2}\mathbf{B}(\mathbf{r},0)-\frac{1}{c^2}\nabla\times\mathbf{E}(\mathbf{r},0)&\\ +\frac{s}{\epsilon_0c^2}\sum_{j=1}^{2}\frac{g_j(\mathbf{r})}{\mathcal{M}_j\mathcal{H}_j(s)}\int_0^\infty d\nu\frac{\rho_{j\nu}\nu^2}{s^2+\nu^2}&\\ \times[s\nabla\times\mathbf{Y}_{j\nu}(\mathbf{r},0)+\nabla\times\mathbf{W}_{j\nu}(\mathbf{r},0)].&\end{aligned} \tag{44}$$

These Eqs. (41) and (44) can be solved formally with the help of the Green's functions $D^E(\mathbf{r},\mathbf{r}';s)$ and $D^M(\mathbf{r},\mathbf{r}';s)$:

$$\begin{aligned}\tilde{\mathbf{E}}(\mathbf{r},s)&=\int d^3r_1D^E(\mathbf{r},\mathbf{r}_1;s)[\tilde{\mathbf{J}}(\mathbf{r}_1,s)+C_{\mathrm{EM}}],\\ \tilde{\mathbf{B}}(\mathbf{r},s)&=\int d^3r_1D^B(\mathbf{r},\mathbf{r}_1;s)[\tilde{\mathbf{J}}(\mathbf{r}_1,s)+C_{\mathrm{EM}}],\end{aligned} \tag{45}$$

where the C_{EM} denotes the terms depending on initial conditions for the electromagnetic fields only. The above Green's functions themselves satisfy

$$\begin{aligned}\nabla\times\nabla\times D^E(\mathbf{r},\mathbf{r}_1;s)+\tilde{\epsilon}(\mathbf{r},s)\frac{s^2}{c^2}D^E(\mathbf{r},\mathbf{r}_1;s)&=-s\,\delta(\mathbf{r}-\mathbf{r}_1),\\ \nabla\times\nabla\times D^M(\mathbf{r},\mathbf{r}_1;s)+\tilde{\epsilon}(\mathbf{r},s)\frac{s^2}{c^2}D^M(\mathbf{r},\mathbf{r}_1;s)&\\ =\nabla\times\mathbf{1}\delta(\mathbf{r}-\mathbf{r}_1),&\end{aligned} \tag{46}$$

where **1** is the unit dyadic. The ''current'' in Eq. (45) is, naturally,

$$\begin{aligned}\tilde{\mathbf{J}}(\mathbf{r},s)=\frac{s}{\epsilon_0c^2}\sum_{j=1}^{2}\frac{g_j(\mathbf{r})}{\mathcal{M}_j\mathcal{H}_j(s)}\int_0^\infty d\nu\frac{\rho_{j\nu}\nu^2}{s^2+\nu^2}&\\ \times[s\mathbf{Y}_{j\nu}(\mathbf{r},0)+\mathbf{W}_{j\nu}(\mathbf{r},0)].&\end{aligned} \tag{47}$$

III. CALCULATION OF THE POYNTING VECTOR BY MEANS OF THE EQUATIONS OF MOTION

Now we are almost ready to write down the expression for the expectation value of the Poynting vector. The very definition of this quantity is, however, a somewhat subtle matter. To begin with, we must not define the Poynting vector simply as

$$\mathbf{S}(\mathbf{r},t)=\mathbf{E}(\mathbf{r},t)\times\mathbf{H}(\mathbf{r},t),$$

because this operator is not Hermitian, while the expectation value of the Poynting vector must be real. One way out could be to use a symmetrized form of the above product. However, we are better guided by the standard quantum field-theoretical definition of the energy-momentum tensor. Namely, taking into account the fact that the electromagnetic vacuum state vector should be invariant with respect to Poincaré transformations, it is necessary to define the total momentum of the electromagnetic field as a normally ordered operator. This definition also guarantees that the momentum (and energy) of the vacuum vanishes. If we now make the natural requirement that the Poynting vector should be the spatial density of the total momentum of the electromagnetic field, we arrive at the definition

$$\mathbf{S}(\mathbf{r},t)=:\mathbf{E}(\mathbf{r},t)\times\mathbf{H}(\mathbf{r},t):, \tag{48}$$

where $:\hat{O}:$ denotes the normally ordered form of the operator $\hat{O}$. It should be noticed that this definition cuts off any possible vacuum contribution.

In order to obtain the expectation value $\langle\mathbf{S}(\mathbf{r},t)\rangle$, we need to invert a double Laplace transformation with respect to its two variables s_1 and s_2. This is achieved by means the formula

$$f(t,t')|_{t'=t}=\frac{1}{(2\pi i)^2}\int_{C_1}ds_1e^{s_1t}\int_{C_2}ds_2e^{s_2t}\tilde{f}(s_1,s_2), \tag{49}$$

where the contours C_1 and C_2 run parallel to the imaginary axes in the complex s_1 and s_2 planes, leaving all singularities of $\tilde{f}(s_1,s_2)$ on their left-hand side. This leads to the expression

$$\langle \mathbf{S}(\mathbf{r},t)\rangle = \frac{1}{(2\pi i)^2}\frac{1}{\mu_0}\int_{C_1} ds_1 e^{s_1 t} \times \int_{C_2} ds_2\, e^{s_2 t}\langle :\mathbf{E}(\mathbf{r},s_1)\times\mathbf{B}(\mathbf{r},s_2):\rangle. \quad (50)$$

Now we can get rid of all the terms containing the initial correlations of the electric and magnetic fields, since we assume that the electromagnetic field is initially in the vacuum state. What remains gives the following contribution to the expectation value of the Poynting vector:

$$\langle S_\alpha(\mathbf{r},t)\rangle = \frac{1}{(2\pi i)^2}\frac{1}{\mu_0}\epsilon_{\alpha\beta\gamma}\int_{C_1} ds_1 e^{s_1 t}\int_{C_2} ds_2 e^{s_2 t}\int d^3 r_1 \times \int d^3 r_2 \int_0^\infty d\nu_1 \int_0^\infty d\nu_2 D^E_{\beta\lambda}(\mathbf{r},\mathbf{r}_1,s_1) \times D^M_{\gamma\mu}(\mathbf{r},\mathbf{r}_2,s_2) R_{\lambda\mu}(\mathbf{r}_1,\mathbf{r}_2,s_1,s_2), \quad (51)$$

where the correlation function $R_{\lambda\mu}$ contains only the contributions from the reservoir fields:

$$R_{\lambda\mu}(\mathbf{r}_1,\mathbf{r}_2,s_1,s_2) = \frac{s_1 s_2}{\epsilon_0^2 c^4}\sum_{j,k}\frac{g_j(\mathbf{r}_1) g_k(\mathbf{r}_2)}{\mathcal{M}_j\mathcal{M}_k\mathcal{H}_j(s_1)\mathcal{H}_k(s_2)} \times \frac{\rho_{j\nu_1}\rho_{k\nu_2}\nu_1^2\nu_2^2}{(s_1^2+\nu_1^2)(s_2^2+\nu_2^2)}\langle :[s_1 Y_{j\nu_1,\lambda}(\mathbf{r}_1,0) + W_{j\nu_1,\lambda}(\mathbf{r}_1,0)][s_2 Y_{k\nu_2,\mu}(\mathbf{r}_2,0) + W_{k\nu_2,\mu}(\mathbf{r}_2,0)]:\rangle. \quad (52)$$

To compute the correlation matrix $R_{\lambda\mu}$ we need the initial states of the reservoir. Naturally, we choose this initial state Γ to be the tensor product of density matrices Γ_1 and Γ_2, with

$$\Gamma_j = \frac{\exp\left(-\beta_j\hbar\int d^3 r\int_0^\infty d\nu\,\nu A^\dagger_{j\nu,\alpha}(\mathbf{r})A_{j\nu,\alpha}(\mathbf{r})\right)}{Tr\left[\exp\left(-\beta_j\hbar\int d^3 r\int_0^\infty d\nu\,\nu A^\dagger_{j\nu,\alpha}(\mathbf{r})A_{j\nu,\alpha}(\mathbf{r})\right)\right]}. \quad (53)$$

The annihilation and creation operators $\mathbf{A}_{j\nu}$ and $\mathbf{A}^\dagger_{j\nu}$ appearing here are defined in terms of the reservoir operators $\mathbf{Y}$ and $\mathbf{Z}$,

$$\mathbf{A}_{j\nu}(\mathbf{r}) = \sqrt{\frac{\rho_{j\nu}\nu}{2\hbar}}\left(\mathbf{Y}_{j\nu}(\mathbf{r},0) + \frac{i}{\rho_{j\nu}\nu}\mathbf{Z}_{j\nu}(\mathbf{r},0)\right), \quad (54)$$

$$\mathbf{A}^\dagger_{j\nu}(\mathbf{r}) = \sqrt{\frac{\rho_{j\nu}\nu}{2\hbar}}\left(\mathbf{Y}_{j\nu}(\mathbf{r},0) - \frac{i}{\rho_{j\nu}\nu}\mathbf{Z}_{j\nu}(\mathbf{r},0)\right), \quad (55)$$

while β_j denotes the inverse temperature of the jth reservoir. The above choice of the initial state then leads to the following formulas for the initial correlations:

$$\langle :Y_{j\nu_1,\lambda}(\mathbf{r}_1,0)Y_{k\nu_2,\mu}(\mathbf{r}_2,0):\rangle = \frac{\hbar}{\rho_{j\nu_1}\nu_1}[\exp(\beta_j\hbar\nu_1)-1]^{-1}\delta_{jk}\delta_{\lambda\mu}\delta(\nu_1-\nu_2) \times \delta(\mathbf{r}_1-\mathbf{r}_2), \quad (56)$$

$$\langle :Y_{j\nu_1,\lambda}(\mathbf{r}_1,0)W_{k\nu_2,\mu}(\mathbf{r}_2,0):\rangle = 0, \quad (57)$$

$$\langle :W_{j\nu_1,\lambda}(\mathbf{r}_1,0)Y_{k\nu_2,\mu}(\mathbf{r}_2,0):\rangle = 0, \quad (58)$$

$$\langle :W_{j\nu_1,\lambda}(\mathbf{r}_1,0)W_{k\nu_2,\mu}(\mathbf{r}_2,0):\rangle = \frac{\hbar\nu_1}{\rho_{j\nu_1}}[\exp(\beta_j\hbar\nu_1)-1]^{-1}\delta_{jk}\delta_{\lambda\mu}\delta(\nu_1-\nu_2)\delta(\mathbf{r}_1-\mathbf{r}_2). \quad (59)$$

Taking everything together, we find that the α component of the Poynting vector can be written in the form

$$\langle S_\alpha(\mathbf{r},t)\rangle = \frac{1}{(2\pi i)^2}\frac{\hbar}{\epsilon_0 c^2}\epsilon_{\alpha\beta\gamma}\int_{C_1} ds_1 e^{s_1 t}\int_{C_2} ds_2 e^{s_2 t} \times \int d^3 r_1\int_0^\infty d\nu D^E_{\beta\lambda}(\mathbf{r},\mathbf{r}_1,s_1)D^M_{\gamma\lambda}(\mathbf{r},\mathbf{r}_1,s_2) \times R'(\mathbf{r}_1,s_1,s_2), \quad (60)$$

where we have performed integration by parts, and where

$$R'(\mathbf{r}_1,s_1,s_2) = s_1 s_2\sum_j \frac{g_j^2(\mathbf{r}_1)}{\mathcal{M}_j^2\mathcal{H}_j(s_1)\mathcal{H}_j(s_2)} \times \frac{\rho_{j\nu}\nu^3}{(s_1^2+\nu^2)(s_2^2+\nu^2)}(s_1 s_2+\nu^2) \times [\exp(\beta_j\hbar\nu)-1]^{-1}. \quad (61)$$

We are now ready to perform the integration over the contours C_1 and C_2, taking into account that only the poles at $s_{1,2}=\pm i\nu$ contribute, since all the singularities of Green's functions have real parts smaller than zero. The latter statement follows from the fact that the integration variable $\mathbf{r}_1$ effectively runs only over the regions of space filled with a (lossy) dielectric. Consecutive integration over s_1 and s_2 eventually yields

$$\langle S_\alpha(\mathbf{r},t)\rangle = \frac{\hbar}{2\epsilon_0 c^2}\epsilon_{\alpha\beta\gamma}\sum_j \int d^3 r_1\int_0^\infty d\nu\frac{g_j^2(\mathbf{r}_1)}{\mathcal{M}_j^2} \times \frac{\rho_{j\nu}\nu^5}{\mathcal{H}_j(i\nu)\mathcal{H}_j(-i\nu)}[\exp(\beta_j\hbar\nu)-1]^{-1} \times \{D^E_{\beta\lambda}(\mathbf{r},\mathbf{r}_1,i\nu)D^M_{\gamma\lambda}(\mathbf{r},\mathbf{r}_1,-i\nu) + D^E_{\beta\lambda}(\mathbf{r},\mathbf{r}_1,-i\nu)D^M_{\gamma\lambda}(\mathbf{r},\mathbf{r}_1,i\nu)\}. \quad (62)$$

Finally, we consider the dielectric function of the jth body,

$$\epsilon_j(\omega)=\tilde{\epsilon}_j(s=-i\omega+\eta)=1+\frac{1}{\epsilon_0\mathcal{M}_j\mathcal{H}_j(-i\omega+\eta)}, \tag{63}$$

where the infinitesimal positive frequency η guarantees causal behavior, and utilize the identities

$$\int_0^\infty d\nu\,\frac{\rho_{j\nu}\nu^2}{\omega^2-\nu^2\pm i\eta\omega}=\pm\frac{\pi}{2i}\omega\rho_{j\omega}-\frac{1}{2\omega}P\int_{-\infty}^{\infty}d\nu\,\frac{\rho_{j\nu}\nu^2}{\nu-\omega} \tag{64}$$

to write its imaginary part in the form

$$\mathrm{Im}[\epsilon_j(\omega)]=\frac{\pi}{2\epsilon_0}\frac{1}{\mathcal{M}_j^2}\frac{\rho_{j\omega}\omega^3}{\mathcal{H}_j(-i\omega+\eta)\mathcal{H}_j(i\omega+\eta)}. \tag{65}$$

This expression (65) then allows us to eliminate the auxiliary function $\rho_{j\nu}$ from the expression (62) for the Poynting vector, in favor of the observable quantity $\mathrm{Im}[\epsilon_j(\omega)]$. In this way, we obtain the principal result

$$\begin{aligned}\langle S_\alpha(\mathbf{r})\rangle=&\frac{\hbar}{\pi c^2}\epsilon_{\alpha\beta\gamma}\sum_j\int d^3r_1\int_0^\infty d\nu\,\nu^2 g_j^2(\mathbf{r}_1)\\&\times[\exp(\beta_j\hbar\nu)-1]^{-1}\mathrm{Im}[\epsilon_j(\nu)]\\&\times\{D^E_{\beta\lambda}(\mathbf{r},\mathbf{r}_1,i\nu)D^M_{\gamma\lambda}(\mathbf{r},\mathbf{r}_1,-i\nu)\\&+D^E_{\beta\lambda}(\mathbf{r},\mathbf{r}_1,-i\nu)D^M_{\gamma\lambda}(\mathbf{r},\mathbf{r}_1,i\nu)\}.\end{aligned} \tag{66}$$

Let us notice that an infinitesimal η (of any sign) has to be added to the argument $\pm i\nu$ of the Green's functions D^E and D^M, since these functions also depend on $\epsilon(\omega)$, so that they contain the singular integral over ν; a way to get round the pole on the positive ν axis should therefore be specified. Regardless of the sign of η, in both terms in the curly bracket of Eq. (66), one of the Green's functions is retarded, and the other one advanced. This characteristic feature is met again, and clarified further, in the following sections, in the framework of the Keldysh formalism.

Up to the chosen system of units, the above result (66) equals a formula already used by Polder and van Hove [9] [cf. the expression between their Eqs. (5) and (6)]; let us notice, however, that their Green's functions have not been defined explicitly.

IV. KELDYSH FORMALISM

In this section we abandon the equations-of-motion approach developed above and employ the closed-time-contour Schwinger-Keldysh technique, which allows one to apply diagrammatic time-dependent perturbation theory to nonequilibrium phenomena. The value of the closed-contour formalism has first been recognized by Schwinger [58], but here we prefer to exploit the diagrammatic version due to Keldysh [59]. Very useful presentations of the Schwinger-Keldysh theory are given in Refs. [60–63]. Although the full strength of the diagrammatic approach becomes apparent only when there is multilinear coupling of the fields (so that perturbation theory is indispensable), it is also quite instructive for the linear model considered in the present paper. In contrast to the general case, here the perturbation series can be summed up exactly, in the sense that the solution reduces to a single linear integral equation of the Wiener-Hopf type, or, simpler yet, to a partial differential equation with nontrivial continuity conditions.

Because of the excellent reviews mentioned above, we shall not provide any general description of the Keldysh formalism, but restrict ourselves to a few basic definitions.

Let us consider a general time-ordered correlation function of two operators $L(t)$ and $M(t)$, denoted as

$$\langle T[L(t)M(t')]\rangle,$$

where the average is taken over an arbitrary state, equilibrium or nonequilibrium. This product can be written as [59]

$$\mathrm{Tr}\{\rho_0\,T_C[S_C L_0(t)M_0(t')]\},$$

where ρ_0 is the density matrix corresponding to the initial state, $L_0(t)$ and $M_0(t)$ are the interaction-picture representations of the operators $L(t)$ and $M(t)$, and T_C is the chronological operator, which arranges the operators it acts on according to the sequence of their arguments on the contour C. The latter is led along the real t axis from $-\infty$ to $+\infty$, and then back from $+\infty$ to $-\infty$. The arguments t, t' lie on the positive branch of this contour (from $-\infty$ to $+\infty$). Finally, the S matrix S_C is given by

$$S_C=T\exp\left(\frac{i}{\hbar}\int_C dt L_{int}(t)\right). \tag{67}$$

The important point to observe is that one can also represent an ordinary correlation function

$$\langle L(t)M(t')\rangle=\mathrm{Tr}\{\rho_0 T_C[S_C L_0(t_-)M_0(t'_+)]\}$$

in this way. The subscript "+" is assigned to all points lying on the positive branch of the contour, and the "−" sign to points lying on the negative branch (from $+\infty$ to $-\infty$). Similarly, one can express a correlation function containing an antichronological product with the help of such a contour C.

Employing a closed-time contour, one can formulate Feynman rules for a nonequilibrium quantum-field theory [59,60,62]. What is very important, one can also write down the Dyson equation resulting from the resummation of graphs. It is convenient to write the Dyson equation in matrix form, rather than in a form employing closed contours. We will need two such matrix forms: First, we consider the matrix

$$G(\mathbf{r},\mathbf{r}',t,t')=\begin{pmatrix}G^{--}(\mathbf{r},\mathbf{r}',t,t') & G^{-+}(\mathbf{r},\mathbf{r}',t,t')\\ G^{+-}(\mathbf{r},\mathbf{r}',t,t') & G^{++}(\mathbf{r},\mathbf{r}',t,t')\end{pmatrix}, \tag{68}$$

the elements of which are Feynman, anti-Feynman, and "ordinary" correlation functions. These functions are defined as follows:

$$G^{--}(\mathbf{r},\mathbf{r}',t,t')=-\frac{i}{\hbar}\langle T[\psi(\mathbf{r},t)\psi^{\dagger}(\mathbf{r}',t')]\rangle, \quad (69)$$

where $\psi(\mathbf{r},t)$ is a boson field operator (we shall not have to deal with fermion operators in this paper);

$$G^{++}(\mathbf{r},\mathbf{r}',t,t')=\frac{i}{\hbar}\langle \tilde{T}[\psi(\mathbf{r},t)\psi^{\dagger}(\mathbf{r}',t')]\rangle; \quad (70)$$

where $\tilde{T}$ is the antichronological operator; and

$$G^{-+}(\mathbf{r},\mathbf{r}',t,t')=-\frac{i}{\hbar}\langle \psi(\mathbf{r},t)\psi^{\dagger}(\mathbf{r}',t')\rangle, \quad (71)$$

$$G^{+-}(\mathbf{r},\mathbf{r}',t,t')=-\frac{i}{\hbar}\langle \psi^{\dagger}(\mathbf{r}',t')\psi(\mathbf{r},t)\rangle. \quad (72)$$

We remark that our notation follows the convention employed by Lifshitz and co-workers [60]. In most other references the usage of the superscripts "+" and "−" adheres to the opposite convention, i.e., G^{--} there denotes Green's function involving the antichronological operator, etc.

In a similar manner, we introduce a matrix containing four self-energies:

$$\Sigma(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)=\begin{pmatrix}\Sigma^{--}(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2) & \Sigma^{-+}(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)\\ \Sigma^{+-}(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2) & \Sigma^{++}(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)\end{pmatrix}. \quad (73)$$

Then we have

$$\begin{aligned}G(\mathbf{r},\mathbf{r}',t,t')&=G^{0}(\mathbf{r},\mathbf{r}',t,t')+\int dt_1\int d^3r_1\int dt_2\int d^3r_2\\ &\times G^{0}(\mathbf{r},\mathbf{r}_1,t,t_1)\cdot\Sigma(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)G(\mathbf{r}_2,\mathbf{r}',t_2,t'), \quad (74)\end{aligned}$$

where $G^{0}(\mathbf{r},\mathbf{r}',t,t')$ denotes the free Green's function matrix.

Second, and alongside this matrix form of the Dyson equation, we shall make use of another form written in terms of the retarded, advanced, and symmetrized correlation functions. Let

$$F(\mathbf{r},\mathbf{r}',t,t')=-\frac{i}{\hbar}\langle \psi(\mathbf{r},t)\psi^{\dagger}(\mathbf{r}',t')+\psi^{\dagger}(\mathbf{r}',t')\psi(\mathbf{r},t)\rangle \quad (75)$$

be the symmetrized Green's function, and let G^R and G^A be the retarded and advanced Green's function, defined in the standard way with the help of commutators:

$$G^{R}(\mathbf{r},\mathbf{r}';t,t')=-\frac{i}{\hbar}\langle[\psi(\mathbf{r},t),\psi^{\dagger}(\mathbf{r}',t')]\rangle\,\theta(t-t'), \quad (76)$$

$$G^{A}(\mathbf{r},\mathbf{r}';t,t')=\frac{i}{\hbar}\langle[\psi(\mathbf{r},t),\psi^{\dagger}(\mathbf{r}',t')]\rangle\,\theta(t'-t), \quad (77)$$

where $\theta(\tau)$ is the Heaviside function,

$$\theta(\tau)=\begin{cases}1 & \text{for } \tau>0,\\ 0 & \text{for } \tau<0.\end{cases}$$

Using these functions, we construct the following matrix G':

$$G'(\mathbf{r},\mathbf{r}',t,t')=\begin{pmatrix}0 & G^{A}(\mathbf{r},\mathbf{r}',t,t')\\ G^{R}(\mathbf{r},\mathbf{r}',t,t') & F(\mathbf{r},\mathbf{r}',t,t')\end{pmatrix}, \quad (78)$$

and define a self-energy matrix Σ' [59] as

$$\Sigma'(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)=\begin{pmatrix}\Omega(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2) & \Sigma^{R}(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)\\ \Sigma^{A}(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2) & 0\end{pmatrix}. \quad (79)$$

Then one finds again a Dyson equation of the form (74), with G replaced by G', and Σ by Σ'.

This matrix representation G' of the Green's functions is more convenient for practical calculations. From the Dyson equation for G' one infers that the equations for G^R and G^A separate from those for the function F, leading to

$$\begin{aligned}G^{R,A}(\mathbf{r},\mathbf{r}',t,t')&=G^{0;R,A}(\mathbf{r},\mathbf{r}',t,t')\\ &+\int d^3r_1\int d^3r_2\int dt_1\int dt_2\\ &\times G^{0;R,A}(\mathbf{r},\mathbf{r}_1,t,t_1)\cdot\Sigma^{R,A}(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)\\ &\times G^{R,A}(\mathbf{r}_2,\mathbf{r}',t_2,t'), \quad (80)\end{aligned}$$

$$\begin{aligned}F(\mathbf{r},\mathbf{r}',t,t')&=\int d^3r_1\int d^3r_2\int dt_1\int dt_2\\ &\times G^{0;R}(\mathbf{r},\mathbf{r}_1,t,t_1)\cdot[\Omega(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)\\ &\times G^{A}(\mathbf{r}_2,\mathbf{r}',t_2,t')+\Sigma^{R}(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)\\ &\times F(\mathbf{r}_2,\mathbf{r}',t_2,t')]. \quad (81)\end{aligned}$$

Naturally, there exists a connection between Green's functions and the self-energies entering the matrices G and G':

$$\begin{aligned}G^{R}&=G^{--}-G^{-+}=G^{+-}-G^{++},\\ G^{A}&=G^{--}-G^{+-}=G^{-+}-G^{++},\\ F&=G^{--}+G^{++}=G^{+-}+G^{-+}, \quad (82)\end{aligned}$$

and

$$\begin{aligned}\Sigma^{R}&=\Sigma^{--}+\Sigma^{-+},\\ \Sigma^{A}&=\Sigma^{--}+\Sigma^{+-},\\ \Omega&=\Sigma^{--}+\Sigma^{++}. \quad (83)\end{aligned}$$

Our strategy in this section is first to list all relevant Green's functions of the reservoir, and then to draw some

simple Feynman graphs to write down their Dyson equation and get corresponding ''dressed'' Green's functions of the polarization field. Finally, we draw a further family of graphs (using these dressed polarization lines) to formulate the Dyson equation for the electromagnetic propagators. In order to perform the computations in this last stage, we need to choose a particular gauge. This is because the interaction Lagrangian which enters the S matrix, and hence the self-energy, is expressed in terms of potentials. It is most convenient to work in the temporal (Pauli-Heisenberg) gauge, in which

$$\Phi(\mathbf{r},t)=0. \tag{84}$$

Then, after eliminating the momenta and the **E** and **B** fields, we obtain the Lagrangian in the following form:

$$\begin{aligned} L=&\sum_{j=1}^{2}\int d^3r\left[\frac{1}{2}\mathcal{M}_j\mathbf{V}_j^2(\mathbf{r})-\frac{1}{2}\mathcal{M}_j\omega_{0j}^2\mathbf{X}_j^2(\mathbf{r})\right]\\ &+\frac{1}{2}\int d^3r\left[\epsilon_0\dot{\mathbf{A}}^2(\mathbf{r})-\frac{1}{\mu_0}(\boldsymbol{\nabla}\times\mathbf{A})^2(\mathbf{r})\right]\\ &+\sum_{j=1}^{2}\int_0^\infty d\nu\int d^3r\left[\frac{1}{2}\rho_{j\nu}\mathbf{W}_{j\nu}^2(\mathbf{r})-\frac{1}{2}\rho_{j\nu}\nu^2(\mathbf{Y}_{j\nu}(\mathbf{r})\right.\\ &\left.-\mathbf{X}_j(\mathbf{r}))^2\right]+\sum_{j=1}^{2}\int d^3r g_j(\mathbf{r})V_j(\mathbf{r})\cdot\mathbf{A}(\mathbf{r}), \end{aligned} \tag{85}$$

where the last term is the interaction Lagrangian.

A. Green's functions of the reservoir fields

Below we provide a summary of the *free* Green's functions of the reservoir fields, valid under the assumption that the reservoirs are in thermal states with inverse temperatures β_j, which are taken to be independent of **r**. (This latter assumption can, in principle, be relaxed, so that we could also take into account temperature differences within one and the same body.)

Green's functions written below are obtained directly from the definitions (69)–(77) of the preceding section, with the general field operators ψ and $\psi^\dagger$ replaced by the (Hermitian) operator $Y_{j\nu,\alpha}$. It is convenient to work with Green's functions which are Fourier transformed with respect to time, so that they are functions of two spatial variables and one frequency variable. Thus, we have the following list:

(a) The retarded Green's function

$$\begin{aligned} H^R_{\alpha\beta,jj'}(\mathbf{r}_1,\mathbf{r}_2,\nu,\nu',\omega)=&\frac{1}{\rho_{j\nu}}\frac{1}{\omega^2-\nu^2+i\eta\omega}\\ &\times\delta_{\alpha\beta}\delta_{jj'}\delta(\nu-\nu')\,\delta(\mathbf{r}_1-\mathbf{r}_2). \end{aligned} \tag{86}$$

(b) The advanced Green's function

$$\begin{aligned} H^A_{\alpha\beta,jj'}(\mathbf{r}_1,\mathbf{r}_2,\nu,\nu',\omega)=&\frac{1}{\rho_{j\nu}}\frac{1}{\omega^2-\nu^2-i\eta\omega}\\ &\times\delta_{\alpha\beta}\delta_{jj'}\delta(\nu-\nu')\,\delta(\mathbf{r}_1-\mathbf{r}_2). \end{aligned} \tag{87}$$

As usual, η is an infinitesimal number added to indicate the location of poles in ω space. Naturally, the retarded and advanced Green's function must not depend on the states of the reservoirs.

(c) The causal (Feynman) Green's function

$$\begin{aligned} H^{--}_{\alpha\beta,jj'}(\mathbf{r}_1,\mathbf{r}_2,\nu,\nu',\omega)=&-\frac{i}{2\rho_{j\nu}}\delta_{\alpha\beta}\delta_{jj'}\delta(\nu-\nu')\,\delta(\mathbf{r}_1-\mathbf{r}_2)\cdot\left[\pi\coth\left(\frac{\beta_j\hbar\nu}{2}\right)\right.\\ &\left.\times[\delta(\omega-\nu)+\delta(\omega+\nu)]+iP\left(\frac{1}{\omega-\nu}-\frac{1}{\omega+\nu}\right)\right]. \end{aligned} \tag{88}$$

(d) The Wightman-type Green's function

$$\begin{aligned} H^{+-}_{\alpha\beta,jj'}(\mathbf{r}_1,\mathbf{r}_2,\nu,\nu',\omega)=&-\frac{i\pi}{2\rho_{j\nu}}\delta_{\alpha\beta}\delta_{jj'}\delta(\nu-\nu')\,\delta(\mathbf{r}_1-\mathbf{r}_2)\cdot\left[\coth\left(\frac{\beta_j\hbar\nu}{2}\right)\right.\\ &\left.\times[\delta(\omega-\nu)+\delta(\omega+\nu)]+\delta(\omega-\nu)-\delta(\omega+\nu)\right]. \end{aligned} \tag{89}$$

(e) The anti-Wightman-type Green's function

$$\begin{aligned} H^{-+}_{\alpha\beta,jj'}(\mathbf{r}_1,\mathbf{r}_2,\nu,\nu',\omega)=&-\frac{i\pi}{2\rho_{j\nu}}\delta_{\alpha\beta}\delta_{jj'}\delta(\nu-\nu')\,\delta(\mathbf{r}_1-\mathbf{r}_2)\cdot\left[\coth\left(\frac{\beta_j\hbar\nu}{2}\right)\right.\\ &\left.\times[\delta(\omega-\nu)+\delta(\omega+\nu)]-\delta(\omega-\nu)+\delta(\omega+\nu)\right]. \end{aligned} \tag{90}$$

(f) The symmetrized Green's function

$$\begin{aligned} F^{(res)}_{\alpha\beta,jj'}(\mathbf{r}_1,\mathbf{r}_2,\nu,\nu',\omega)=&-\frac{i\pi}{\rho_{j\nu}}\delta_{\alpha\beta}\delta_{jj'}\delta(\nu-\nu')\,\delta(\mathbf{r}_1-\mathbf{r}_2)\cdot\coth\left(\frac{\beta_j\hbar\nu}{2}\right)\\ &\times[\delta(\omega-\nu)+\delta(\omega+\nu)]. \end{aligned} \tag{91}$$

These functions will be used in the following subsection for obtaining the self-energies and the dressed polarization Green's functions.

B. The dressed polarization-field Green's functions

Taking into account the form of the polarization-reservoir interaction, as specified by the Lagrangian (85), we formulate the following Feynman rules to get diagrammatic expansions of Green's functions.

(1) Any free polarization propagator, i.e., a correlation function of the polarization fields $\mathbf{X}_j(\mathbf{r})$, is represented by a dashed line.

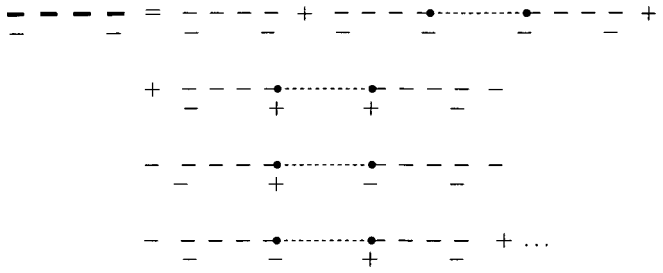
(2) Free reservoir correlations are represented by dotted lines.

(3) Since the interaction is bilinear, all graphs have a chainlike shape: the vertices connect only two lines.

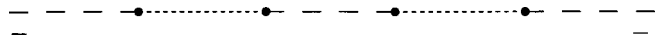
(4) Each vertex is associated with an index j and a factor $\rho_{j\nu}\nu^2$.

(5) To obtain a diagram, we integrate over ν and $\mathbf{r}$ for all internal lines, and sum over all internal vectorial indices.

For instance, the Feynman propagator K^{--} of the polarization field is represented, to second order, by the graph

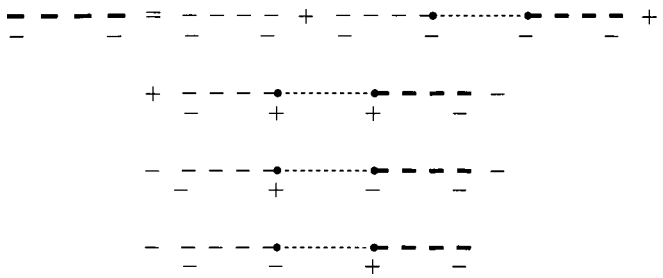


Here the bold dashed line denotes the dressed polarization propagator. With each vertex we have to associate one of the two signs, "+" or "−" [60]; each "+" vertex yields a factor "−1" to multiply the corresponding expression with. Therefore, in fourth order we obtain 16 similar linear graphs, each one containing three dashed lines and two dotted lines. They all share the elements



In addition, to each of the four vertices either "+" or "−" has to be assigned, resulting in 16 different graphs.

All graphs can be summed up exactly: They all are obtained by graphical iteration of the diagrammatic Dyson equation,



We conclude that the self-energies required for dressing the polarization-field Green's functions are easily expressed in terms of the reservoir Green's functions, namely,

$$\begin{pmatrix} \Sigma^{--} & \Sigma^{-+} \\ \Sigma^{+-} & \Sigma^{++} \end{pmatrix} = \begin{pmatrix} H^{--} & -H^{-+} \\ -H^{+-} & H^{++} \end{pmatrix}, \tag{92}$$

where we have omitted indices and arguments for simplicity of notation.

From the above matrix equation it follows that the expressions for the self-energies needed to compute the retarded, advanced, and symmetrized polarization propagators read

$$\Omega = \Omega^{res} = H^{--} + H^{++},$$

$$\Sigma^R = \Sigma^{R,res} = H^{--} - H^{-+},$$

$$\Sigma^A = \Sigma^{A,res} = H^{--} - H^{+-}. \tag{93}$$

Thus, we obtain the following Dyson equation for the retarded Green's function:

$$\begin{aligned} K^R_{\alpha\beta,jk}(\mathbf{r}_1,\mathbf{r}_2;t_1,t_2) &= K^{0,R}_{\alpha\beta,jk}(\mathbf{r}_1,\mathbf{r}_2;t_1,t_2) + \sum_{l,m} \int_{-\infty}^{\infty} dt_3 \int_{-\infty}^{\infty} dt_4 \int d^3r_3 \\ &\times \int d^3r_4 \int_0^{\infty} d\nu_3 \rho_{l\nu_3} \nu_3^2 \int_0^{\infty} d\nu_4 \rho_{m\nu_4} \nu_4^2 \\ &\times K^{0,R}_{\alpha\gamma,jl}(\mathbf{r}_1,\mathbf{r}_3;t_1,t_3) H^R_{\gamma\lambda,lm}(\mathbf{r}_3,\mathbf{r}_4;\nu_3,\nu_4;t_3,t_4) \\ &\times K^R_{\lambda\beta,mk}(\mathbf{r}_4,\mathbf{r}_2;t_4,t_2), \end{aligned} \tag{94}$$

where $K^{0,R}$ denotes the free retarded polarization propagators, and we have used the identity $H^{--} - H^{-+} = H^R$. In principle, the above Green's functions depend on two times t_1 and t_2 separately. However, as both $K^{0,R}$ and the self-energy Σ^R depend only on time differences, we find that the twofold Fourier transform of K^R, being a function of the two frequencies ω_1 and ω_2, contains the overall factor $\delta(\omega_1 + \omega_2)$. This implies that K^R actually is a function of $t_1 - t_2$ only, and the above Dyson equation can be solved by just one Fourier transformation with respect to $\tau = t_1 - t_2$. We obtain, therefore, a linear equation for K^R which, because of the simple structure of both $K^{0,R}$ and H^R, leads to the following solution:

$$K^R_{\alpha\beta,jk}(\mathbf{r}_1,\mathbf{r}_2;\omega) = \delta_{jk}\delta_{\alpha\beta}\delta(\mathbf{r}_1 - \mathbf{r}_2) K^R_j(\omega), \tag{95}$$

with

$$K^R_j(\omega) = \left[\mathcal{M}_j(\omega^2 - \omega_0^2) - \omega^2 \int_0^{\infty} d\nu \frac{\rho_{j\nu}\nu^2}{\omega^2 - \nu^2 + i\eta\omega} \right]^{-1}, \tag{96}$$

where the infinitesimal imaginary term in the denominator of the integrand stems from the retarded character of H^R.

This result can be compared with that is provided by the equations of motion. The retarded and advanced propagators do not depend on the initial states; they are given by certain inhomogeneous solutions of (integro-)differential equations. From the equations of motion for the polarization field coupled to the reservoir field we find that the retarded polarization propagators satisfy

$$\left(\frac{d^2}{dt^2}+\omega_{1j}^2\right)K^R_{\alpha\beta,jj'}(\mathbf{r}_1,\mathbf{r}_2,t,t') = \frac{1}{\mathcal{M}_j}\int_0^\infty d\nu\rho_{j\nu}\nu^2 f_{\alpha,j}(\mathbf{r}_1,\mathbf{r}_2,\nu,t,t'), \tag{97}$$

where the function $f_{\alpha,j}$ is defined as

$$f_{\alpha,j}(\mathbf{r}_1,\mathbf{r}_2,\nu,t,t') = -\frac{i}{\hbar}\theta(t-t') \times\langle[Y_{j\nu,\alpha}(\mathbf{r}_1,t),X_{j,\beta}(\mathbf{r}_2,t')]\rangle,$$

and

$$\omega_{1j}^2=\omega_{0j}^2+\frac{1}{\mathcal{M}_j}\int_0^\infty d\nu\rho_{j\nu}\nu^2.$$

Now the equation of motion for $f_{\alpha,j}$ reads

$$\left(\frac{d^2}{dt^2}+\nu^2\right)f_{\alpha,j}(\mathbf{r}_1,\mathbf{r}_2,\nu,t,t')=\nu^2K^R_{\alpha\beta,j,j'}(\mathbf{r}_1,\mathbf{r}_2,t,t'), \tag{98}$$

so that after taking the Fourier transform and solving for K^R, Eq. (96) is recovered. The infinitesimal imaginary term in the denominator of the integrand appears because $f_{\alpha,j}$ itself is a retarded correlation function.

Similar to Eq. (96), the advanced Green's function is given by

$$K^A_{\alpha\beta,jk}(\mathbf{r}_1,\mathbf{r}_2;\omega)=\delta_{jk}\delta_{\alpha\beta}\delta(\mathbf{r}_1-\mathbf{r}_2)K^A_j(\omega), \tag{99}$$

with

$$K^A_j(\omega)=\left[\mathcal{M}_j(\omega^2-\omega_0^2)-\omega^2\int_0^\infty d\nu\frac{\rho_{j\nu}\nu^2}{\omega^2-\nu^2-i\eta\omega}\right]^{-1}. \tag{100}$$

The third important Green's function, the symmetrized Green's function, satisfies the following Dyson equation:

$$\begin{aligned}F^{pol}_{\alpha\beta,jk}(\mathbf{r}_1,\mathbf{r}_2;t_1,t_2) &= \sum_{l,m}\int_{-\infty}^\infty dt_3\int_{-\infty}^\infty dt_4\int d^3r_3\int d^3r_4\int_0^\infty d\nu_3\rho_{j\nu_3}\nu_3^2\\ &\times\int_0^\infty d\nu_4\rho_{j\nu_4}\nu_4^2K^{0,R}_{\alpha\gamma,jl}(\mathbf{r}_1,\mathbf{r}_3;t_1,t_3)\\ &\times[\Omega^{pol}_{\gamma\lambda,lm}(\mathbf{r}_3,\mathbf{r}_4;\nu_3,\nu_4;t_3,t_4)K^A_{\lambda\beta,mk}(\mathbf{r}_4,\mathbf{r}_2,t_4,t_2)\\ &+H^R_{\gamma\lambda,lm}(\mathbf{r}_3,\mathbf{r}_4;\nu_3,\nu_4;t_3,t_4)F^{pol}_{\lambda\beta,mk}(\mathbf{r}_4,\mathbf{r}_2;t_4,t_2)].\end{aligned} \tag{101}$$

Once again, we realize that these Green's functions depend only on the difference $\tau=t_1-t_2$. On taking the Fourier transform, we find the solution in the form

$$F^{pol}_{\alpha\beta,jk}(\mathbf{r}_1,\mathbf{r}_2;\omega)=\delta_{jk}\,\delta_{\alpha\beta}\delta(\mathbf{r}_1-\mathbf{r}_2)F^{pol}_j(\omega), \tag{102}$$

where

$$F^{pol}_j(\omega)=-\pi iK^R_j(\omega)\rho_{j\omega}\omega^3\coth\left(\frac{\beta_j\hbar\omega}{2}\right)K^A_j(\omega). \tag{103}$$

The above correlation functions can now be used to obtain the dressed Green's functions K^{--}, K^{+-}, K^{-+}, and K^{++}. More importantly, they will be employed in the following section to derive the Green's functions of the electromagnetic field.

C. The correlation functions of the electromagnetic field

The correlation functions of the electromagnetic field are again obtained from the general formulas (69)–(77), with the electromagnetic potential operator A_α replacing the general field operator ψ. The correlation functions of the free electromagnetic field which are most important for our purposes, namely, the retarded, the advanced, and the symmetrized correlation function, satisfy the following wave equations [64]:

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\hat{D}^{R,A}(\mathbf{r},\mathbf{r}';\omega)-\frac{\omega^2}{c^2}\hat{D}^{R,A}(\mathbf{r},\mathbf{r}';\omega)=-\mu_0\delta(\mathbf{r}-\mathbf{r}'), \tag{104}$$

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\hat{F}^{EM}(\mathbf{r},\mathbf{r}';\omega)-\frac{\omega^2}{c^2}\hat{F}^{EM}(\mathbf{r},\mathbf{r}';\omega)=0. \tag{105}$$

In the latter equation the superscript "EM" indicates that the symmetrized propagator $\hat{F}^{EM}$ pertains to the electromagnetic field. The retarded and the advanced Green's functions (as well as the Feynman and anti-Feynman Green's functions) satisfy one and the same equation and differ only in the boundary conditions. We will now construct the dressed correlation functions with the help of the diagrammatic technique.

Since the interaction Lagrangian has the form

$$L_{int}=\sum_{j=1}^2\int d^3r g_j(\mathbf{r})\mathbf{V}_j(\mathbf{r})\cdot\mathbf{A}(\mathbf{r}),$$

we infer that in the graphical expansion every vertex will join two lines only, and the only connected graphs are linear ones. The perturbation expansion involves the correlations functions of the velocities $\mathbf{V}_j$, but not those of the "displacements" $\mathbf{X}_j$. However, all the two-point Green's functions defined in terms of velocities can be obtained from those defined in terms of displacements simply by differentiation with respect to the two time variables involved:

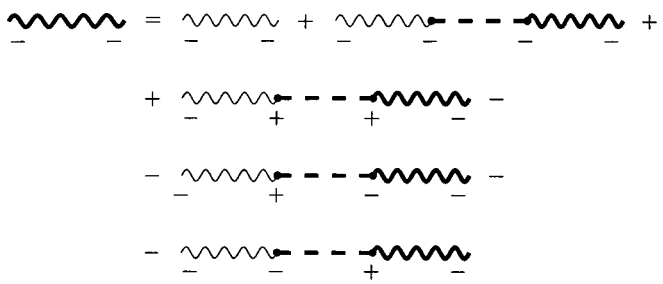
$$K_{VV}(t_1,t_2)=\frac{\partial^2}{\partial t_1\partial t_2}K_{XX}(t_1,t_2), \tag{106}$$

where K_{VV} denotes any of the dressed "velocity" Green's functions of the polarization field (that is, K^{--}, K^{-+}, etc.), and K_{XX} is the corresponding "displacement" Green's function. In the language of the Fourier transform, this means that

$$K_{VV}(\omega)=\omega^2 K_{XX}(\omega). \tag{107}$$

Thus, when building the graphs to obtain the electromagnetic Green's function, we must only remember that every dressed polarization line should be multiplied by ω^2 when taking the Fourier transform.

Drawing the graphs for the electromagnetic correlation functions is fully analogous to that same procedure for the polarization Green's functions: we have four graphs in the second order, 16 graphs in fourth order, etc. Moreover, one has to associate with every vertex the factor $g_j(\mathbf{r})$, and one of the two signs. To compute the graphs, there is no need to integrate over the ν variables, as the lines are already ν independent, but now there is an additional summation over j. It is easy to realize that all the graphs can be obtained by iteration of the following graphical Dyson equation:



The undressed (dressed) photon propagators are denoted here by thin (bold) wavy lines. We then find that the self-energies, considered as functions of ω, are given by

$$\begin{pmatrix} \Sigma^{--} & \Sigma^{-+} \\ \Sigma^{+-} & \Sigma^{++} \end{pmatrix} = \omega^2 \begin{pmatrix} K^{--} & -K^{-+} \\ -K^{+-} & K^{++} \end{pmatrix}, \tag{108}$$

where every K is to be understood as K_{XX}. (We use the same symbols here to denote the self-energies as in our Dyson equation for polarization propagators, hoping that this will not lead to any misunderstanding.) This equation also implies

$$\Sigma^{R,A}_{\alpha\beta,jk}(\mathbf{r}_1,\mathbf{r}_2;\omega)=\omega^2 K_j^{R,A}(\omega)\,\delta_{\alpha\beta}\delta_{jk}\delta(\mathbf{r}_1-\mathbf{r}_2), \tag{109}$$

$$\Omega_{\alpha\beta,jk}(\mathbf{r}_1,\mathbf{r}_2;\omega)=\omega^2 F_j^{pol}(\omega)\,\delta_{\alpha\beta}\delta_{jk}\delta(\mathbf{r}_1-\mathbf{r}_2). \tag{110}$$

The Dyson equation for the retarded and advanced Green's functions becomes quite simple after Fourier transformation with respect to time:

$$\begin{aligned} D^{R,A}_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2;\omega)=D^{0;R,A}_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2;\omega)+\omega^2\sum_j \int d^3r_3 g_j^2(\mathbf{r}_3) \\ \times D^{0;R,A}_{\alpha\gamma}(\mathbf{r}_1,\mathbf{r}_3;\omega)K_j^{R,A}(\omega) \\ \times D^{R,A}_{\gamma\beta}(\mathbf{r}_3,\mathbf{r}_2;\omega). \end{aligned} \tag{111}$$

On the other hand, the Dyson equation for the symmetrized electromagnetic propagator reads

$$\begin{aligned} F^{EM}_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2;\omega)=\sum_j \int d^3r_3 g_j^2(\mathbf{r}_3) D^{0,R}_{\alpha\gamma}(\mathbf{r}_1,\mathbf{r}_3;\omega)\cdot\omega^2 \\ \times[F_j^{pol}(\omega)D^A_{\gamma\beta}(\mathbf{r}_3,\mathbf{r}_2;\omega) \\ +K_j^R(\omega)F^{EM}_{\lambda\beta}(\mathbf{r}_3,\mathbf{r}_2;\omega)]. \end{aligned} \tag{112}$$

Next, applying the differential operator $\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times-\omega^2/c^2$ to both sides of Eqs. (111) and (112), we obtain

$$\begin{aligned} \left(\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times-\frac{\omega^2}{c^2}\right)\hat{D}^{R,A}(\mathbf{r}_1,\mathbf{r}_2;\omega) \\ =-\mu_0\delta(\mathbf{r}_1-\mathbf{r}_2)-\mu_0\omega^2\sum_j g_j^2(\mathbf{r}_1)K_j^{R,A}(\omega) \\ \times\hat{D}^{R,A}(\mathbf{r}_1,\mathbf{r}_2;\omega) \end{aligned} \tag{113}$$

and

$$\begin{aligned} \left(\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times-\frac{\omega^2}{c^2}\right)\hat{F}^{EM}(\mathbf{r}_1,\mathbf{r}_2;\omega) \\ =-\mu_0\omega^2\sum_j g_j^2(\mathbf{r}_1)[F_j^{pol}(\omega)\hat{D}^A(\mathbf{r}_1,\mathbf{r}_2;\omega) \\ +K_j^R(\omega)\hat{F}^{EM}(\mathbf{r}_1,\mathbf{r}_2;\omega)]. \end{aligned} \tag{114}$$

From Eq. (113) we then recover the refreshingly well-known differential equations for the dressed retarded and advanced Green's functions, namely,

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\hat{D}^R(\mathbf{r},\mathbf{r}',\omega)-\frac{\omega^2}{c^2}\boldsymbol{\epsilon}(\mathbf{r},\omega)\hat{D}^R(\mathbf{r},\mathbf{r}',\omega)=\mu_0\delta(\mathbf{r}-\mathbf{r}'), \tag{115}$$

together with

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\hat{D}^A(\mathbf{r},\mathbf{r}',\omega)-\frac{\omega^2}{c^2}\overline{\boldsymbol{\epsilon}}(\mathbf{r},\omega)\hat{D}^A(\mathbf{r},\mathbf{r}',\omega)=\mu_0\delta(\mathbf{r}-\mathbf{r}'), \tag{116}$$

where

$$\boldsymbol{\epsilon}(\mathbf{r},\omega)=1-\frac{1}{\epsilon_0}\sum_j g_j^2(\mathbf{r})K_j^R(\omega) \tag{117}$$

has no singularities in the upper ω half plane, whereas

$$\overline{\boldsymbol{\epsilon}}(\mathbf{r},\omega)=1-\frac{1}{\epsilon_0}\sum_j g_j^2(\mathbf{r})K_j^A(\omega) \tag{118}$$

has no singularities in the lower ω half plane. Similarly, Eq. (114) leads to

$$\begin{aligned} F^{EM}_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2;\omega)=\omega^2\sum_j \int d^3r_3 g_j^2(\mathbf{r}_3)D^R_{\alpha\gamma}(\mathbf{r}_1,\mathbf{r}_2;\omega) \\ \times F_j^{pol}(\omega)D^A_{\gamma\beta}(\mathbf{r}_3,\mathbf{r}_2;\omega). \end{aligned} \tag{119}$$

Exploiting the fact that the Wightman-type Green's function $\hat{D}^{+-}$ is related to these propagators through the formula

$$\hat{D}^{+-}=\frac{1}{2}(F^{EM}+D^R-D^A), \tag{120}$$

Eqs. (111) and (112) yield the wave equation for $\hat{D}^{+-}$:

$$\nabla\times\nabla\times\hat{D}^{+-}(\mathbf{r},\mathbf{r}';\omega)-\frac{\omega^2}{c^2}\hat{D}^{+-}(\mathbf{r},\mathbf{r}';\omega)$$
$$=-\mu_0\omega^2\sum_j g_j^2(\mathbf{r}_1)[K_j^R(\omega)\hat{D}^{+-}(\mathbf{r}_1,\mathbf{r}_2;\omega)$$
$$+K_j^{+-}(\omega)\hat{D}^A(\mathbf{r}_1,\mathbf{r}_2,\omega)], \tag{121}$$

where, in analogy to Eq. (120),

$$K_j^{+-}(\omega)=\frac{1}{2}[F_j^{pol}(\omega)+K_j^R(\omega)-K_j^A(\omega)]. \tag{122}$$

This wave equation (121) can be solved with the help of the retarded Green's function, so that we finally obtain

$$\hat{D}^{+-}(\mathbf{r}_1,\mathbf{r}_2,\omega)=\omega^2\sum_j\int d^3r_3 g_j^2(\mathbf{r}_3)\hat{D}^R(\mathbf{r}_1,\mathbf{r}_3;\omega)$$
$$\times K_j^{+-}(\omega)\hat{D}^A(\mathbf{r}_3,\mathbf{r}_2;\omega). \tag{123}$$

For the sake of comparison, we shall also employ additional auxiliary electric ($\hat{D}^E$) and magnetic ($\hat{D}^M$) Green's functions of the retarded and advanced type which correspond directly to the Green's functions used in the pioneering work by Polder and van Hove [9]. They satisfy the following differential equations:

$$\nabla\times\nabla\times D^{E,R}(\mathbf{r},\mathbf{r}',\omega)-\frac{\omega^2}{c^2}\epsilon(\mathbf{r},\omega)D^{E,R}(\mathbf{r},\mathbf{r}',\omega)$$
$$=i\omega\delta(\mathbf{r}-\mathbf{r}'),$$

$$\nabla\times\nabla\times D^{E,A}(\mathbf{r},\mathbf{r}',\omega)-\frac{\omega^2}{c^2}\overline{\epsilon}(\mathbf{r},\omega)D^{E,A}(\mathbf{r},\mathbf{r}',\omega)$$
$$=-i\omega\delta(\mathbf{r}-\mathbf{r}'),$$

$$\nabla\times\nabla\times D^{M,R}(\mathbf{r},\mathbf{r}',\omega)-\frac{\omega^2}{c^2}\epsilon(\mathbf{r},\omega)D^{M,R}(\mathbf{r},\mathbf{r}',\omega)$$
$$=\nabla\times\mathbf{1}\delta(\mathbf{r}-\mathbf{r}'),$$

$$\nabla\times\nabla\times D^{M,A}(\mathbf{r},\mathbf{r}',\omega)-\frac{\omega^2}{c^2}\overline{\epsilon}(\mathbf{r},\omega)D^{M,A}(\mathbf{r},\mathbf{r}',\omega)$$
$$=\nabla\times\mathbf{1}\delta(\mathbf{r}-\mathbf{r}'). \tag{124}$$

From Eqs. (115) and (116) we then have the relations

$$D^R(\mathbf{r},\mathbf{r}';\omega)=\frac{i\mu_0}{\omega}D^{E,R}(\mathbf{r},\mathbf{r}';\omega),$$

$$D^A(\mathbf{r},\mathbf{r}';\omega)=-\frac{i\mu_0}{\omega}D^{E,R}(\mathbf{r},\mathbf{r}';\omega),$$

$$\nabla\times D^R(\mathbf{r},\mathbf{r}';\omega)=-\mu_0 D^{M,R}(\mathbf{r},\mathbf{r}';\omega),$$

$$\nabla\times D^A(\mathbf{r},\mathbf{r}';\omega)=-\mu_0 D^{M,A}(\mathbf{r},\mathbf{r}';\omega) \tag{125}$$

between the various Green's functions. It is also obvious that there are trivial relations between $D^{E,R}$, $D^{E,A}$, $D^{M,R}$, and $D^{M,A}$ on the one hand, and the Green's functions D^E and D^M depending on the Laplace variable as introduced in the section on the other (we omit spatial arguments):

$$D^{E,R}(\omega)=D^E(s=-i\omega+\eta),$$

$$D^{E,A}(\omega)=D^E(s=i\omega+\eta),$$

$$D^{M,R}(\omega)=D^M(s=-i\omega+\eta),$$

$$D^{M,A}(\omega)=D^M(s=i\omega+\eta), \tag{126}$$

where η is an infinitesimal positive constant which guarantees the correct treatment of the poles in the complex ω plane.

V. CALCULATION OF THE POYNTING VECTOR BY MEANS OF THE DIAGRAMMATIC EXPANSION

The "α" component of the Poynting vector can be obtained from the above symmetrized and retarded propagators, if we manage to express the normally ordered correlation function in terms of them. This is by no means trivial. Let us start with the definition

$$\langle S_\alpha(\mathbf{r},t)\rangle=\epsilon_{\alpha\beta\gamma}\lim_{\mathbf{r}'\to\mathbf{r}}\lim_{t'\to t}\langle:E_\beta(\mathbf{r},t)H_\gamma(\mathbf{r}',t'):\rangle. \tag{127}$$

If we decompose the electric and magnetic fields into their positive and negative frequency parts,

$$\psi=\psi^{(p)}+\psi^{(n)},$$

where ψ is any of the operators E_β and H_γ, and the superscripts (n) and (p) denote the negative- and positive-frequency parts of an operator [instead of "$(-)$" and "$(+)$" superscripts, in order to avoid confusion with "$+-$" and "$-+$" superscripts labeling particular Green's functions], we find

$$\langle:E_\beta(\mathbf{r},t)H_\gamma(\mathbf{r}',t'):\rangle$$
$$=\langle E_\beta(\mathbf{r},t)H_\gamma(\mathbf{r}',t')\rangle+\langle H_\gamma^{(n)}(\mathbf{r}',t')E_\beta^{(p)}(\mathbf{r},t)\rangle$$
$$-\langle E_\beta^{(p)}(\mathbf{r},t)H^{(n)}(\mathbf{r}',t')\rangle. \tag{128}$$

As our diagrammatic expansion involves the electromagnetic vector potential, but not the fields themselves, we cast the expression for the normally ordered product of fields into the form

$$\langle :E_\beta(\mathbf{r},t)H_\gamma(\mathbf{r}',t'):\rangle = -\frac{i\hbar}{\mu_0}\epsilon_{\gamma\lambda\mu}\frac{\partial}{\partial t}\nabla'_\lambda\{D^{+-}_{\beta\mu}(\mathbf{r},\mathbf{r}';t,t') + D^{(N)}_{\mu\beta}(\mathbf{r}',\mathbf{r};t',t) - D^{(A)}_{\beta\mu}(\mathbf{r},\mathbf{r}';t,t')\}, \tag{129}$$

where

$$D^{(N)}_{\mu\beta}(\mathbf{r}',\mathbf{r};t',t) = -\frac{i}{\hbar}\langle A^{(n)}_\mu(\mathbf{r}',t')A^{(p)}_\beta(\mathbf{r},t)\rangle,$$

$$D^{(A)}_{\beta\mu}(\mathbf{r},\mathbf{r}';t,t') = -\frac{i}{\hbar}\langle A^{(p)}_\mu(\mathbf{r},t)A^{(n)}_\beta(\mathbf{r}',t')\rangle. \tag{130}$$

We already know that our Green's functions depend on the difference $t-t'$ only. This means that the above Green's functions allow for a Fourier representation containing just one frequency. Taking care of the fact that in $D^{(N)}$ the sequence of its time arguments is inverted, we can express the above normally ordered product of the electric- and magnetic-field operators as

$$\langle :E_\beta(\mathbf{r},t)H_\gamma(\mathbf{r}',t):\rangle = -\frac{\hbar}{2\pi\mu_0}\epsilon_{\gamma\lambda\mu}\nabla'_\lambda\int_{-\infty}^{\infty}d\omega\,\omega[D^{+-}_{\beta\mu}(\mathbf{r},\mathbf{r}';\omega) - D^{(N)}_{\mu\beta}(\mathbf{r}',\mathbf{r};\omega) - D^{(A)}_{\beta\mu}(\mathbf{r},\mathbf{r}';\omega)], \tag{131}$$

where the time differentiation has already been performed, and the limit $t'\to t$ has been taken.

The way to extract the positive- and negative-frequency parts of an electromagnetic field operator without employing a mode decomposition has already been shown by Schwinger [65]. With the help of Schwinger's method, Agarwal [13] has expressed $\hat{D}^{(N)}(\omega)$ and $\hat{D}^{(A)}(\omega)$ in terms of $D^{+-}(\omega)$:

$$\hat{D}^{(N)}(\mathbf{r},\mathbf{r}';\omega) = \theta(-\omega)\hat{D}^{+-}(\mathbf{r},\mathbf{r}';\omega),$$

$$\hat{D}^{(A)}(\mathbf{r},\mathbf{r}';\omega) = \theta(\omega)\hat{D}^{+-}(\mathbf{r},\mathbf{r}';\omega). \tag{132}$$

Inserting these expressions into our solution for $D^{+-}(\mathbf{r},\mathbf{r}';\omega)$, we arrive at

$$\langle :E_\beta(\mathbf{r},t)H_\gamma(\mathbf{r}',t):\rangle = -\frac{\hbar}{2\pi\mu_0}\epsilon_{\gamma\lambda\mu}\sum_j\int d^3r_1 g_j^2(\mathbf{r}_1)\int_{-\infty}^{\infty}d\omega\,\omega^3 \times\theta(-\omega)K^{+-}(\omega)\cdot[D^R_{\beta\rho}(\mathbf{r},\mathbf{r}_1;\omega)D^A_{\rho\mu}(\mathbf{r}_1,\mathbf{r}';\omega) - D^R_{\mu\rho}(\mathbf{r}',\mathbf{r}_1;\omega)D^A_{\rho\beta}(\mathbf{r}_1,\mathbf{r};\omega)]. \tag{133}$$

The presence of the Heaviside function now allows us to restrict the integration to positive (or negative) frequencies. Let us note that this restriction to positive frequencies in the integration over ω has not been justified in either Ref. [9] or Ref. [10]; though physically plausible, it is rather difficult to understand from the mathematical viewpoint in these papers. Here this restriction is a result, rather than an assumption, and it is a direct consequence of the fact that the Poynting vector is defined in terms of the normally ordered product of operators.

Now we change the integration variable from ω to $-\omega$, and use the fact that $D^A(\omega)=D^R(-\omega)$ and $D^R(\omega)=D^A(-\omega)$. Then relations (125) and (126) allow us to express the expectation value of the Poynting vector in terms of the auxiliary electric and magnetic functions introduced in Eq. (124):

$$\langle S_\alpha(\mathbf{r},t)\rangle = \frac{i\hbar}{2\pi}\epsilon_{\alpha\beta\gamma}\sum_j\int d^3r_1\int_0^\infty d\omega\,\omega^2 g_j^2(\mathbf{r}_1)\,K_j^{+-}(-\omega) \times[D^{E,A}_{\beta\rho}(\mathbf{r},\mathbf{r}_1;\omega)D^{M,R}_{\gamma\rho}(\mathbf{r},\mathbf{r}_1;\omega) + D^{M,A}_{\gamma\rho}(\mathbf{r},\mathbf{r}_1;\omega)D^{E,R}_{\beta\rho}(\mathbf{r},\mathbf{r}_1;\omega)]. \tag{134}$$

Here we have used the reciprocity relations $D_{\mu\nu}(\mathbf{r},\mathbf{r}';\omega)=D_{\nu\mu}(\mathbf{r}',\mathbf{r};\omega)$, which hold for the electromagnetic Green's functions satisfying the wave equations of the preceding section, and have taken the limit $\mathbf{r}'\to\mathbf{r}$.

Finally, we make a simple calculation to express the imaginary part of the dielectric function of the jth body in terms of $K^{+-}(\omega)$ (in a manner fully analogous to the procedure in Sec. III) and obtain, with the help of Eqs. (96), (99), and (102),

$$K^{+-}(-\omega) = -2i\,\epsilon_0\mathrm{Im}[\epsilon_j(\omega)][\exp(\beta_j\hbar\omega)-1]^{-1}. \tag{135}$$

Using this, our final solution for the Poynting vector, i.e.,

$$\langle S_\alpha(\mathbf{r},t)\rangle = \frac{\hbar}{\pi c^2}\epsilon_{\alpha\beta\gamma}\sum_j\int d^3r_1\int_0^\infty d\omega\,\omega^2 g_j^2(\mathbf{r}_1)\mathrm{Im}[\epsilon_j(\omega)] \times[\exp(\beta_j\hbar\omega)-1]^{-1}[D^{E,A}_{\beta\rho}(\mathbf{r},\mathbf{r}_1;\omega) \times D^{M,R}_{\gamma\rho}(\mathbf{r},\mathbf{r}_1;\omega)+D^{M,A}_{\gamma\rho}(\mathbf{r},\mathbf{r}_1;\omega) \times D^{E,R}_{\beta\rho}(\mathbf{r},\mathbf{r}_1;\omega)], \tag{136}$$

is, in view of Eq. (126), identical to the expression (66) provided by the equations-of-motion approach.

It is obvious by now that for the present linear model the equations-of-motion approach is more straightforward, and by far simpler than the Keldysh formalism. The algebra involved in the latter, while not complicated, turns out to be somewhat tiring. However, as soon as the physics forces one to introduce any nonlinearity into the model, it is the Keldysh formalism which provides a fairly natural framework for the perturbation expansion. In that case the operator equations of motion lose much of their value, as physically reasonable approximations are, in general, difficult to implement, and the Laplace transformation ceases to work.

VI. SUMMARY AND CONCLUDING REMARKS

In this paper we have developed a quantum approach to the problem of electromagnetic energy transfer between dispersive and absorptive dielectric bodies. Using the Schwinger quantum action principle, we have derived the equations of motion for the electromagnetic field operators, and for two auxiliary kinds of quantum fields: the polarization field and the reservoir field. We have then performed the Laplace transformation and expressed the Poynting vector, which measures the electromagnetic energy flux between the bodies, in terms of the correlation functions of the initial values of the fields involved. For sufficiently large times the initial correlations of the polarization field have died out, and the remaining contribution stems from fluctuating currents in the media caused by the stochastic forces associated with the presence of reservoirs. It does depend on both their dielectric properties and their temperatures.

As an alternative to this approach based on the equations of motion, we have also demonstrated that the nonequilibrium diagrammatic technique due to Keldysh can be employed to solve the problem of the radiative heat transfer. We have calculated a large number of Green's functions for the reservoir and for the polarization fields and shown that the electromagnetic correlation functions can be found almost exactly, in the sense that the calculation of these quantities can be reduced—via a set of Dyson equations—to the solution of a partial differential equation with suitable continuity (and, possibly, boundary) conditions. This partial differential equation allows one to find the retarded and advanced Green's functions of the electromagnetic field. *Both* these Green's functions are necessary (and sufficient) to obtain the required expectation value of the Poynting vector. This latter result follows from both the equations of motions and the diagrammatic Keldysh approach.

We have shown that both quantum approaches lead to the same result for the Poynting vector, as expressed by Eqs. (66) and (136). This result coincides with a well-known formula provided by the classical theory of electromagnetic fluctuations [11], which has been taken for granted in the earlier works [9,10]. Nonetheless, what we have done in the present study amounts to more than a mere, quite arduous quantum-mechanical rederivation of the classical formulas. Our treatment not only proves that previous classical approaches have been free of internal inconsistencies, but it also specifies the conditions under which the ''classical'' result holds. Formulas (66) and (136) assume normal ordering of the field operators entering the Poynting vector, and require reservoir fields coupled to the polarization fields at each point of the dielectric bodies. If these propositions are satisfied in an actual experimental setup, the heat flux is guaranteed to be described by the theory developed in this paper. If, therefore, serious discrepancies between measured data and Eq. (66) or (136) should be found, this could not be ascribed to an intrinsically inconsistent theory, but would inevitably point to an imperfect realization of the underlying model. For instance, one might well ask whether the small tip of a typical scanning thermal microscope really serves as a reservoir in the sense assumed here. Comparison of experimental data with what follows from Eq. (66) or (136) for microscope geometries will therefore be of crucial importance for the further development of a quantitative theory.

More generally speaking, the above considerations fit into the framework of macroscopic quantum electrodynamics. Our model of the dielectric is not a ''realistic'' one; its purpose is to quantize the electromagnetic field inside lossy and dispersive dielectrics. It does, however, allow for modification in several directions. On the one hand, one could, e.g., include a higher-order polynomial polarization Lagrangian to simulate nonlinear properties of the dielectric. On the other hand, it is, in principle, also possible to use realistic rather than phenomenological models to describe atoms which constitute the dielectric surfaces—the perturbative treatment based on the Keldysh diagrams will then again provide a convenient route to obtaining good approximations. Finally, one might also change the strategy and calculate the current correlation functions in the dielectric, instead of the Poynting vector.

In a forthcoming paper we are going to apply the results of this work to several geometrical arrangements of interest.

ACKNOWLEDGMENTS

One of us (M.J.) gratefully acknowledges financial support from the Alexander von Humboldt Foundation. The authors would like to thank Professor Jürgen Parisi and Dr. Achim Kittel for fruitful discussions and important bibliographical directions. M.J. also thanks Professor Daniel Boyanovsky and Professor Chris van Weert for their kind explanations of certain details concerning the Keldysh formalism.

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