Generalized radiation-field quantization method and the Petermann excess-noise factor

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We propose a generalized radiation-field quantization formalism, where quantization does not have to be referenced to a set of power-orthogonal eigenmodes as conventionally required. This formalism can be used to directly quantize the true system eigenmodes, which can be non-power-orthogonal due to the open nature of the system or the gain/loss medium involved in the system. We apply this generalized field quantization to the laser linewidth problem, in particular, lasers with non-power-orthogonal oscillation modes, and derive the excess-noise factor in a fully quantum-mechanical framework. We also show that, despite the excess-noise factor for oscillating modes, the total spatially averaged decay rate for the laser atoms remains unchanged.

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I. INTRODUCTION

The standard approach for the quantization of electromagnetic radiation is to express the field in a set of power-orthogonal modes, and then quantize each mode as a simple harmonic oscillator [1]. These power-orthogonal modes are in general defined by an enclosed physical boundary or a fictious quantization box. While this quantization formulation has been widely acknowledged and used in many important physical problems with great success, its applications to recent quantum optics problems possess certain mathematical difficulties due to the orthogonality requirement of the basis used in the field quantization. These problems include, in particular, the quantum noise problems in laser resonators and amplifiers where a set of power-orthogonal modes does not clearly exist. In fact, many optical systems in practice are most naturally described by modes that are non-power-orthogonal due to the open nature of the cavity or nonuniform gain or loss involved in wave propagation. Prominent examples are unstable resonator lasers and gain-guided amplifiers [2,3]. Each is best described by the non-power-orthogonal eigenmodes of its non-Hermitian propagation operator. The eigenmodes of such non-Hermitian systems are instead biorthogonal to a set of adjoint functions [4,5], which physically correspond to the backward propagation eigenmodes of optical system.

For the question whether a set of power-orthogonal modes exists in a quantum optics problem, the answer really depends on the chosen boundary condition, which in some cases can have certain flexibilities. For the classic problems like blackbody radiation, spontaneous emission, and Lamb shift problems, where the radiation field is either the subject of interest (system) or no interest (reservoir), the boundary condition is assumed to be a closing box. Regardless of it's detail shape, it naturally defines a set of orthogonal modes that can be used by the standard quantization formulation. For laser resonator problems, where the radiation field is partly of interest (laser oscillation modes) and partly of no interest (external reservoir), the choice of boundary condition has some flexibility. Ideally, one can still picture that the laser device is enclosed by, again, a large artificial quantization box and use the orthogonal plane-wave eigenmodes as the basis for field quantization. To describe the laser oscillation, one then has to calculate the interaction between the quantized plane-wave components and laser atoms as well as the wave propagation inside laser cavity [6–8]. The cavity eigenmodes are often also assumed to be plane-wave modes to avoid further complicating the calculation by the diffraction effect in wave propagation. However, the laser oscillation eigenmodes are seldom plane-wave modes in practice. Because eventually only the resonator eigenmodes, a subset of the entire radiation-field components, are of interest, an alternative approach is to directly quantize the laser oscillation eigenmodes from the beginning [9,10]. Conventionally, quantum operators similar to those derived from the standard quantization approach are directly assigned to the oscillation eigenmodes [10]. Even though this method is justified in cases where the cavity eigenmodes are indeed orthogonal, there are examples in which the oscillation eigenmodes can be noticeably nonorthogonal and the direct application of the conventional operator to nonorthogonal eigenmodes is questionable.

To resolve this problem, a basis-independent quantization formalism has been proposed [11], where the amplitude operator of an arbitrary mode is defined as a linear combination of the operators of orthonormal modes with the expansion coefficients equal to the overlap integrals between the arbitrary mode and the orthonormal modes. This approach, however, still implies that the arbitrary mode belongs to a set of orthogonal modes. Another approach has also been reported, where the quantization is focused on the open system eigenmodes with loss [12]. Here, we focus on the mode orthogonality and propose a generalized system eigenmode quantization in Sec. II, which is applicable to orthogonal as well as nonorthogonal eigenmodes.

One of the very important and interesting problems in

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quantum optics is the quantum noise in laser oscillators and amplifiers [13]. In conventional quantum optics, it is well accepted that the noise added into an amplification process for each eigenmode is equivalent to as if it is induced by one noise photon in the same mode. This statement is in essence a mathematical result of the conventional unity commutation relation between the creation and annihilation operators of each eigenmode. In our proposed quantization formalism, this commutation relation is, however, greater than unity for systems having nonorthogonal modes, which then suggests the existence of an excess-noise factor for each eigenmode. This excess-noise factor was first proposed by Petermann [14] in a semiclassical analysis for the fundamental linewidth of gain-guided semiconductor lasers. Haus and Kawakami [15] then extended this result to demonstrate that there was a correlation between the excess-noise emissions into different transverse modes of these gain-guided lasers. Subsequently, it was further generalized by one of the authors that this excess-noise factor actually exists for all laser resonators and amplifiers with nonorthogonal eigenmodes [2,3]. This excess-noise factor has been under serious investigations by several research groups both theoretically [15–29] and experimentally [30–38]. Even though it has been experimentally confirmed, it is still somewhat intriguing from the theoretical point of view because it mysteriously contradicts to the conventional one noise photon per mode statement. Recent efforts have been focused on the full quantum-mechanical analyses [24–29]. However, the analyses are mostly limited to linear regime without taking into account the gain saturation [24–28], or considering a simplified two mode system to include gain saturation [29]. The proposed quantization procedure together with quantum Langevin formulation enable us to provide a rather general theoretical basis for this excess-noise factor in a fully quantum-mechanical context. The analysis, detailed in Sec. III, also includes gain saturation effect that is missing in the recent linear regime analyses. A discussion is followed in Sec. IV.

II. QUANTIZATION OF ELECTROMAGNETIC FIELD

In the conventional quantization formulation, the electromagnetic radiation is often expressed in terms of a set of orthonormal modes, where each mode is quantized as a simple harmonic oscillator. Following this approach, the quantized electric-field operator $\hat{E}$ referenced to the plane-wave mode expansion basis has the expression

$$\hat{E}=\sum_{k,\varepsilon}\sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}\varepsilon_k\{\hat{a}_k\exp(-i\omega_k t+i\mathbf{k}\cdot\mathbf{x})+\hat{a}_k^{\dagger}\exp(i\omega_k t-i\mathbf{k}\cdot\mathbf{x})\},\tag{1}$$

where $\mathbf{k}$ is the wave vector of each plane-wave mode, ε_k ($\varepsilon=1,2$) are the two orthogonal polarization states for each plane-wave mode $\mathbf{k}$, ω_k is the corresponding oscillation frequency, and V is the volume of the quantization box. The creation and annihilation operators $\hat{a}_k^{\dagger}$ and $\hat{a}_k$ satisfy the conventional commutation relations $[\hat{a}_n,\hat{a}_m^{\dagger}]=\delta_{nm}$ and $[\hat{a}_n,\hat{a}_m]=[\hat{a}_n^{\dagger},\hat{a}_m^{\dagger}]=0$. To further simplify the notation, the electric-field operator is rewritten as

$$\hat{E}=\sum_k\sqrt{\frac{\hbar\omega_k}{2\epsilon_0}}\{\hat{a}_{e_k}e_k+\hat{a}_{e_k}^{\dagger}e_k^*\},\tag{2}$$

where e_k,

$$e_k=\sqrt{\frac{1}{V}}\exp(i\mathbf{k}\cdot\mathbf{x})\varepsilon_k,\tag{3}$$

is the power normalized plane-wave mode with two orthogonal polarization states, $\varepsilon=1,2$, implicitly included in index k. The constant $\sqrt{\hbar\omega_k/2\epsilon_0}$ is to lead to a familiar simple harmonic oscillator Hamiltonian $H_k=\hbar\omega_k(\hat{a}_{e_k}\hat{a}_{e_k}^{\dagger}+1/2)$. The subscript e_k is to explicitly identify its corresponding quantization mode. The time dependent function $\exp(-i\omega_k t)$ is absorbed into annihilation operator $\hat{a}_{e_k}$ from now on, likewise for creation operator $\hat{a}_{e_k}^{\dagger}$.

To describe the dynamics of an optical system, it is often more convenient to express the electromagnetic field in the system eigenmode basis. In general, the eigenmode basis defined by an optical system does not necessarily expand the whole functional space defined by the plane-wave mode basis of the quantization box. For example, the eigenmode basis of a conventional standing wave or traveling wave laser resonator cannot expand plane-wave modes propagating perpendicular to the resonator axis. Let us denote the system eigenmode basis as $\{u_i^s\}$, which is the eigensolution defined by the system propagation equation and boundary condition. It expands only the space within system boundary. To establish a complete set of basis, mathematically it is possible to construct an additional set of basis $\{u_j^r\}$, which expands the functional space not covered by the system eigenmode basis $\{u_i^s\}$. $\{u_j^r\}$ can be regarded as reservoir. This basis is orthogonal to $\{u_i^s\}$ and together with $\{u_i^s\}$ form a complete basis $\{u_n\}\equiv\{u_i^s\}+\{u_j^r\}$ expanding the same functional space defined by the plane-wave mode basis $\{e_k\}$. Note that the eigenmode polarization states are also implicitly included in the mode index n, i.e., index n is for each spatial eigenmode and polarization state. We further confine each mode of $\{u_n\}$ to be consisting of a single frequency component ω_n. Therefore, u_n can be expressed as a linear combination of e_k,

$$u_n=\sum_k c_{n,k}e_k,\tag{4}$$

where the summation is taken over only for $|\mathbf{k}|=\omega_n/c$ and u_n is power normalized just like e_k. This formulation, together with the fact that the two basis systems $\{e_k\}$ and $\{u_n\}$ expand the same space, allows us to express the electric-field operator $\hat{E}$ in the new basis similar to that in the plane-wave mode basis,

$$\hat{E}=\sum_k \sqrt{\frac{\hbar\omega_k}{2\epsilon_0}}\{\hat{a}_{e_k}e_k+\hat{a}^\dagger_{e_k}e_k^*\}, \tag{5}$$

$$=\sum_n \sqrt{\frac{\hbar\omega_n}{2\epsilon_0}}\{\hat{a}_{u_n}u_n+\hat{a}^\dagger_{u_n}u_n^*\}, \tag{6}$$

where $\hat{a}_{u_n}$ and $\hat{a}^\dagger_{u_n}$ are the new creation and annihilation operators assigned to mode u_n. The second equality is basically an E field quantization referenced to mode basis $\{u_n\}$, which is not necessarily power orthogonal.

The next step is to find the commutation relations for the new mode operators $\hat{a}_{u_n}$ and $\hat{a}^\dagger_{u_n}$. This can be easily accomplished if one can find the transformation between the creation and annihilation operators in these two different expansion basis systems. To carry out the derivation, we need to introduce another set of basis $\{\phi_n\}$, called adjoint modes, which expands the same functional space defined by $\{u_n\}$ and satisfies the biorthogonal relation

$$(\phi_n|u_m)\equiv\int \phi_n^* u_n d\mathbf{x}=\delta_{nm}. \tag{7}$$

Mathematically, this basis $\{\phi_n\}$ is said to be *reciprocal* to basis $\{u_n\}$. Projecting the annihilation and creation parts of the above two electric-field operator expressions $\hat{E}$ in this reciprocal basis system by operation $\int_{\mathbf{x}}\phi_n^*$ and $\int_{\mathbf{x}}\phi_n$, respectively, we readily obtain the following transformation relations:

$$\hat{a}_{u_n}=\sum_k (\phi_n|e_k)\hat{a}_{e_k}, \tag{8}$$

$$\hat{a}^\dagger_{u_n}=\sum_k (\phi_n|e_k)^*\hat{a}^\dagger_{e_k}. \tag{9}$$

The creation (annihilation) operator of a general mode u_n is expressed as a linear combination of the conventional plane-wave mode creation (annihilation) operators, weighted by the overlap integrals between the corresponding adjoint mode ϕ_n and the plane-wave mode e_k. The above crucial result is different from other quantization approaches [18,24,25,27,28], where different expansion coefficients are proposed.

One can check the validity of the above derived transformation relations by evaluating the expectation value of the electric-field operator in an arbitrary coherent state $|\alpha\rangle$. The resulting value should be independent of the expansion basis used for mode amplitude quantization. Since a continuous set of coherent state forms a complete basis for the state space of simple harmonic oscillators [39], it is sufficient to constitute a proof by using a coherent state. Assume that $|\alpha\rangle$ is a coherent state and has an expression

$$|\alpha\rangle=|\alpha_1,\alpha_2,\alpha_3,\ldots,\alpha_k,\ldots\rangle \tag{10}$$

in $\{e_k\}$ basis, where α_k is the amplitude coefficient of the e_k mode component of the coherent state $|\alpha\rangle$. The expectation value of $\hat{a}_{e_k}$ in this state is

$$\langle\alpha|\hat{a}_{e_k}|\alpha\rangle=\alpha_k. \tag{11}$$

In this state, the expectation value of the electric-field operator expressed in the plane-wave mode basis, Eq. (5), is

$$\langle\alpha|\hat{E}|\alpha\rangle=\sum_k \sqrt{\frac{\hbar\omega_k}{2\epsilon_0}}\{\alpha_k e_k+\text{H.c.}\}\equiv\alpha+\text{H.c.}, \tag{12}$$

where H.c. stands for Hermitian conjugate. If one uses the expression of $\hat{E}$ operator in the $\{u_n\}$ basis, Eq. (6), together with the transformation relations Eqs. (8) and (9), the expectation value is

$$\begin{aligned}\langle\alpha|\hat{E}|\alpha\rangle&=\langle\alpha|\sum_n \sqrt{\frac{\hbar\omega_n}{2\epsilon_0}}\{\hat{a}_{u_n}u_n+\hat{a}^\dagger_{u_n}u_n^*\}|\alpha\rangle\\&=\sum_{n,k}\sqrt{\frac{\hbar\omega_n}{2\epsilon_0}}\{(\phi_n|e_k)\alpha_k u_n+\text{H.c.}\}\\&=\sum_n\left\{\left(\phi_n\middle|\sum_k\sqrt{\frac{\hbar\omega_k}{2\epsilon_0}}\alpha_k e_k\right)u_n+\text{H.c.}\right\}\\&=\sum_n\{(\phi_n|\alpha)u_n+\text{H.c.}\}=\alpha+\text{H.c.},\end{aligned} \tag{13}$$

where the closure relation $\Sigma_n u_n(\phi_n|\equiv\Sigma_n u_n\int d\mathbf{x}\phi_n=1$ is used in the last equality. Since the reciprocal basis function ϕ_n consists of only plane-wave components with $|\mathbf{k}|=\omega_n/c$ just like u_n does, $(\phi_n|e_k)$ is not zero only for $\omega_k=\omega_n$ and we have $\sqrt{\omega_n}(\phi_n|e_k)=\sqrt{\omega_k}(\phi_n|e_k)$. This relation is used in the above third equality. Thus, we have shown that the expectation value of electric-field operator for an arbitrary state is indeed independent of the choice of the expansion basis and have justified the derived transformation relations between different basis systems.

The commutation relations for operators in an arbitrary basis system can be readily obtained

$$[\hat{a}_{u_n},\hat{a}^\dagger_{u_m}]=\sum_{i,j}(\phi_n|e_i)(e_j|\phi_m)[\hat{a}_{e_i},\hat{a}^\dagger_{e_j}]=(\phi_n|\phi_m), \tag{14}$$

$$[\hat{a}_{u_n},\hat{a}_{u_m}]=[\hat{a}^\dagger_{u_n},\hat{a}^\dagger_{u_m}]=0, \tag{15}$$

where the closure relation $\Sigma_i|e_i)(e_i|=1$ is used. Note that, in the above derived formalism, we have so far let the basis $\{u_n\}$ to be as general as possible, in particular, without the orthogonality requirement. If $\{u_n\}$ is an orthonormal basis, it can be easily shown that $(\phi_n|\phi_m)=\delta_{nm}$ and $[\hat{a}_{u_n},\hat{a}^\dagger_{u_m}]$ reduces to the conventional commutation relation. If $\{u_n\}$ is not an orthogonal basis, then $(\phi_n|\phi_m)\neq\delta_{nm}$, and more interestingly, the overlap integral $(\phi_n|\phi_n)>1$ [2]. At the very fundamental level, the conventional unity commutation relation leads to the well-known one noise photon per mode statement in quantum optics. Given the above derived result, the noise photon becomes larger than one if the system eigenmode is nonorthogonal. However, one should also note that noise photons between different modes are not totally independent due to $(u_n|u_m)\neq\delta_{nm}$.

III. QUANTUM NOISE IN LASER OSCILLATORS

We now apply the above results to laser quantum noise analysis. Here, we use the quantum Langevin formalism to describe the dynamics of laser oscillators. However, in contrast to the conventional approach [10], where standard bosonian commutation relations are directly assigned to the amplitude operators of the laser oscillation eigenmodes, we take a more generalized approach without this presumption since these eigenmodes are not necessarily orthogonal as mentioned earlier.

The general aspect of quantum Langevin theory is to describe the equation of motion of the operators of a system under the influence of surrounding reservoir. Its format is often expressed as

$$\frac{d}{dt}\hat{o}=-\frac{1}{2}\gamma\hat{o}+F_{\hat{o}},\tag{16}$$

where $\hat{o}$ is the system operator, γ is the corresponding damping coefficient due to the system-reservoir interaction, and $F_{\hat{o}}$ is the accompanying noise operator that introduces fluctuations into the system. The presence of the noise operator $F_{\hat{o}}$ is essential here to ensure the conservation of the commutation relation of system operator $\hat{o}$ and satisfies relation

$$[F_{\hat{o}}(t),F^{\dagger}_{\hat{o}}(t')]=\gamma[\hat{o},\hat{o}^{\dagger}]\delta(t-t').\tag{17}$$

The reservoir noise is considered as δ correlated with respect to system relaxation time $1/\gamma$ because a reservoir by definition consists of a very large number of degrees of freedom [9]. This condition is generally satisfied by the environment surrounding most of laser systems. Because we are mainly interested in the laser system variables, the mode field notation u_n primarily refers to laser eigenmodes $\{u^s\}$ in the following Langevin analysis. When $\hat{o}$ is the amplitude operator of laser oscillation eigenmode u_n, the above commutation relation becomes $[F_{\hat{a}_{u_n}}(t),F^{\dagger}_{\hat{a}_{u_n}}(t')]=\gamma[\hat{a}_{u_n},\hat{a}^{\dagger}_{u_n}]\delta(t-t')$, where $[\hat{a}_{u_n},\hat{a}^{\dagger}_{u_n}]$ is equal to unity if $\{u_n\}$ is an orthogonal basis and equal to $(\phi_n|\phi_n)>1$ otherwise.

Before writing down the quantum Langevin equations, let us first introduce the relevant operator notations. We use $|e\rangle$ and $|g\rangle$ to denote the upper and lower states of laser transition, where the lower state of laser transition is assumed to be different from the laser atom ground state. The system operators that are of interest are atomic upper and lower population density operators $\hat{\sigma}_e\equiv|e\rangle\langle e|$ and $\hat{\sigma}_g\equiv|g\rangle\langle g|$, atomic dipole operator $\hat{\sigma}\equiv|g\rangle\langle e|$, and laser mode field amplitude operator $\hat{a}_{u_n}$. The interaction between atom and field operators are introduced by the electric-dipole interaction Hamiltonian,

$$H_I=\int d\mathbf{x}\sum_n\sqrt{\frac{\hbar\omega_n}{2\epsilon_0}}\{\hat{a}^{\dagger}_{u_n}u_n^*(\mathbf{x})\cdot\hat{\sigma}(\mathbf{x})e\mathbf{D}+\hat{\sigma}^{\dagger}(\mathbf{x})e\mathbf{D}\cdot\hat{a}_{u_n}u_n(\mathbf{x})\}\tag{18}$$

$$=\int d\mathbf{x}\sum_n g_n\hbar\{\hat{a}^{\dagger}_{u_n}u_n^*(\mathbf{x})\hat{\sigma}(\mathbf{x})+\hat{\sigma}^{\dagger}(\mathbf{x})\hat{a}_{u_n}u_n(\mathbf{x})\},\tag{19}$$

where $e\mathbf{D}=e\langle e|\vec{r}|g\rangle$ is the atomic dipole moment between the laser transition states $|e\rangle$ and $|g\rangle$. The coupling factor g_n is

$$g_n=\sqrt{\frac{\omega_n}{2\epsilon_0\hbar}}\varepsilon_n\cdot e\mathbf{D},\tag{20}$$

where ε_n is the mode polarization state. Applying the Langevin theory to the atom and field operators along with the above interaction Hamiltonian, we have the following coupled quantum Langevin equations

$$\frac{d}{dt}\hat{\sigma}_e(\mathbf{x})=\Lambda-\gamma_e\hat{\sigma}_e(\mathbf{x})+i\sum_n g_n[\hat{a}_n^{\dagger}u_n^*\hat{\sigma}(\mathbf{x})-\hat{\sigma}^{\dagger}(\mathbf{x})\hat{a}_n u_n]+F_{\hat{\sigma}_e}(\mathbf{x}),\tag{21}$$

$$\frac{d}{dt}\hat{\sigma}_g(\mathbf{x})=-\gamma_g\hat{\sigma}_g(\mathbf{x})-i\sum_n g_n[\hat{a}_n^{\dagger}u_n^*\hat{\sigma}(\mathbf{x})-\hat{\sigma}^{\dagger}(\mathbf{x})\hat{a}_n u_n]+F_{\hat{\sigma}_g}(\mathbf{x}),\tag{22}$$

$$\frac{d}{dt}\hat{\sigma}(\mathbf{x})=-[\gamma+i(\omega-\nu)]\hat{\sigma}(\mathbf{x})+i\sum_n g_n\times[\hat{\sigma}_e(\mathbf{x})-\hat{\sigma}_g(\mathbf{x})]\hat{a}_n u_n+F_{\hat{\sigma}}(\mathbf{x}),\tag{23}$$

$$\frac{d}{dt}\hat{a}_{u_n}=-\left[\frac{1}{2}\gamma_{c,n}+i(\Omega_n-\nu)\right]\hat{a}_{u_n}-ig_n\int d\mathbf{x}\times\phi_n^*(\mathbf{x})\hat{\sigma}(\mathbf{x})+F_{\hat{a}_{u_n}},\tag{24}$$

where Λ is the pump rate to the laser upper state $|e\rangle$ and is assumed to be constant in time, γ_e and γ_g are the spontaneous decay rates of upper and lower laser states to other states, γ is the atomic dipole decay rate, and $\gamma_{c,n}$ is the system eigenmode u_n decay rate. For the simplicity of analysis, we assume that the laser is in single mode oscillation and the oscillation frequency ν lines up with the dipole transition frequency ω and the lasing resonator eigenmode frequency Ω_0. The coupling terms containing both atomic and mode field operators in the above Langevin operator rate equations are obtained from interaction Hamiltonian by $i/\hbar[H_I,\hat{o}]$, where $\hat{o}$ is the atomic or mode field operators of interest. The presence of adjoint mode ϕ_n in the mode amplitude operator rate Eq. (24) might look odd at the first glance but indeed it is also derived from the same interaction Hamiltonian

$$\frac{i}{\hbar}[H_I,\hat{a}_{u_n}]=i\int d\mathbf{x}\sum_m g_m[\hat{a}^\dagger_{u_m}u^*_m(\mathbf{x})\hat{\sigma}(\mathbf{x}),\hat{a}_{u_n}] \tag{25}$$

$$=-i\int d\mathbf{x}\sum_m g_m(\phi_n|\phi_m)u^*_m(\mathbf{x})\hat{\sigma}(\mathbf{x}) \tag{26}$$

$$=-ig_n\int d\mathbf{x}\phi^*_n(\mathbf{x})\hat{\sigma}(\mathbf{x}), \tag{27}$$

where $g_m(\phi_n|\phi_m)=g_n(\phi_n|\phi_m)$ is used because $(\phi_n|\phi_m)$ is not zero only for those $\omega_m=\omega_n$. The noise operators $F_{\hat{\sigma}_e}$, $F_{\hat{\sigma}_g}$, $F_{\hat{\sigma}}$, and $F_{\hat{a}_{u_n}}$ are considered to be δ correlated in time compared to the time scale of interest in the system. The atomic noise operators $F_{\hat{\sigma}_e}$, $F_{\hat{\sigma}_g}$, and $F_{\hat{\sigma}}$ are also assumed to be δ correlated in space because laser atoms are normally fairly far apart in gain medium compared to the characteristic dimension of atom-atom interaction. Their correlation relations can be obtained from the Einstein relation [9],

$$\langle F_{\hat{\sigma}_e}(t)F_{\hat{\sigma}_e}(t')\rangle_R=(\langle\Lambda\rangle_R+\gamma_e\langle\hat{\sigma}_e\rangle_R)\,\delta(t-t')\,\delta(x-x'), \tag{28}$$

$$\langle F_{\hat{\sigma}_g}(t)F_{\hat{\sigma}_g}(t')\rangle_R=\gamma_g\langle\hat{\sigma}_g\rangle_R\,\delta(t-t')\,\delta(x-x'), \tag{29}$$

$$\langle F^\dagger_{\hat{\sigma}}(t)F_{\hat{\sigma}}(t')\rangle_R=(2\gamma\langle\hat{\sigma}_e\rangle_R+\langle\Lambda\rangle_R-\gamma_e\langle\hat{\sigma}_e\rangle_R)\times\delta(t-t')\,\delta(x-x'), \tag{30}$$

$$\langle F_{\hat{\sigma}}(t)F^\dagger_{\hat{\sigma}}(t')\rangle_R=(2\gamma\langle\hat{\sigma}_g\rangle_R-\gamma_g\langle\hat{\sigma}_g\rangle_R)\,\delta(t-t')\,\delta(x-x'), \tag{31}$$

$$\langle F^\dagger_{a_{u_n}}(t)F_{a_{u_n}}(t')\rangle_R=\gamma_{c,n}\langle a^\dagger_{u_n}a_{u_n}\rangle_R\,\delta(t-t'), \tag{32}$$

$$\langle F_{a_{u_n}}(t)F^\dagger_{a_{u_n}}(t')\rangle_R=\gamma_{c,n}\langle a_{u_n}a^\dagger_{u_n}\rangle_R\,\delta(t-t'), \tag{33}$$

where $\langle\cdots\rangle_R$ stands for reservoir average. In this paper, the reservoir average is taken over zero point vacuum state, which gives the following results for mode amplitude operator $\hat{a}_{u_n}$,

$$\langle a^\dagger_{u_n}a_{u_n}\rangle_R=\sum_{i,j}(\phi_n|e_i)(e_j|\phi_n)\langle 0|\hat{a}^\dagger_{e_i}\hat{a}_{e_j}|0\rangle \tag{34}$$

$$=0, \tag{35}$$

$$\langle a_{u_n}a^\dagger_{u_n}\rangle_R=\sum_{i,j}(\phi_n|e_i)(e_j|\phi_n)\langle 0|\hat{a}_{e_i}\hat{a}^\dagger_{e_j}|0\rangle \tag{36}$$

$$=(\phi_n|\phi_n). \tag{37}$$

We now suppose that the dipole relaxation rate γ is much faster than the atomic population decay rates γ_e and γ_g and the cavity decay rate $\gamma_{c,n}$, which is a valid approximation for most solid-state lasers. The atomic dipole then closely follows the phase of cavity field, and Eq. (23) can be simplified to

$$\hat{\sigma}(\mathbf{x})=\frac{i\sum_n g_n[\hat{\sigma}_e(\mathbf{x})-\hat{\sigma}_g(\mathbf{x})]\hat{a}_n u_n+F_{\hat{\sigma}}(\mathbf{x})}{\gamma}. \tag{38}$$

Substituting this expression into the atomic population and field operator rate Eqs. (21), (22), and (24), we obtain the simplified coupled equations

$$\frac{d}{dt}\hat{\sigma}_e(\mathbf{x})=\Lambda-\gamma_e\hat{\sigma}_e(\mathbf{x})-\frac{2}{\gamma}\sum_{n,n'}g_ng_{n'}\hat{a}^\dagger_{u_n}\hat{a}_{u'_n}u^*_nu_{n'}\times[\hat{\sigma}_e(\mathbf{x})-\hat{\sigma}_g(\mathbf{x})]+\frac{i}{\gamma}\sum_n g_n\hat{a}^\dagger_{u_n}u^*_nF_{\hat{\sigma}}(\mathbf{x})+\text{H.c.}+F_{\hat{\sigma}_e}(\mathbf{x}), \tag{39}$$

$$\frac{d}{dt}\hat{\sigma}_g(\mathbf{x})=-\gamma_g\hat{\sigma}_g(\mathbf{x})+\frac{2}{\gamma}\sum_{n,n'}g_ng_{n'}\hat{a}^\dagger_{u_n}\hat{a}_{u'_n}u^*_nu_{n'}\times[\hat{\sigma}_e(\mathbf{x})-\hat{\sigma}_g(\mathbf{x})]-\frac{i}{\gamma}\sum_n g_n\hat{a}^\dagger_{u_n}u^*_nF_{\hat{\sigma}}(\mathbf{x})\times+\text{H.c.}+F_{\hat{\sigma}_g}(\mathbf{x}), \tag{40}$$

$$\frac{d}{dt}\hat{a}_{u_n}=-\frac{1}{2}\gamma_{c,n}\hat{a}_{u_n}+\frac{g_n}{\gamma}\int d\mathbf{x}\,\phi^*_n(\mathbf{x})\sum_{n'}g_{n'}\times[\hat{\sigma}_e(\mathbf{x})-\hat{\sigma}_g(\mathbf{x})]\hat{a}_{u_{n'}}u_{n'}(\mathbf{x})-i\frac{g_n}{\gamma}\int d\mathbf{x}\times\phi^*_n(\mathbf{x})F_{\hat{\sigma}}(\mathbf{x})+F_{\hat{a}_{u_n}}. \tag{41}$$

The above operator rate equations are in a form very similar to their classical counterparts except that the additional noise operator terms are uniquely derived from the quantum Langevin theory. In addition, the spatial dependence is explicitly spelled out and eigenmode $\{u_n\}$ is a true system eigenmode basis, which does not have to be an orthogonal basis.

To carry the analysis further, we rewrite the atomic population and field amplitude operators into complex numbers and small signal operators,

$$\hat{\sigma}_e=\sigma_e+\tilde{\sigma}_e, \tag{42}$$

$$\hat{\sigma}_g=\sigma_g+\tilde{\sigma}_g, \tag{43}$$

$$\hat{a}_{u_n}=A_{u_n}+\tilde{a}_{u_n}. \tag{44}$$

The complex numbers are the reservoir averaged steady-state solutions, satisfying the following equations

$$\Lambda-\gamma_e\sigma_e(\mathbf{x})-\frac{2}{\gamma}\sum_{n,n'} g_n g_{n'} A^{\dagger}_{u_n} A_{u_{n'}} u_n^* u_{n'} \times[\sigma_e(\mathbf{x})-\sigma_g(\mathbf{x})]=0, \quad (45)$$

$$-\gamma_g\sigma_g(\mathbf{x})+\frac{2}{\gamma}\sum_{u_n,u_{n'}} g_n g_{n'} A^{\dagger}_{u_n} A_{u_{n'}} u_n^{\dagger} u_{n'} \times[\sigma_e(\mathbf{x})-\sigma_g(\mathbf{x})]=0, \quad (46)$$

$$-\frac{1}{2}\gamma_{c,n}A_{u_n}+\frac{g_{u_n}^2}{\gamma}[\sigma_e(\mathbf{x})-\sigma_g(\mathbf{x})]A_{u_n}=0, \quad (47)$$

where we have assumed that the atomic population inversion, $\sigma_e(\mathbf{x})-\sigma_g(\mathbf{x})$, is uniformly distributed in space for mathematical simplicity. Now, we assume the laser is in a single mode oscillation and label the oscillation mode as u_0. From Eq. (47), this gives $A_{u_n}=0$ except A_{u_0} and $g_{u_0}^2/\gamma[\sigma_e(\mathbf{x})-\sigma_g(\mathbf{x})]=\frac{1}{2}\gamma_{c,0}$. Given the above steady-state solutions, the linearized rate equations for the small signal operators are

$$\frac{d}{dt}\tilde{\sigma}_e(\mathbf{x})=-\gamma_e\tilde{\sigma}_e(\mathbf{x})-\frac{2}{\gamma}\sum_{n,n'} g_n g_{n'} A^{\dagger}_{u_n} A_{u_{n'}} u_n^* u_{n'} \times[\tilde{\sigma}_e(\mathbf{x})-\tilde{\sigma}_g(\mathbf{x})]-\frac{2}{\gamma}\sum_{n,n'} g_n g_{n'} \times(\tilde{a}_n^{\dagger}A_{n'}+A_n^{\dagger}\tilde{a}_{n'})u_n^* u_{n'}[\sigma_e(\mathbf{x})-\sigma_g(\mathbf{x})] +\frac{i}{\gamma}\sum_n g_n A_n^{\dagger} u_n^* F_{\hat{\sigma}}(\mathbf{x})+\text{H.c.}+F_{\hat{\sigma}_e}(\mathbf{x}), \quad (48)$$

$$\frac{d}{dt}\tilde{\sigma}_g(\mathbf{x})=-\gamma_g\tilde{\sigma}_g(\mathbf{x})+\frac{2}{\gamma}\sum_{n,n'} g_n g_{n'} A^{\dagger}_{u_n} A_{u_{n'}} u_n^* u_{n'} \times[\tilde{\sigma}_e(\mathbf{x})-\tilde{\sigma}_g(\mathbf{x})]+\frac{2}{\gamma}\sum_{n,n'} g_n g_{n'} \times(\tilde{a}_n^{\dagger}A_{n'}+A_n^{\dagger}\tilde{a}_{n'})u_n^* u_{n'}[\sigma_e(\mathbf{x})-\sigma_g(\mathbf{x})] -\frac{i}{\gamma}\sum_n g_n A_n^{\dagger} u_n^* F_{\hat{\sigma}}(\mathbf{x})+\text{H.c.}+F_{\hat{\sigma}_g}(\mathbf{x}), \quad (49)$$

$$\frac{d}{dt}\tilde{a}_n=\frac{g_n^2}{\gamma}[\tilde{\sigma}_e(\mathbf{x})-\tilde{\sigma}_g(\mathbf{x})]A_n-i\frac{g_n}{\gamma}\int d\mathbf{x} \times\phi_n^*(\mathbf{x})F_{\hat{\sigma}}(\mathbf{x})+F_{\hat{a}_{u_n}}. \quad (50)$$

These small signal rate equations basically describe the fluctuations added into the steady-state solutions by the reservoir noise. If we choose a phase reference such that A_0 is real, the in-phase and quadrature-phase components of amplitude rate, Eq. (50), are respectively,

$$\frac{d}{dt}\tilde{a}_{01}=\frac{g_0^2}{\gamma}[\tilde{\sigma}_e(\mathbf{x})-\tilde{\sigma}_g(\mathbf{x})]A_0 +\text{Re}\left[-i\frac{g_0}{\gamma}(\phi_0|F_{\hat{\sigma}})+F_{\hat{a}_{u_0}}\right], \quad (51)$$

$$\frac{d}{dt}\tilde{a}_{02}=\text{Im}\left[-i\frac{g_0}{\gamma}(\phi_0|F_{\hat{\sigma}})+F_{\hat{a}_{u_0}}\right], \quad (52)$$

where subscripts 1 and 2 denote the in-phase and quadrature-phase components, respectively,

$$\tilde{a}_0=\tilde{a}_{01}+i\tilde{a}_{02}, \quad (53)$$

$$F_{\hat{\sigma}}=F_{\hat{\sigma}1}+iF_{\hat{\sigma}2}, \quad (54)$$

$$F_{\hat{a}_{u_0}}=F_{\hat{a}_{u_0}1}+iF_{\hat{a}_{u_0}2}. \quad (55)$$

As one can see, the in-phase amplitude rate, Eq. (51), and the atomic population rate, Eqs. (48) and (49), are coupled together and always try to retain the fluctuations around their steady-state values. This effect can also be understood from the atom-field interaction term in the atomic and field operator rate Eqs. (39)–(41), where its contribution to the change rates of the atomic population inversion and the field amplitude are out of phase. The quadrature-phase amplitude fluctuation rate, Eq. (52), on the other hand, does not have any constrain force and basically is a random-walk process driven by the dipole and reservoir field fluctuations. This quadrature-phase fluctuation contributes to the laser frequency noise and is the dominant source for the fundamental laser linewidth.

The phase fluctuation rate equation of laser oscillator is

$$\frac{d}{dt}\phi=\frac{1}{A_0}\frac{d}{dt}\tilde{a}_{02} \quad (56)$$

$$=\frac{g_0}{A_0\gamma}\text{Im}[-i(\phi_0|F_{\hat{\sigma}1})]+\frac{1}{A_0}\text{Im}[F_{\hat{a}_{u_0}2}]. \quad (57)$$

Because both dipole and amplitude noise operators are δ correlated in time, the laser frequency fluctuation noise has a white power spectrum and its noise power spectral density is

$$S_f=\frac{g_0^2}{4A_0^2\gamma^2}\langle((\phi_0|F_{\hat{\sigma}})+(\phi_0|F_{\hat{\sigma}})^{\dagger})^2\rangle_R -\frac{1}{4A_0^2}\langle(F_{\hat{a}_{u_0}}-F^{\dagger}_{\hat{a}_{u_0}})^2\rangle_R \quad (58)$$

$$=\frac{g_0^2}{2A_0^2\gamma}(\phi_0|\phi_0)(\sigma_e+\sigma_g)+\frac{1}{4A_0^2}\langle F^{\dagger}_{a_{u_0}}F_{a_{u_0}}+F_{a_{u_0}}F^{\dagger}_{a_{u_0}}\rangle_R \quad (59)$$

$$=\frac{1}{4A_0^2}\gamma_{c,0}(\phi_0|\phi_0)\frac{\sigma_e+\sigma_g}{\sigma_e-\sigma_g}+\frac{1}{4A_0^2}\gamma_{c,0}(\phi_0|\phi_0) \quad (60)$$

$$=\frac{1}{4A_0^2}\gamma_{c,0}(\phi_0|\phi_0)\left(1+\frac{\sigma_e+\sigma_g}{\sigma_e-\sigma_g}\right), \tag{61}$$

where Eq. (47) is used to reach the last equality. Finally, we obtain the full width at half maximum quantum limited laser linewidth

$$\Delta\omega_l=S_f \tag{62}$$

$$=(\phi_0|\phi_0)\frac{\hbar\omega_l\gamma_c^2}{P_l}\frac{1}{4}\left(1+\frac{\sigma_e+\sigma_g}{\sigma_e-\sigma_g}\right) \tag{63}$$

$$\simeq(\phi_0|\phi_0)\frac{\hbar\omega_l\gamma_c^2}{2P_l}, \tag{64}$$

where laser power $P_l=A_0^2\hbar\omega_l\gamma_c$ is used in the second equality. The term $(\sigma_e+\sigma_g)/(\sigma_e-\sigma_g)$ describes the noise dependence on atomic inversion level. The last expression is obtained for the highly inverted case $\sigma_e\gg\sigma_g$, which is a valid approximation for lasers with fast lower laser transition state decay rate γ_g. This is the well-known quantum laser linewidth equation with the excess-noise factor $(\phi_0|\phi_0)$ now derived in a fully quantum-mechanical framework.

IV. DISCUSSION

From the above derived laser linewidth equation, there is excess quantum noise coupled into the laser oscillation mode if resonator eigenmodes are not orthogonal. Since this result is very different from the well accepted one noise photon per mode statement, it might raise the question whether the atomic spontaneous decay rate will also be enhanced by the same excess-noise factor. To answer this question, we now take a closer look at the atomic rate Eqs. (39) and (40). The spontaneous decay is embedded in the noise terms containing amplitude operator $\hat{a}_{u_n}$ and dipole noise operator $F_{\hat{\sigma}}$, i.e., $i/\gamma\Sigma_n g_n\hat{a}_{u_n}^\dagger u_n^* F_{\hat{\sigma}}(\mathbf{x})+\text{H.c.}$ The spontaneous decay rate is in fact described by the ensemble average of this noise term. Because amplitude operator $\hat{a}_{u_n}$ is in part driven by dipole noise operator $F_{\hat{\sigma}}$ in Eq. (41), the average of this term is not zero and can be calculated by first computing the average

$$\langle F_{\hat{\sigma}}^\dagger\hat{a}_{u_n}\rangle=\langle F_{\hat{\sigma}}^\dagger(t)\hat{a}_{u_n}(t-\Delta t)\rangle+\left\langle F_{\hat{\sigma}}^\dagger\int_{t-\Delta t}^{t}\dot{\hat{a}}_{u_n}(t')dt'\right\rangle \tag{65}$$

$$=\left\langle -i\frac{g_n}{\gamma}\int_{t-\Delta t}^{t}F_{\hat{\sigma}}^\dagger(x,t)\int_{x'}\phi_n^*(x')F_{\hat{\sigma}}(x',t')dt'\right\rangle \tag{66}$$

$$=-i\frac{g_n}{\gamma}\int_{t-\Delta t}^{t}\int_{x'}\phi_n^*(x')\langle F_{\hat{\sigma}}^\dagger(x,t)F_{\hat{\sigma}}(x',t')\rangle dt' \tag{67}$$

$$=-i\frac{g_n}{\gamma}\int_{t-\Delta t}^{t}\int_{x'}\phi_n^*(x')\langle\hat{\sigma}_e(x)\rangle \times\delta(t-t')\delta(x-x')dt' \tag{68}$$

$$=-ig_n\phi_n^*(x)\sigma_e(x), \tag{69}$$

where Eq. (30) is used to evaluate $\langle F_{\hat{\sigma}}^\dagger F_{\hat{\sigma}}\rangle$ and the assumption that dipole decay rate is much faster than other system decay constants is also used in the fourth equality to reach the final result. The first term in the first equality is zero because $\hat{a}_{u_n}$ is uncorrelated to future $F_{\hat{\sigma}}$. The second equality is obtained from the fact that $F_{\hat{\sigma}}$ are uncorrelated to all the terms for $\dot{\hat{a}}_{u_n}$ in Eq. (41) except $-i(g_n/\gamma)\int_x\phi_n^*F_{\hat{\sigma}}$. Given the above result, we readily obtain the ensemble average

$$\left\langle\frac{i}{\gamma}\sum_n g_n\hat{a}_{u_n}^\dagger u_n^*F_{\hat{\sigma}}+\text{H.c.}\right\rangle=\frac{2}{\gamma}\sum_n g_n^2\phi_n^*u_n\sigma_e. \tag{70}$$

Now, we compare this term with respect to the stimulated emission term in Eq. (39), i.e.,

$$\frac{2}{\gamma}\sum_{n,n'}g_ng_{n'}\hat{a}_{u_n}^\dagger\hat{a}_{u_{n'}}u_n^*u_{n'}(\sigma_e-\sigma_g).$$

For an orthogonal basis, where $(u_n|u_m)=\delta_{nm}$ and $\phi_n=u_n$, the above ensemble average and the stimulated emission term are reduced to $2/\gamma\Sigma_n g_n^2u_n^*u_n\sigma_e$ and $2/\gamma\Sigma_n g_n^2\hat{a}_{u_n}^\dagger\hat{a}_{u_n}u_n^*u_n(\sigma_e-\sigma_g)$, respectively. We see that Eq. (70) indeed represents the spontaneous decay and describes one noise photon per mode for spontaneous decay rate. For a nonorthogonal basis, it is less clear to talk about the photon number per mode because eigenmodes are nonorthogonal. It is interesting to see that the spontaneous decay has a spatial dependence $\phi_n^*u_n$ compared to the stimulated decay dependence $u_n^*u_n$. The spontaneous emission can be enhanced or suppressed depending on location. On the other hand, we can evaluate the average atomic spontaneous decay rate by taking spatial integral

$$\int_x\frac{2}{\gamma}\sum_n g_n^2\phi_n^*u_n\sigma_e=\frac{2}{\gamma}\sum_n g_n^2\sigma_e, \tag{71}$$

where we assume a uniform distribution of σ_e. The above result is independent of whether the eigenmode basis is orthogonal or nonorthogonal and shows that the spatially averaged atomic decay rate is preserved.

Another question that might be asked is that since the the excess-noise factor results from the nonorthogonal eignemode basis and only the oscillating mode is concerned for a single mode laser oscillation, theoretically, one can reconstruct those nonoscillating eigenmodes to become orthogonal to each other and to the lasing mode. Then, the excess-noise factor can be artificially reduced to unity. The answer to this seeming paradox is that in the above derivation the eigenmode basis defined by the optical system has to be used in order to obtain the decoupled Langevin laser mode equation, Eq. (24). If laser propagation eigenmode basis is not used, it is not possible to write such a decoupled amplitude operator rate equation. Instead, the amplitude operator rate equation will be coupled among different modes and make the calculation totally different and more complex. However, even if such an approach is taken [7,8], the laser system is often simplified to an one-dimensional cavity model to make the calculation manageable and the result of the excess-noise factor is still the same.

V. CONCLUSION

In this paper, we have derived a generalized radiation-field quantization formalism, where quantization does not have to be referenced to a set of orthogonal eigenmodes. This formalism can be applied to the quantization of the true system propagation eigenmodes especially for those which are nonorthogonal. We use this generalized quantization approach to provide a fully quantum-mechanical derivation for the excess-noise factor in the fundamental laser linewidth.

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