Temporal beam splitter and temporal interference

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The effect of photon beam splitting in a time-varying medium is described by classical and quantum theoretical models. It generalizes the concept of time refraction, introduced recently by the authors as a natural extrapolation of the usual concepts of refraction and reflection into the time domain. Total time reflection is shown to exist. A sequence of time refraction processes is shown to lead to temporal interference effects. The concept of temporal beam splitter is introduced. Bogoliubov transformations for the temporal beam splitter are derived. Resonant amplification of light by change in time in the optical medium is shown to exist.

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I. INTRODUCTION

In recent years, the development of intense and ultrafast lasers [1] opened the way to study new phenomena concerning the interaction of radiation with matter [2]. With these laser pulses we are now able to produce very significant nonlinear changes of the refractive index of the optical media in a very short time scale. Of particular relevance is the occurrence of frequency shifts of radiation propagating in nonstationary media. In Plasma Physics this effect was called photon acceleration [3–5]. Similar effects in Nonlinear Optics are usually associated with self- (and crossed-) phase modulations, and can lead to the formation of supercontinuum radiation [6,7].

In a recent work [8,9] the most elementary physical process leading to phase modulation and to photon acceleration was identified as time refraction, or an instantaneous temporal jump of the refractive index of a medium. Refraction is a well-known elementary optical process, which occurs when light interacts with the boundary between two distinct optical media. It is related to nonconservation of the photon momentum, while the photon energy (or frequency) is conserved. If instead we have a single uniform medium that suddenly changes its refractive index at a given instant of time, the photons will suffer, not a spatial jump as previously, but a temporal jump that conserves the momentum but results in a frequency shift (or energy nonconservation). This effect, exactly symmetric to refraction, is time refraction.

It is the purpose of the present work to generalize our previous results in order to include multiple time refraction events and to show that temporal interference is an important aspect of photon propagation in nonstationary media. The concept of a temporal beam splitter will also be introduced. This is a four-port optical device, resulting from two successive and opposite time refraction events, and can be seen as the temporal analog of the well-known optical beam splitter [10].

This work is organized in the following way. In Sec. II we give the classical description of time refraction, and derive the relevant field and mode frequency transformations. Total time reflection is shown to exist. The classical approach is generalized to the case of successive time refraction events, showing the appearance of temporal interference due to the interaction of successive secondary waves. The case of a medium that is suddenly perturbed and after some time returns to its initial condition, after suffering two opposite time refraction events, will be considered in detail. This process will be seen as a temporal beam splitter, with properties similar to the usual beam splitter. It will be shown that, in such a case, the energy of the electromagnetic field can increase at the expense of the energy of the medium, leading to a new kind of amplification process. In Sec. III, quantization of the field will allow to establish the quantum definition of a temporal beam splitter. Apart from temporal interference, it will be shown that photon pair creation is possible, thus appearing as a specific quantum effect that complements the amplification process described in the preceding section. Similar pair creation and amplification processes were already noticed in vibrating cavities [11–13], but the present work shows that the existence of moving space boundaries (typical of the dynamical Casimir effect) is not a necessary ingredient. Finally, in Sec. IV, we will state the conclusions.

II. A FOUR-PORT TEMPORAL DEVICE

Let us first consider a classical description for the electromagnetic field. We assume that an optical medium, with initial refractive index n_0, suffers a series of successive changes at times $t_0, t_1, \ldots, t_j$ to the values $n_1, n_2, \ldots, n_{j+1}$. Because such a change occurs all over the medium, there are no gradients of the refractive index, and the wave number k remains constant. In contrast, the mode frequency will have to change in order to adjust to the new dispersion relation of the medium.

The electric field associated with the two modes propagating in opposite directions [14], with an initial frequency ω_0 and a given polarization, along Ox, will be given by

$$\vec{E}(x,t) = [\vec{e}_j(t) + \vec{e}\,'_j(t)]e^{ikx} + \text{c.c.}, \tag{1}$$

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with $\vec{e}_j(t)=\vec{E}_j e^{-i\omega_j t}$ and $\vec{e}'_j(t)=\vec{E}'_j e^{+i\omega_j t}$, for $t_{j-1}<t<t_j$ and $j=0,1,2,\ldots$. The frequencies are determined by $\omega_j = kc/n_j=\omega_0 n_0/n_j$.

Now, considering that Maxwell's equations remain valid for all times, including $t=0$, the following continuity relations for the vector displacement field and for the induction magnetic field can be established [13,8]:

$$\vec{D}(x,t_j^-)=\vec{D}(x,t_j^+),\quad \vec{B}(x,t_j^-)=\vec{B}(x,t_j^+). \tag{2}$$

Assuming the same polarization direction for all the fields, we can establish the following relations for the complex field amplitudes:

$$e_{j+1}(t_j)=\frac{\alpha_j^2}{2}[e_j(t_j)+e'_j(t_j)]+\frac{\alpha_j}{2}[e_j(t_j)-e'_j(t_j)],$$

$$e'_{j+1}(t_j)=\frac{\alpha_j^2}{2}[e_j(t_j)+e'_j(t_j)]-\frac{\alpha_j}{2}[e_j(t_j)-e'_j(t_j)], \tag{3}$$

where $\alpha_j=n_j/n_{j+1}=\omega_{j+1}/\omega_j$. If we replace e_j and e'_j by their expressions in terms of e_{j-1} and e'_{j-1}, we obtain

$$\begin{aligned} e_{j+1}(t_j)=&\frac{\alpha_j}{2}(1+\alpha_j)e^{-i\omega_j\Delta_j}\left[\frac{\alpha_{j-1}}{2}(1+\alpha_{j-1})e_{j-1}(t_{j-1})\right.\\ &\left.-\frac{\alpha_{j-1}}{2}(1-\alpha_{j-1})e'_{j-1}(t_{j-1})\right]-\frac{\alpha_j}{2}(1\\ &-\alpha_j)e^{+i\omega_j\Delta_j}\left[\frac{\alpha_{j-1}}{2}(1+\alpha_{j-1})e'_{j-1}(t_{j-1})\right.\\ &\left.-\frac{\alpha_{j-1}}{2}(1-\alpha_{j-1})e_{j-1}(t_{j-1})\right], \end{aligned} \tag{4}$$

where $\Delta_j=t_j-t_{j-1}$, with a similar expression for $e'_{j+1}(t_j)$.

This iteration procedure leads to the appearance of a complicated interference pattern of the secondary waves generated at each temporal jump. The duration of the distance between each of these jumps, measured by Δ_j, plays the same role as the width of a dielectric plate in the usual (space) interference pattern. We are then clearly describing a similar interference effect in the temporal domain, thus opening the possibility for realizing temporal interferometers.

Using the above continuity conditions (3) for a single jump between n_0 and n_1, we get, for $t=0$ and for arbitrary x, the following relations between the electric-field amplitudes

$$E_1=\frac{\alpha}{2}[(1+\alpha)E_0-(1-\alpha)E'_0],$$

$$E'_1=\frac{\alpha}{2}[(1+\alpha)E'_0-(1-\alpha)E_0], \tag{5}$$

where $\alpha=n_0/n_1$.

These equations relate the two incident fields with the two emerging ones. If, instead of two waves, initially we have only one wave propagating along the positive direction of the axis Ox, we assume $E'_0=0$, and the corresponding reflection and transmission coefficients are

$$R=\frac{E'_1}{E_0}=\frac{\alpha}{2}(\alpha-1),\quad T=\frac{E_1}{E_0}=\frac{\alpha}{2}(\alpha+1). \tag{6}$$

These expressions can be called the Fresnel formulas for time refraction, by analogy with the case of (space) refraction. They are not identical to those of Ref. [5] due to a minor algebraic error. We also notice that $R+T=\alpha^2$. This contrasts with the usual (space) Fresnel formulas for normal wave incidence, where $R+T=1$ as a consequence of energy nonconservation for nonstationary media.

It is also possible to envisage a situation where, due to the temporal change of the medium, the transmitted wave is completely canceled: $E_1=0$. Equations (5) show that this occurs when the two initial wave amplitudes are such that $E_0=(1-\alpha)E'_0/(1+\alpha)$. The total field of the reflected wave will be given by

$$E'_1=\frac{2\alpha^2}{1+\alpha}E'_0. \tag{7}$$

Similarly, we can have the opposite case of a totally transmitted field ($E'_1=0$) if the initial field are such that $E_0=(1+\alpha)E'_0/(1-\alpha)$.

Next, we consider the case of two successive jumps from n_0 to the values n_1 and n_2 at times $t=0$ and $t=\tau$. We start with the initial conditions: $e_0=E_0$ and $e'_0=0$ at $t=0$. The final field amplitude can now be derived from Eq. (4)

$$\begin{aligned} E_2=&\frac{\alpha_0\alpha_1}{4}[(1+\alpha_0)(1+\alpha_1)e^{-i\omega_1\tau}\\ &+(1-\alpha_0)(1-\alpha_1)e^{+i\omega_1\tau}]e^{+i\omega_2\tau}E_0,\end{aligned}$$

$$\begin{aligned} E'_2=&-\frac{\alpha_0\alpha_1}{4}[(1-\alpha_0)(1+\alpha_1)e^{+i\omega_1\tau}\\ &+(1+\alpha_0)(1-\alpha_1)e^{-i\omega_1\tau}]e^{-i\omega_2\tau}E_0. \end{aligned} \tag{8}$$

Here we have used the relations $e_2(t)=E_2\exp(-i\omega_2 t)$ and $e'_2(t)=E'_2\exp(+i\omega_2 t)$.

A very interesting physical situation occurs when, after this time interval τ, the medium returns to its initial state: $n_2=n_0$. Such a situation can be called a temporal beam splitter, because of its obvious analogies with the usual (space) beam splitter. In this case the wave frequency also returns to its initial value: $\omega_2=\omega_0$. In contrast, the field will not return to its initial state, because of temporal interference. Noticing that $\alpha_0\alpha_1=1$, and using $\alpha_1=\alpha$, we can easily establish the final amplitudes of the transmitted and reflected waves;

$$E_2=\left[\cos(\omega_1\tau)-\frac{i}{2\alpha}(1+\alpha^2)\sin(\omega_1\tau)\right]e^{i\omega_0\tau}E_0,$$

$$E'_2=\frac{i}{2\alpha}(1-\alpha^2)\sin(\omega_1\tau)e^{-i\omega_0\tau}E_0. \tag{9}$$

The amplitude of the incident wave is larger than the initial wave, in order to compensate for the appearance of the reflected one. This means that we can have a new kind of amplification of light by the time-varying medium. This light amplification process attains its maximum value for time intervals such that $\omega_1\tau=(2n+1)\pi/2$ and $n=0,1,2,\ldots$.

For a small perturbation of the refractive index, we can use $\alpha=1+\delta$, with $|\delta|\ll 1$, and we get $E_{2max}\simeq i(1+\delta)\exp(i\omega_0\tau)E_0$ and $E'_{2max}\simeq i\delta\exp(-i\omega_0\tau)E_0$. This means that the wave energy W will increase in both directions by an amount $\Delta W\simeq\delta^2 W_0$, where W_0 is the initial energy. If we have Ω successive events of this kind per second, we will have an exponential increase of the wave energy: $W(t)\simeq W_0\exp(\delta^2\Omega t)$. In practical terms the growth rate $\delta^2\Omega$ can be considerably large and lead to the concept of a temporal optical resonator. This will be discussed in a future work.

III. QUANTUM MODEL FOR TEMPORAL BEAM SPLITTERS

We consider now the quantum theory of time refraction, by extending the results of our previous work [8] to the case of a temporal beam splitter. Let us represent the field operators. For purely transverse modes and for a real polarization vector $\vec{e}(k)=\vec{e}^*(k)$, we can write the electric-field operator valid inside the time interval $t_{j-1}<t<t_j$ as

$$\vec{E}(x,t)=i\sqrt{\frac{\hbar\omega}{2\epsilon}}[a_j(k,t)e^{ikx}-a_j^\dagger(k,t)e^{-ikx}]\vec{e}(k), \tag{10}$$

with the time-dependent operators

$$a_j(k,t)=a(k)e^{-i\omega t},\quad a_j^\dagger(k,t)=a^\dagger(k)e^{i\omega t}. \tag{11}$$

Applying the above continuity conditions, we are led to the following Bogoliubov transformation:

$$a_{j-1}(k,t_j)=A_{j-1}a_j(k,t_j)-B_{j-1}a_j^\dagger(-k,t_j),$$

$$a_{j-1}^\dagger(-k,t)=-B_{j-1}a_j(k,t_j)+A_{j-1}a_j^\dagger(-k,t_j), \tag{12}$$

where

$$A_{j-1}=\frac{1}{2}\frac{(1+\alpha_{j-1})}{\sqrt{\alpha_{j-1}}},\quad B_{j-1}=\frac{1}{2}\frac{(1-\alpha_{j-1})}{\sqrt{\alpha_{j-1}}}. \tag{13}$$

Let us first consider the simplest case of a single time jump at $t_0=0$. The operator transformations will reduce to $a_0=A_0a_1-B_0a_1^\dagger$ and $a_0^\dagger=-B_0a_1+A_0a_1^\dagger$, where we have written a and $a^\dagger$ instead of $a(k)$ and $a^\dagger(-k)$. This single-step process was considered in detail in our previous work on time refraction [8]. The corresponding coefficients are

$$A_0=\frac{1}{2}\frac{(1+\alpha_0)}{\sqrt{\alpha_0}},\quad B_0=\frac{1}{2}\frac{(1-\alpha_0)}{\sqrt{\alpha_0}}. \tag{14}$$

Let us now consider a second time step at $t_1=\tau$. From Eqs. (27) we can write

$$a_2(k,\tau)=A_1a_1(k,\tau)+B_1a_1^\dagger(-k,\tau),$$

$$a_2^\dagger(-k,\tau)=B_1a_1(k,\tau)+A_1a_1^\dagger(-k,\tau). \tag{15}$$

Expressing the operators a_1 and $a_1^\dagger$ in terms of a_0 and $a_0^\dagger$, we can find, after simple calculations,

$$a_2=A_{10}a_0+B_{10}a_0^\dagger,$$

$$a_2^\dagger=B_{10}^*a_0+A_{10}^*a_0^\dagger, \tag{16}$$

where we write a and $a^\dagger$ instead of $a(k)$ and $a^\dagger(-k)$. This new Bogoliubov transformation is a consequence of Eq. (12) applied for two consecutive time steps or temporal jumps. The corresponding coefficients contain the interference effects associated with the temporal phase shifts, already encountered in the above classical description,

$$A_{10}=A_1A_0e^{-i(\omega_1-\omega_2)\tau}+B_1B_0e^{+i(\omega_1+\omega_2)\tau},$$

$$B_{10}=A_1B_0e^{-i(\omega_1-\omega_2)\tau}+B_1A_0e^{+i(\omega_1+\omega_2)\tau}. \tag{17}$$

Let us return to the case where $n_2=n_0$, which corresponds to the temporal beam splitter. We have now $\omega_2=\omega_0$ and $\alpha_0=1/\alpha_1=\alpha$, which implies that $A_1=A_0$ and $B_1=-B_0$. The coefficients A_{10} and B_{10} defining the double Bogoliubov transformations (16) then take a very interesting form

$$A_{10}=\left[\cos(\omega_1\tau)-\frac{i}{2\alpha}(1+\alpha^2)\sin(\omega_1\tau)\right]e^{i\omega_0\tau},$$

$$B_{10}=-\frac{i}{2\alpha}(1-\alpha^2)\sin(\omega_1\tau)e^{i\omega_0\tau}. \tag{18}$$

The oscillations appearing in these expressions result from temporal interference associated with the secondary transmitted and reflected waves at the two successive temporal jumps. If the resonant condition $\cos(\omega_1\tau)=0$ is satisfied, we can obtain, from Eqs. (16) and (18), the following operator transformations:

$$a_2(k)=-\frac{i}{2\alpha}e^{i\omega_0\tau}[(1+\alpha^2)a_0(k)+(1-\alpha^2)a_0^\dagger(-k)],$$

$$a_2^\dagger(-k)=i2\alpha e^{-i\omega_0\tau}[(1-\alpha^2)a_0(k)+(1+\alpha^2)a_0^\dagger(-k)]. \tag{19}$$

These expressions describe the maximum possible coupling between field modes propagating in opposite directions. On the other hand, if the antiresonance condition $\sin(\omega_1\tau)=0$ is satisfied, we simply get $A_{10}=\exp(i\omega_0\tau)$ and $B_{10}=0$.

The establishment of the double Bogoliubov transformation (16) can be used to determine the temporal evolution of the quantum state of the field propagating in the nonstationary medium. For that purpose, let us introduce the symmetric Fock state $|n,n'\rangle_j$, for n photons propagating with wave number k, and n' photons with wave number $-k$, with frequency $\omega_j = kc/n_j$. It can be built from the corresponding vacuum state $|0,0\rangle_j$ by the operation

$$|n,n'\rangle_j = F_j(n,n')|0,0\rangle_j, \tag{20}$$

with

$$F_j(n,n') = \frac{1}{\sqrt{n!n'!}}[a_j^\dagger(k)]^n[a_j^\dagger(-k)]^{n'}. \tag{21}$$

If we start with the initial vacuum state $|0,0\rangle_0$, we can represent it in terms of the final states resulting from the temporal beam splitter $|m,m'\rangle_2$, in the form

$$|0,0\rangle_0 = \sum_{m,m'} C_{m,m'}|m,m'\rangle_2. \tag{22}$$

Due to momentum conservation, we have $C_{m,m'} = C_m \delta_{m,m'}$. Applying annihilation operators $a_0(k)$ to Eq. (20), and using normalized vector states, we can easily arrive at the following recurrence relation $C_{m+1} = \beta C_m = \beta^m C_0 = \beta^m e^{i\phi}\sqrt{1-\beta^2}$, where ϕ is an arbitrary phase and $\beta = B_{10}/A_{10}$. We can then calculate the probability for photon pair creation from vacuum by a temporal beam splitter with a duration τ. Calling $p(m)$ the probability for the creation of a symmetric Fock state of m photon pairs, we obtain

$$p(m) = |{}_2\langle m,m|0,0\rangle_0|^2 = |C_0|^2\beta^{2m} = (1-\beta^2)\beta^{2m}, \tag{23}$$

where the dependence on the width of the temporal beam splitter τ, and on the refractive index variation is implicitly given by the value of β.

IV. CONCLUSION

It was shown that a temporal beam splitter can result from two consecutive temporal changes of the optical properties of a medium, due to the interference of partially reflected and partially transmitted signals resulting from these changes. This is a straightforward generalization of the process of time refraction [8], and can be seen as a four-port device which is the temporal analog of the usual (spatial) optical beam splitter.

This process was described here by using both classical and quantum formulations. The classical approach allowed us to derive appropriate Fresnel laws for temporal refraction and to discover a possible new mechanism for optical amplification, due to successive temporal changes of the medium. This was described as an electromagnetic instability where some external agent, responsible for these temporal jumps, produces work on the photon field and converts a fraction of its energy into radiation. Here, this external agent (a laser beam or an intense electric field, for instance) was assumed as an independent parameter and an infinite source of energy. Obviously, such a description is valid only when the energy flowing into the electromagnetic field is significantly lower than the energy associated with the optical change in the medium.

The present results are only valid for perturbations on a time scale shorter than the period of the wave modes, but they can easily be extended to arbitrary time scales. Future models will have to specify the appropriate physical conditions where a temporal beam splitter can occur. This will eventually imply the use of laser pulses with a size shorter than the medium where the optical changes are taking place. For very short pulses, with durations below 100 fs, a temporal beam splitter can be achieved by using a second short laser pulse, acting as a pump, and applied to a submillimeter-size nonlinear optical medium (a crystal or simple piece of glass).

On the other hand, the quantum approach was able to confirm our classical modeling, and led to general forms of Bogoliubov transformations where temporal interference was included. The electromagnetic instability derived from the classical model was replaced here by the possible creation of pairs of photons from vacuum, propagating with the same frequency and in opposite directions in order to preserve momentum conservation. We have also shown that the photon creation process is present for arbitrary temporal beam splitters with a duration τ, and that the rate of creation depends resonantly on the photon frequency and this time interval. The existence of such a temporal resonance can then be used to build up a temporal resonance cavity for light amplification, which will be analyzed elsewhere [15].

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