# Joint measurement of photon-number sum and phase-difference operators in a two-mode field

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(Received 25 June 2003; published 14 October 2003)

We present an experimental scheme that realizes joint measurement of photon-number sum and phasedifference operators on a two-mode field. The proposed scheme only involves linear optical elements and photon detectors with single-photon sensitivity. Furthermore, we demonstrate that such a measurement setup can be applied to generate two-mode *N*-photon entangled states from a pair of squeezed vacuum states. These *N*-photon entangled states are useful resources for quantum-information processing, high-precision frequency measurement, and quantum optical lithography.

DOI: 10.1103/PhysRevA.68.043819

PACS number(s): 42.50.Dv, 42.50.Ar, 42.50.St

#### I. INTRODUCTION

An important problem in quantum theory is the quantummechanical description of the phase of the radiation field [1]. Various ingenious ways have been explored in order to understand the quantum nature of phase of a single-mode bosonic field [2-4,6]. First, some of these approaches were motivated by the aim of expressing phase as the complement of the photon number [2], and we mention particularly the work of Pegg and Barnett [3] on the limit approach based on a finite Hilbert space. A second conception of quantum phase is based on examining quantum phase probability distribution from the s-parametrized quasiprobability distribution functions [4]. In Ref. [5], several schemes have been proposed to measure canonical quantum phase probability distribution function of single-mode field. The third conception, the operational approach to quantum phase by Noh et al. started with an analysis of what is usually measured in classical optics when the phase difference is to be determined, and then translated the formalism into the quantum domain [6]. Although all experimental tests of operational approach so far have led to a good agreement between theory and experiment [6], this operational approach has led to the conclusion that there is no unique phase operator, but that different measurement schemes correspond to different operators.

Now it has been recognized that the absence of a proper phase operator in single-mode case is mainly due to the semiboundedness of spectrum of the number operator [7]. This motivated an intensive research in finding the operator corresponding to the phase difference of two-mode fields [8,9]. In the two-mode case, the conjugate variable to the phase difference is the number difference, which is not bounded from below. So, it is reasonable to expect that the phase difference will be free of the problems arising in the single-mode case. In Ref. [8], Luis *et al.* introduced the Hermitian phase-difference operator

$$\Phi_{12} = \sum_{N=0}^{\infty} \sum_{r=0}^{N} \phi_r^N |\phi_r^N\rangle \langle \phi_r^N| \tag{1}$$

 $|\phi_r^N\rangle = \frac{1}{\sqrt{N+1}} \sum_{n=0}^N e^{in\phi_r^N} |n\rangle_1 |N-n\rangle_2, \qquad (2)$ 

$$\phi_r^N = \theta + \frac{2\pi r}{N+1},\tag{3}$$

where  $|n\rangle$  denotes the *n*-photon Fock state and  $\theta$  is an arbitrary angle. It is obvious that the phase-difference operator  $\hat{\Phi}_{12}$  commutes with the total photon-number operator  $\hat{n}$  $=\hat{n}_1+\hat{n}_2$ , where  $\hat{n}_1$  and  $\hat{n}_2$  are the photon-number operators for each mode. Therefore, the joint measurement of the photon-number sum and phase-difference operators on twomode field  $|\Psi\rangle$  projects the quantum state into the state  $|\phi_r^N\rangle$ . The success probability of the measurement is  $|\langle \Psi | \phi_r^N \rangle|^2$ , which is the joint probability distribution function for the total number and the phase difference [8]. In Ref. [10], it is shown that the joint measurement of the photonnumber sum and phase-difference operators play a role of Bell measurement in quantum teleportation of photonnumber states. In this paper, we show that the joint measurement of the photon-number sum and phase-difference operators can be used to generate two-mode N-photon entangled states of the form

$$\sum_{n=0}^{N} C_{n} |n\rangle |N-n\rangle \tag{4}$$

from a pair of squeezed vacuum states. Here  $C_n$  are arbitrary complex parameters. If the amplitudes of all the parameters  $C_n$  are equal, states (4) are maximally entangled state, which are useful resources in quantum teleportation processing [10]. If the amplitudes of parameters  $C_0$  and  $C_N$  are equal to  $1/\sqrt{2}$  and other parameters are zero, states (4) are reduced to

$$\frac{1}{\sqrt{2}}(|0\rangle|N\rangle + e^{i\varphi}|N\rangle|0\rangle), \tag{5}$$

which have been used to improve the sensitivity of interferometric measurements [11] and form a key ingredient of quantum optical lithography [12]. Recently several experimental setups for the generation of two-mode *N*-photon entangled states have been suggested [13]. But these schemes require

with

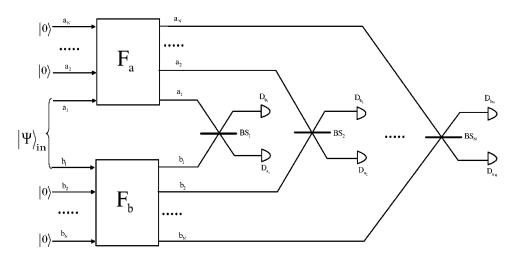


FIG. 1. Schematic experimental setup for joint measurement of photon-number sum and phasedifference operators on two field modes  $|\Psi_{in}\rangle$ , which includes two symmetric *N*-port devices  $F_a$  and  $F_b$ , *N* beam splitters BS<sub>i</sub>, and 2*N* photon detectors  $D_{a_i}$  and  $D_{b_i}$ .

many single-photon resources as auxiliary resources. Triggered single-photon sources operate by means of fluorescence from a single molecule [14] or a single quantum dot [15,16] with terrible spatial properties. With the currently available technology of single-photon resources, the requirements of these schemes are difficult to satisfy.

The paper is organized as follows. In Sec. II, we present an experimental setup to implement the joint measurement of the photon-number sum and phase-difference operators on the two-mode fields with linear optical elements and photon detectors. Furthermore, we demonstrate that the experimental setup can measure quantum state overlaps of one twomode radiation field and one arbitrary two-mode *N*-photon entangled state (4) by appropriately choosing the parameters of the setup. In Sec. III, we show that the measurement setup proposed in Sec. II can be used to generate the two-mode *N*-photon entangled state (4) from a pair of squeezed vacuum states. In addition, a simple scheme is proposed for the generation of the entangled state (5). Finally, the conclusion are given in Sec. IV.

## II. LINEAR OPTICAL IMPLEMENTATION OF PHOTON-NUMBER SUM AND PHASE-DIFFERENCE OPERATORS ON A TWO-MODE FIELD

Our main goal in this section is to present a scheme that can measure the joint probability distribution function for the total photon number and the phase difference on one twomode field of the form

$$|\Psi_{in}\rangle = \sum_{n,m=0}^{\infty} C_{n,m} |n\rangle_{a_1} |m\rangle_{b_1}, \qquad (6)$$

where  $C_{n,m}$  are arbitrary complex parameters. The experimental setup is depicted in Fig. 1, which consists of two symmetric *N*-port devices  $F_a$  and  $F_b$ , *N* beam splitters, and 2*N* photon detectors. An extended introduction to the symmetric multiport device is given in Ref. [17]. The action of the symmetric *N*-port device can be described by the unitary operator  $U^N$ . The matrix element of  $U^N$  is given by

$$U_{ij}^{N} = \frac{1}{\sqrt{N}} \gamma_{N}^{(i-1)(j-1)}, \qquad (7)$$

where  $\gamma_N = \exp(i2\pi/N)$  and indices *i* and *j* denote the input and the exit port, respectively. The matrix element  $U_{ij}^N$  gives the probability amplitude for a single photon entering via input *i* and leaving the device by output *j* (*i*,*j*=1,...,*N*). Reck *et al.* [18] have shown that it is possible to construct a multiport device from mirrors, beam splitters, and phase shifters that will transform the input modes into the output modes in accord with any  $N \times N$  unitary matrix.

Now we present a detailed analysis of the proposed scheme shown in Fig. 1. We assume that the mode  $a_1$  of state (6) is mixed with the N-1 vacuum modes  $a_2, \ldots, a_N$  at the symmetric N-port device  $F_a$ , while the mode  $b_1$  of state (6) is mixed with the N-1 vacuum modes  $b_2, \ldots, b_N$  at the symmetric N-port device  $F_b$ . After these modes pass through two symmetric N-port devices  $F_a$  and  $F_b$ , the state of the system becomes

$$U_a^N U_b^N |\Psi_{in}\rangle \prod_{j=2}^N |0\rangle_{a_j} |0\rangle_{b_j}.$$
 (8)

Then the modes  $a_i$  and  $b_i$  are mixed at a beam splitters  $BS_i$  $(i=1,\ldots,N)$ . The action of the beam splitters  $BS_i$  is described by the unitary operator

$$U_i^{bs} = \exp[\theta_i(a_i b_i^{\dagger} e^{\varphi_i} - a_i^{\dagger} b_i e^{-\varphi_i})], \qquad (9)$$

where parameters  $\theta_i$  and  $\varphi_i$  characterize the beam splitter BS<sub>i</sub>, which will be determined later. After these photon modes passing through beam splitters, we can obtain the state of the system:

$$\left(\prod_{i=1}^{N} U_{i}^{bs}\right) U_{a}^{N} U_{b}^{N} |\Psi_{in}\rangle \prod_{j=2}^{N} |0\rangle_{a_{j}} |0\rangle_{b_{j}}.$$
 (10)

Now let the photodetectors  $PD_{a_i}$  and  $PD_{b_i}$  measure the photon numbers in the modes  $a_i$  and  $b_i$  ( $i=1,\ldots,N+1$ ), respectively. Consider the case where one photon is detected in the detectors  $PD_{a_i}$  and the detectors  $PD_{b_i}$  do not detect any

photon (i = 1, ..., N). This means that all detectors can distinguish between zero photon, one photon, and more than one photon. The probability for the detection event is

$$P_{N} = \left| \left( \prod_{j=1}^{N} a_{j} \langle 1 | b_{j} \langle 0 | \right) \times \left( \prod_{i=1}^{N} U_{i}^{bs} \right) U_{a}^{N} U_{b}^{N} | \Psi_{in} \rangle \prod_{j=2}^{N+1} | 0 \rangle_{a_{j}} | 0 \rangle_{b_{j}} \right|^{2}$$
$$= \langle \Psi_{in} | \Psi_{N} \rangle \langle \Psi_{N} | \Psi_{in} \rangle, \qquad (11)$$

where

$$|\Psi_{N}\rangle = \left(\prod_{j=2}^{N} a_{j}\langle 0|_{b_{j}}\langle 0| \right) U_{b}^{N\dagger} U_{a}^{N\dagger} \left(\prod_{i=1}^{N} U_{i}^{bs\dagger}\right) \times \left(\prod_{j=1}^{N} |1\rangle_{a_{j}}|0\rangle_{b_{j}}\right).$$
(12)

If we write the photon creation operators acting on the modes  $a_i$  and  $b_i$  as  $a_i^{\dagger}$  and  $b_i^{\dagger}$ , respectively, Eq. (12) can be rewritten as

$$\begin{split} |\Psi_{N}\rangle &= \left(\prod_{j=2}^{N} {}_{a_{j}}\langle 0|_{b_{j}}\langle 0|\right) \\ &\times \left[\prod_{j=1}^{N} \left(U_{b}^{N\dagger}U_{a}^{N\dagger}U_{j}^{bs\dagger}a_{j}^{\dagger}U_{j}U_{a}U_{b}\right)\right]U_{b}^{N\dagger}U_{a}^{N\dagger} \\ &\times \prod_{j=1}^{N} \left(U_{j}^{bs\dagger}\right) \left(\prod_{j=1}^{N} |0\rangle_{a_{j}}|0\rangle_{b_{j}}\right). \end{split}$$
(13)

By using the unitary transformation of the operators  $U_a^N$ ,  $U_b^N$ , and BS<sub>i</sub>,

$$U_{j}^{bs\dagger}a_{j}^{\dagger}U_{j}^{bs} = \cos \theta_{j}a_{j}^{\dagger} + \sin \theta_{j}e^{i\varphi_{j}}b_{j}^{\dagger},$$

$$U_{j}^{bs\dagger}b_{j}^{\dagger}U_{j}^{bs} = \cos \theta_{j}b_{j}^{\dagger} - \sin \theta_{j}e^{-i\varphi_{j}}a_{j}^{\dagger},$$

$$U_{j}^{bs\dagger}b_{k}^{\dagger}U_{j}^{bs} = b_{k}^{\dagger}, \quad U_{j}^{bs\dagger}a_{k}^{\dagger}U_{j}^{bs} = a_{k}^{\dagger} \quad (j \neq k),$$

$$U_{a}^{N\dagger}a_{j}^{\dagger}U_{a}^{N} = \sum_{i=1}^{N} U_{ij}^{N}a_{i}^{\dagger},$$

$$U_{b}^{N\dagger}b_{j}^{\dagger}U_{b}^{N} = \sum_{i=1}^{N} U_{ij}^{N}b_{i}^{\dagger},$$

$$U_{a}^{N\dagger}b_{j}^{\dagger}U_{a}^{N} = b_{j}^{\dagger}, \quad U_{b}^{N\dagger}a_{j}^{\dagger}U_{b}^{N} = a_{j}^{\dagger}, \quad (14)$$

we obtain

$$|\Psi_{N}\rangle = \left(\prod_{j=2}^{N} a_{j}\langle 0|_{b_{j}}\langle 0|\right) \left[\prod_{j=1}^{N} \left(\cos \theta_{j} \sum_{i=1}^{N} U_{ij}^{N} a_{i}^{\dagger} + \sin \theta_{j} e^{i\varphi_{j}} \sum_{i=1}^{N} U_{ij}^{N} b_{i}^{\dagger}\right)\right] \left(\prod_{j=1}^{N} |0\rangle_{a_{j}}|0\rangle_{b_{j}}\right)$$
$$= \prod_{j=1}^{N} \left(\cos \theta_{j} a_{1}^{\dagger} + \sin \theta_{j} e^{i\varphi_{j}} b_{1}^{\dagger}\right) |0\rangle_{a_{1}} |0\rangle_{b_{1}}, \quad (15)$$

where we have used the relation

$$U_{a}^{N}U_{b}^{N}\prod_{j=1}^{N} (\mathbf{B}\mathbf{S}_{j})\left(\prod_{j=1}^{N} |0\rangle_{a_{j}}|0\rangle_{b_{j}}\right) = \left(\prod_{j=1}^{N} |0\rangle_{a_{j}}|0\rangle_{b_{j}}\right),$$
$$\left(\prod_{j=2}^{N} {}_{a_{j}}\langle 0|\right)a_{k}^{\dagger} = 0, \quad \left(\prod_{j=2}^{N} {}_{b_{j}}\langle 0|\right)b_{k}^{\dagger} = 0, \quad k \ge 2.$$
(16)

In order to realize the joint measurement of photon-number sum and phase-difference operators on two field modes (6), we require that the measurement probability  $|\langle \Psi_{in} | \Psi_N \rangle|^2$  be proportional to the joint probability distribution function  $|\langle \Psi_{in} | \phi_r^N \rangle|^2$  for the total photon-number N and the phase difference. This requirement can be satisfied by appropriately choosing parameters  $\theta_j$  and  $\varphi_j$ . If the parameters  $\tan \theta_1 e^{i\varphi_1}, \ldots, \tan \theta_N e^{i\varphi_N}$  are the N complex roots of the characteristic polynomial

$$\sum_{n=0}^{N} \frac{e^{in\psi_r^N}}{\sqrt{n!(N-n)!}} (\tan\theta e^{i\varphi})^n = 0, \qquad (17)$$

expression (15) is proportional to the state  $|\phi_r^N\rangle$ . This demonstrates that the proposed setup, which is shown in Fig. 1, definitely implements the joint measurement of photon-number sum and phase-difference operators on the two-mode field  $|\Psi_{in}\rangle$ .

Further, if the parameters  $\tan \theta_1 e^{i\varphi_1}, \ldots, \tan \theta_N e^{i\varphi_N}$  are the *N* complex roots of the characteristic polynomial

$$\sum_{n=0}^{N} \frac{C_n}{\sqrt{n!(N-n)!}} (\tan \theta e^{i\varphi})^n = 0, \qquad (18)$$

where  $C_n$  are determined by Eq. (4), expression (15) is proportional to state (4). This demonstrates that the proposed setup can also be used to measure quantum state overlap of one two-mode field state and arbitrary two-mode *N*-photon entangled state (4).

# III. GENERATION OF TWO-MODE *N*-PHOTON ENTANGLED STATE FROM A PAIR OF SQUEEZED VACUUM STATES

In this section, we show that the experimental setup proposed in Sec. II can be used to generate two-mode N-photon entangled states (4) from a pair of squeezed vacuum states. In principle, the scheme proposed here is closely related to quantum entanglement swapping [19] in which the particles that have never interacted directly are entangled and can be

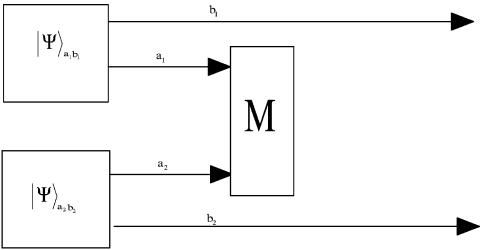


FIG. 2. Schematic experimental setup for generation of *N*-photon entangled states from the a pair of squeezed state  $|\Psi_s\rangle_{a_1,b_1}$  and  $|\Psi_s\rangle_{a_2,b_2}$ . Here *M* denotes the setup proposed in Fig. 1.

$$\frac{1}{\sqrt{N+1}} \sum_{n=0}^{N} e^{-in\phi_{r}^{N}} |N-n\rangle_{b_{1}} |n\rangle b_{2}, \qquad (21)$$

which are the maximally *N*-photon entangled states. These states have found applications in quantum teleportation [10]. The present scheme provides an experimental realization of the specific version of entanglement concentration protocol by using entanglement swapping [20].

On the other hand, if the parameters of the setup are chosen to satisfy

$$\sum_{n=0}^{N} \frac{C_{n}^{*}}{\sqrt{n!(N-n)!}} [\tan(\theta)e^{\varphi}]^{n} = 0, \qquad (22)$$

where  $C_n$  are determined by Eq. (4), the expression (15) is proportional to  $\sum_{n=0}^{N} C_n^* |n\rangle_{b_1} |N-n\rangle_{b_2}$ . In this case, the state of the modes  $b_1$  and  $b_2$  is projected into state (4). In particular, if the parameters  $\theta_j$  and  $\varphi_j$  are chosen to satisfy  $\theta_j$  $= \pi/4$  and  $\varphi_j = 2j\pi/N$ , the state of the modes  $b_1$  and  $b_2$  is projected into state (5).

Note that if we only consider the generation of the entangled state (5), we can simplify our scheme and reduce the number of the linear optical elements and photon detectors. The simplified scheme is shown in Fig. 3, which consists of one symmetric *N*-port device  $F_a$  and *N*-photon detectors. Now we present a detailed analysis of the proposed simple scheme. Let the modes  $a_1$  and  $a_2$  of state (20) inject into two input ports of the symmetric *N*-port devices  $F_a$ . The other input modes of this device are in the vacuum state. After the modes  $a_1$  and  $a_2$  pass through  $F_a$ , the state of the system becomes

$$U_a^N |\Psi_s\rangle_{a_1,b_1} |\Psi_s\rangle_{a_2,b_2} \left(\prod_{j=3}^N |0\rangle_{a_j}\right).$$
(23)

If each detector detects one photon, the state of the modes  $b_1$  and  $b_2$  is projected into

regarded as a specific version of entanglement concentration protocol by using entanglement swapping [20]. In Ref. [21], an experimental purification scheme has been proposed for generating two-mode maximally N-photon entangled states from a pair of squeezed vacuum states. However, the scheme requires the nonlinear Kerr medium. With the current technology, sufficiently strong nonlinear Kerr interactions are not available. Compared with the scheme [21], the present scheme still requires a nonlinear process such as parameteric down conversion to prepare the squeezed vacuum states, but avoids the necessity of nonlinear Kerr interaction. Figure 2 shows the required experimental setup, which can generate a two-mode N-photon entangled state from a pair of squeezed states. We assume that we have generated a pair of squeezed vacuum states  $|\Psi_s\rangle_{a_1,b_1}$  and  $|\Psi_s\rangle_{a_2,b_2}$ , which in the number basis can be written in the form

$$|\Psi_s\rangle_{a_i,b_i} = \sqrt{1-\lambda^2} \sum_{n=0}^{\infty} \lambda^n |n\rangle_{a_i} |n\rangle_{b_i}, \qquad (19)$$

where  $\lambda < 1$  is the squeezing parameter. Based on Eq. (19), we can rewrite the state of total system as follows:

$$\begin{split} |\Psi_{s}\rangle_{a_{1},b_{1}}|\Psi_{s}\rangle_{a_{2},b_{2}} \\ &= (1-\lambda^{2})\sum_{n,m=0}^{\infty} \lambda^{n+m}|n\rangle_{a_{1}}|n\rangle_{b_{1}}|m\rangle_{a_{2}}|m\rangle_{b_{2}} \\ &= (1-\lambda^{2})\sum_{N=0}^{\infty} \lambda^{N}\sum_{j=0}^{N} |j\rangle_{a_{1}}|N-j\rangle_{a_{2}}|j\rangle_{b_{1}}|N-j\rangle_{b_{2}}. \end{split}$$

$$(20)$$

The modes  $a_1$  and  $a_2$  of state (20) act as input modes of the experimental setup shown in Fig. 1. We appropriately choose the parameters of the beam splitters BS<sub>i</sub> to satisfy Eq. (17) and implement joint measurement of photon-number sum and phase-difference operators on these two field modes. If the result of measurement is the state  $|\phi_r^N\rangle$ , the states of the modes  $b_1$  and  $b_2$  is projected into

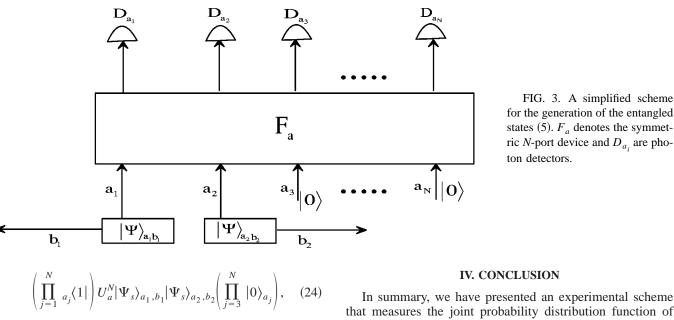


FIG. 3. A simplified scheme for the generation of the entangled states (5).  $F_a$  denotes the symmetric N-port device and  $D_{a_i}$  are photon detectors.

### **IV. CONCLUSION**

that measures the joint probability distribution function of photon-number sum and phase-difference operators on twomode fields. We also demonstrate that such an experimental setup can be used to measure the quantum state overlaps of one two-mode radiation field and arbitrary two-mode N-photon entangled states. As an example of application of the proposed experimental setup, we consider the generation of arbitrary two-mode N-photon entangled states from a pair of squeezed vacuum states. If we only consider the generation of the entangled state (5), we can simplify the scheme and reduce the number of the linear optical elements and photon detectors. Since the present scheme does not need single-photon resource as auxiliary resources, the main difficulty of the scheme in respect of an experimental demonstration consists in the requirement on the sensitivity of the detectors. These detectors should be capable of distinguishing between no photon, one photon, or more photons. Recently, experimental techniques for single-photon detection have made tremendous progress. A photon detector based on visible-light photon counter has been reported, which can distinguish between a single-photon incidence and twophoton incidence with high quantum efficiency, good time resolution, and low bit-error rate [22]. More recently, high efficiency photon counting has been proposed by combining the techniques of photonic quantum memory and ion-trap fluorescence detection. It is possible to achieve photon counting with efficiency approaching 100% [23,24]. However, experimental realization to detect very weak optical fields with high efficiency and to distinguish the number of photons in a given time interval is still a very challenging technical problem.

which can be rewritten as

$$\left(\prod_{j=1}^{N} {}_{a_{j}}\langle 0|\right) \prod_{j=1}^{N} (U_{a}^{N\dagger} a_{j} U_{a}^{N}) |\Psi_{s}\rangle_{a_{1},b_{1}} |\Psi_{s}\rangle_{a_{2},b_{2}} \times \left(\prod_{j=3}^{N} |0\rangle_{a_{j}}\right).$$

$$(25)$$

By using relation (14), we obtain the state of the modes  $b_1$ and  $b_2$ :

$$a_{1}\langle 0|_{a_{2}}\langle 0|\prod_{j=1}^{N} (a_{1}+\gamma_{N}^{*j}a_{2})|\Psi_{s}\rangle_{a_{1},b_{1}}|\Psi_{s}\rangle_{a_{2},b_{2}}$$

$$=a_{1}\langle 0|_{a_{2}}\langle 0|[a_{1}^{N}-(-1)^{N}a_{2}^{N}]|\Psi_{s}\rangle_{a_{1},b_{1}}|\Psi_{s}\rangle_{a_{2},b_{2}}.$$
(26)

Substituting Eq. (20) into Eq. (26), we obtain the state of modes  $b_1$  and  $b_2$ :

$$\frac{1}{\sqrt{2}}(|0\rangle_{b_1}|N\rangle_{b_2} - (-1)^N|N\rangle_{b_1}|0\rangle_{b_2}), \qquad (27)$$

which is expired entangled state (5).

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