

# Generation of atom-photon entangled states in atomic Bose-Einstein condensate via electromagnetically induced transparency

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In this paper, we present a method to generate continuous-variable-type entangled states between photons and atoms in atomic Bose-Einstein condensate (BEC). The proposed method involves an atomic BEC with three internal states, a weak quantized probe laser, and a strong classical coupling laser, which form a three-level  $\Lambda$ -shaped BEC system. We consider a situation where the BEC is in electromagnetically induced transparency with the coupling laser being much stronger than the probe laser. In this case, the upper and intermediate levels are unpopulated, so that their adiabatic elimination enables an effective two-mode model involving only the atomic field at the lowest internal level and the quantized probe laser field. Atom-photon quantum entanglement is created through laser-atom and interatomic interactions, and two-photon detuning. We show how to generate atom-photon entangled coherent states and entangled states between photon (atom) coherent states and atom-(photon-) macroscopic quantum superposition (MQS) states, and between photon-MQS and atom-MQS states.

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## I. INTRODUCTION

For decades, quantum entanglement has been the focus of much work in the foundations of quantum mechanics, being particularly associated with quantum nonseparability, the violation of Bell's inequalities, and the so-called Einstein-Pololsky-Rosen paradox. Beyond this fundamental aspect, creating and manipulating of entangled states are essential for quantum information applications. Among these applications are quantum computation [1,2], quantum teleportation [3,4], quantum dense coding [5], quantum cryptography [6], and quantum positioning and clock synchronization [7]. Hence, quantum entanglement has been viewed as an essential resource for quantum information processing.

Recently much attention has been paid to continuous-variable quantum information processing in which continuous-variable-type entangled pure states play a key role. For instance, two-state entangled coherent states are used to realize efficient quantum computation [8,9] and quantum teleportation [10]. Two-mode squeezed vacuum states have been applied to quantum dense coding [11]. Especially, continuous-variable teleportation [12] has been experimentally demonstrated for coherent states of a light field [13] by using entangled two-mode squeezed vacuum. It also has been shown that a two-state entangled squeezed vacuum state can be used to realize quantum teleportation of an arbitrary coherent superposition state of two equal-amplitude and opposite-phase squeezed vacuum states [14]. Continuous-variable-type entangled states including squeezed states and coherent states have also been widely applied to quantum cryptography [15–20]. Therefore, it is an interesting topic to create continuous-variable-type entangled pure states.

On the other hand, it is well known that atoms and photons can be viewed as carriers of storing and transmitting quantum information. Atoms are suited for storing quantum

information at a fixed location, and they are sources of local entanglement for quantum information processing while photons are natural sources for communications of quantum information since they can traverse over long distances. One topic of particular interest in this context is the possibility to generate atom-photon entangled states where one member is readily used to couple a distant system while the other is stored by the local sender. In this aspect, some progress has been made, and possible schemes have been proposed in recent years. For instance, Moore and Meystre [21] proposed a scheme to generate atom-photon pairs via off-resonant light scattering from an atomic Bose-Einstein condensate (BEC). Deb and Agarwal [22] showed that it is possible to entangle three different many-particle states by Bragg spectroscopy with nonclassical light in an atomic BEC. Gasenzer and co-workers [23] investigated quantum entanglement characterized by the relative number squeezing between photons and atoms coupled out from an atomic BEC. Optical control over the BEC quantum statistics and atom-photon quantum correlation in an atomic BEC [24–27] have been widely studied.

In this paper, we describe a method to produce atom-photon-entangled states of continuous-variable-type pure states in an atomic BEC by electromagnetically induced transparency (EIT) [28]. EIT is a kind of quantum interference effects [29]. It arises in three-level (or multilevel) systems and consists of the cancellation of the absorption on one transition induced by simultaneous coherent driving of another transition. The phenomenon can be understood as a destructive interference of the two pathways to the excited level and has been used to demonstrate ultraslow light propagation [30–34] and light storage [35,36] in many systems including atomic BECs [30,31]. The proposed system in the present paper consists of an atomic BEC with three internal states, one weak probe laser and one relatively strong coupling laser with appropriate frequencies. The probe-laser

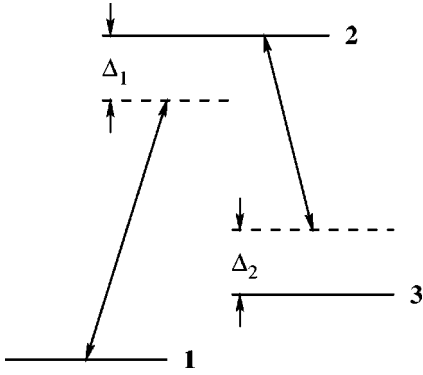


FIG. 1. Three-level  $\Lambda$ -shaped atoms coupled to a quantized probe laser and a classical coupling laser with the detunings  $\Delta_1$  and  $\Delta_2$ , respectively.

field is quantized while the coupling-laser field is treated classically. They form a three-level  $\Lambda$  configuration. When the BEC is in EIT with the coupling laser being much stronger than the probe laser, the upper and intermediate levels are unpopulated due to quantum interference. Adiabatic elimination of the two unpopulated levels enables an effective two-mode model involving only atoms in the lowest internal state and photons in the probe laser. We show that it is possible to generate atom-photon continuous-variable-type entangled pure states in the atomic BEC. This paper is organized as follows. In Sec. II, we present the physical system under our consideration, review the reduction of the problem from four to two modes, followed by an analytic solution of the model, and then discuss the theoretical mechanism to create quantum entanglement in the proposed scheme. In Sec. III, we focus on a dynamical approach to creating a variety of atom-photon entangled pure states. Two-state, three-state, and four-state atom-photon continuous-variable-typed entangled pure states are generated explicitly. We shall conclude our paper with discussions and remarks in the last section.

## II. PHYSICAL MODEL AND SOLUTION

We consider a cloud of BEC atoms that have three internal states labeled by  $|1\rangle$ ,  $|2\rangle$ , and  $|3\rangle$  with energies  $E_1$ ,  $E_2$ , and  $E_3$ , respectively. The two lower states  $|1\rangle$  and  $|3\rangle$  are metastable states in each of which the atoms can live for a long time. They are Raman coupled to the upper state  $|2\rangle$  via, respectively, a quantized probe-laser field and a classical coupling-laser field of frequencies  $\omega_1$  and  $\omega_2$  in the  $\Lambda$  configuration. The interaction scheme is shown in Fig. 1. The atoms in these internal states are subject to isotropic harmonic trapping potentials  $V_i(\mathbf{r})$  for  $i=1,2,3$ , respectively. Furthermore, the atoms in the BEC interact with each other via elastic two-body collisions with the  $\delta$ -function potentials  $V_{ij}(\mathbf{r}-\mathbf{r}')=U_{ij}\delta(\mathbf{r}-\mathbf{r}')$ , where  $U_{ij}=4\pi\hbar^2 a_{ij}/m$  with  $m$  and  $a_{ij}$ , respectively, being the atomic mass and the  $s$ -wave scattering length between atoms in states  $i$  and  $j$ . A good experimental candidate of this system is the sodium atom condensate for which there exist appropriate atomic internal levels and external laser fields to form the  $\Lambda$  configuration which is needed for reaching EIT under our consideration.

The sodium atom condensate in EIT has been used to demonstrate ultraslow light propagation [30] and amplification of light and atoms [31] in atomic BECs.

The second quantized Hamiltonian to describe the system at zero temperature is given by

$$\hat{H}=\hat{H}_p+\hat{H}_a+\hat{H}_{a-l}+\hat{H}_c, \quad (1)$$

where  $\hat{H}_p$  and  $\hat{H}_a$  give the free evolution of the probe-laser field and the atomic fields respectively,  $\hat{H}_{a-l}$  describes the dipole interactions between the atomic fields and the probe and coupling fields, and  $\hat{H}_c$  represents interatom two-body collisional interactions.

The free atomic Hamiltonian is given by

$$\hat{H}_a=\sum_{i=1}^3\int d\mathbf{x}\hat{\psi}_i^\dagger(\mathbf{x})\left[-\frac{\hbar^2}{2m}\nabla^2+V_i(\mathbf{x})+E_i\right]\hat{\psi}_i(\mathbf{x}), \quad (2)$$

where  $E_i$  are internal energies for the three internal states,  $\hat{\psi}_i(\mathbf{x})$  and  $\hat{\psi}_i^\dagger(\mathbf{x})$  are the boson annihilation and creation operators for the  $|i\rangle$ -state atoms at position  $\mathbf{x}$ , respectively, they satisfy the standard boson commutation relation  $[\hat{\psi}_i(\mathbf{x}),\hat{\psi}_j^\dagger(\mathbf{x}')]=\delta_{ij}\delta(\mathbf{x}-\mathbf{x}')$  and  $[\hat{\psi}_i(\mathbf{x}),\hat{\psi}_j(\mathbf{x}')]=0=[\hat{\psi}_i^\dagger(\mathbf{x}),\hat{\psi}_j^\dagger(\mathbf{x}')]$ .

The free evolution of the probe-laser field is governed by the Hamiltonian

$$\hat{H}_p=\hbar\omega_1\hat{a}_1^\dagger\hat{a}_1, \quad (3)$$

where  $\omega_1$  is the frequency of the probe laser, and  $\hat{a}_1^\dagger$  and  $\hat{a}_1$  are the photon creation and annihilation operators for the probe-laser field, satisfying the boson communication relation  $[\hat{a}_1,\hat{a}_1^\dagger]=1$ .

The atom-laser interactions in the dipole approximation can be described by the following Hamiltonian

$$\hat{H}_{a-l}=-\hbar\int d\mathbf{x}\left[g\hat{a}_1\hat{\psi}_2^\dagger(\mathbf{x})\hat{\psi}_1(\mathbf{x})e^{i(\mathbf{k}_1\cdot\mathbf{x}-\omega_1 t)}+\frac{1}{2}\Omega\hat{\psi}_2^\dagger(\mathbf{x})\hat{\psi}_3(\mathbf{x})e^{i(\mathbf{k}_2\cdot\mathbf{x}-\omega_2 t)}+\text{H.c.}\right], \quad (4)$$

where the two dipole coupling constants are defined by  $g=\mu_{21}\mathcal{E}_1/\hbar$  and  $\Omega=\mu_{23}E_2/\hbar$  with  $\mu_{ij}$  denoting a transition dipole-matrix element between states  $|i\rangle$  and  $|j\rangle$ ,  $\mathcal{E}_1=\sqrt{\hbar\omega_1/2\epsilon_0V}$  being the electric field per photon for the quantized probe light of frequency  $\omega_1$  in a mode volume  $V$ , and  $E_2$  being the amplitude of the electric field for the classical coupling light of frequency  $\omega_2$ ,  $\mathbf{k}_1$  and  $\mathbf{k}_2$  are wave vectors of corresponding laser fields.

The collisional Hamiltonian is taken to be of the following form:

$$\hat{H}_c = \frac{2\pi\hbar^2}{m} \int d\mathbf{x} \left[ \sum_{i=1}^3 a_i^{sc} \hat{\psi}_i^\dagger(\mathbf{x}) \hat{\psi}_i^\dagger(\mathbf{x}) \hat{\psi}_i(\mathbf{x}) \hat{\psi}_i(\mathbf{x}) + \sum_{i \neq j} 2a_{ij}^{sc} \hat{\psi}_i^\dagger(\mathbf{x}) \hat{\psi}_j^\dagger(\mathbf{x}) \hat{\psi}_i(\mathbf{x}) \hat{\psi}_j(\mathbf{x}) \right], \quad (5)$$

where  $a_i^{sc}$  is the  $s$ -wave scattering length of condensed atoms in the internal state  $|i\rangle$  and  $a_{ij}^{sc}$  that between condensed atoms in the internal states  $|i\rangle$  and  $|j\rangle$ .

For a weakly interacting BEC at zero temperature there are no thermally excited atoms and the quantum depletion is negligible, the motional state is frozen to be approximately the ground state. One may neglect all modes except for the condensate mode and approximately factorize the atomic field operators as  $\hat{\psi}_i(\mathbf{x}) \approx \hat{b}_i \phi_i(\mathbf{x})$  where  $\phi_i(\mathbf{x})$  is a normalized wave function for the atoms in the BEC in the internal state  $|i\rangle$ , which is given by the ground state of the following Schrödinger equation:

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V_i(\mathbf{x}) + E_i \right] \phi_i(\mathbf{x}) = \hbar \nu_i \phi_i(\mathbf{x}), \quad (6)$$

where  $\hbar \nu_i$  is the energy of mode  $i$ , and  $\nu_i$  denotes the frequency of the free evolution of the condensate in the internal state  $|i\rangle$ .

Substituting the single-mode expansions of the atomic field operators into Eqs. (2), (4), and (5), we arrive at the following four-mode approximate Hamiltonian:

$$\begin{aligned} \hat{H} = & \hbar \omega_1 \hat{a}_1^\dagger \hat{a}_1 + \hbar \sum_{i=1}^3 \nu_i \hat{b}_i^\dagger \hat{b}_i - \hbar [g_1 \hat{a}_1 \hat{b}_2^\dagger \hat{b}_1 e^{-i\omega_1 t} \\ & + g_2 \hat{b}_2^\dagger \hat{b}_3 e^{-i\omega_2 t} + \text{H.c.}] + \hbar \sum_{i=1}^3 \lambda_i \hat{b}_i^{\dagger 2} \hat{b}_i^2 \\ & + \hbar \sum_{i \neq j} \lambda_{ij} \hat{b}_i^\dagger \hat{b}_i \hat{b}_j^\dagger \hat{b}_j, \end{aligned} \quad (7)$$

where one mode corresponds to the probe-laser field, the other three correspond to atomic fields in the three internal state. Here  $g_1$  and  $g_2$  denote the laser-atom dipole interactions, respectively, defined by

$$g_1 = g \int d\mathbf{x} \phi_2^*(\mathbf{x}) \phi_1(\mathbf{x}) e^{i\mathbf{k}_1 \cdot \mathbf{x}}, \quad (8)$$

$$g_2 = \frac{1}{2} \Omega \int d\mathbf{x} \phi_2^*(\mathbf{x}) \phi_3(\mathbf{x}) e^{i\mathbf{k}_2 \cdot \mathbf{x}}, \quad (9)$$

and  $\lambda_i$  and  $\lambda_{ij}$  ( $i, j = 1, 2, 3$ ) describe interatomic interactions given by

$$\lambda_i = \frac{2\pi\hbar^2 a_i^{sc}}{m} \int d\mathbf{x} |\phi_i(\mathbf{x})|^4, \quad (10)$$

$$\lambda_{ij} = \frac{4\pi\hbar^2 a_{ij}^{sc}}{m} \int d\mathbf{x} |\phi_i(\mathbf{x})|^2 |\phi_j(\mathbf{x})|^2 \quad (i \neq j). \quad (11)$$

Hamiltonian (7) can be decomposed to the sum of an interaction Hamiltonian and a free evolution Hamiltonian defined by

$$H_0 = \hbar \omega_1 \hat{a}_1^\dagger \hat{a}_1 + \hbar \nu_1 \sum_{i=1}^3 \hat{b}_i^\dagger \hat{b}_i + \hbar (\omega_1 - \omega_2) \hat{b}_3^\dagger \hat{b}_3 + \hbar \omega_1 \hat{b}_2^\dagger \hat{b}_2. \quad (12)$$

Going over to an interaction picture with respect to  $H_0$ , we transfer the time-dependent Hamiltonian (7) to the following time-independent Hamiltonian:

$$\begin{aligned} \hat{H}_I = & \hbar (\Delta_1 - \Delta_2) \hat{b}_3^\dagger \hat{b}_3 + \Delta_1 \hat{b}_2^\dagger \hat{b}_2 - \hbar [g_1 \hat{a}_1 \hat{b}_2^\dagger \hat{b}_1 + g_2 \hat{b}_2^\dagger \hat{b}_3 \\ & + \text{H.c.}] + \hbar \sum_{i=1}^3 \lambda_i \hat{b}_i^{\dagger 2} \hat{b}_i^2 + \hbar \sum_{i \neq j} \lambda_{ij} \hat{b}_i^\dagger \hat{b}_i \hat{b}_j^\dagger \hat{b}_j, \end{aligned} \quad (13)$$

where  $\Delta_1 = \nu_2 - \nu_1 - \omega_1$  and  $\Delta_2 = \nu_2 - \nu_3 - \omega_2$  are the detunings of the two laser beams, respectively.

We consider the situation of the ideal EIT which is attained only when the system is at the two-photon resonance with the two-photon detuning  $\Delta = \Delta_1 = \Delta_2$ . Initially the lasers are outside the BEC medium in which all atoms are in their ground state, i.e., the internal state  $|1\rangle$ . The condensed atoms are generally in a superposition state of the state  $|1\rangle$  and the state  $|3\rangle$  when they are in EIT. However, when the coupling laser is much stronger than the probe laser, atomic population at the intermediate level approaches zero while the upper level is unpopulated [37]. Hence, under the condition of  $(g_1/g_2)^2 \ll 1$ , the terms that involve  $\hat{b}_2^\dagger \hat{b}_2$  and  $\hat{b}_3^\dagger \hat{b}_3$  in Hamiltonian (13) may be neglected, and from Hamiltonian (13) we can adiabatically eliminate the atomic field operators in the internal states  $|2\rangle$  and  $|3\rangle$ . First, the atomic operators in the internal state  $|2\rangle$   $\hat{b}_2$  and  $\hat{b}_2^\dagger$  can be adiabatically eliminated with the replacements  $\hat{b}_2 = (g_1 \hat{a}_1 \hat{b}_1 + g_2 \hat{b}_3) / \Delta$  and  $\hat{b}_2^\dagger = (g_1^* \hat{a}_1^\dagger \hat{b}_1^\dagger + g_2^* \hat{b}_3^\dagger) / \Delta$ , which come from the adiabatic elimination process in Heisenberg equations  $i\hat{b}_2 = [\hat{b}_2, \hat{H}_I] = 0$  and  $i\hat{b}_2^\dagger = [\hat{b}_2^\dagger, \hat{H}_I] = 0$ , respectively. From Eq. (13) we can obtain the following effective Hamiltonian

$$\begin{aligned} \hat{H}'_I = & 2\omega'_1 \hat{b}_1^\dagger \hat{b}_1 + \omega'_3 \hat{b}_3^\dagger \hat{b}_3 + (g' \hat{b}_3^\dagger \hat{a}_1 \hat{b}_1 + g'^* \hat{b}_1^\dagger \hat{a}_1 \hat{b}_3) \\ & + \lambda_1 \hat{b}_1^{\dagger 2} \hat{b}_1^2 + \omega'_1 \hat{a}_1^\dagger \hat{a}_1 \hat{b}_1^\dagger \hat{b}_1, \end{aligned} \quad (14)$$

which contains only atomic field operators in internal states  $|1\rangle$  and  $|3\rangle$  and the probe field operators. Here we have set  $\hbar = 1$  and introduced

$$\omega'_1 = -\frac{|g_1|^2}{\Delta}, \quad \omega'_3 = -\frac{|g_2|^2}{\Delta}, \quad g' = -\frac{g_1 g_2^*}{\Delta}. \quad (15)$$

The first part  $\hat{a}_1 \hat{b}_3^\dagger \hat{b}_1$  in the third term of Eq. (14) describes such a quantum transition process where an atom in the lowest internal state  $|1\rangle$  absorbs a photon in the probe laser and then it transits to the intermediate internal state  $|3\rangle$ . The second part  $\hat{b}_1^\dagger \hat{a}_1 \hat{b}_3$  describes the inverse process of this process.

Then from Hamiltonian (14) the atomic field operators in the third internal state can be adiabatically eliminated through the replacements  $\hat{b}_3 = -g_1 \hat{a}_1 \hat{b}_1 / g_2$  and  $\hat{b}_3^\dagger = -g_1^* \hat{a}_1^\dagger \hat{b}_1^\dagger / g_2^*$ , and the final effective Hamiltonian is given by

$$\hat{H}_{eff} = 2\omega'_1 \hat{b}_1^\dagger \hat{b}_1 + 4\omega'_1 \hat{b}_1^\dagger \hat{b}_1 \hat{a}_1^\dagger \hat{a}_1 + \lambda_1 \hat{b}_1^{\dagger 2} \hat{b}_1^2. \quad (16)$$

The effective Hamiltonian (16) is diagonal in the Fock space with the bases defined by

$$|n, m\rangle = \frac{1}{\sqrt{n!m!}} \hat{a}_1^{\dagger n} \hat{b}_1^{\dagger m} |0, 0\rangle, \quad (17)$$

which are eigenstates of the effective Hamiltonian with the eigenvalues given by the following expression:

$$E(n, m) = 2\omega'_1 m + 4\omega'_1 n m + \lambda_1 m(m-1), \quad (18)$$

where  $n$  and  $m$  take non-negative integers.

From the effective Hamiltonian (16) we can well understand the theoretical mechanism to create atom-photon quantum entanglement in the present scheme. Comparing Eq. (16) with Eq. (13) we can see that for the BEC in the EIT, through adiabatically eliminating the atomic field operators at the upper and intermediate levels, the laser-atom dipole interactions are converted as an effective collisional interaction between photons in the probe laser and condensed atoms in the internal state  $|1\rangle$  which can be described as effective elastic collisions between photons in the probe laser and condensed atoms in the internal state  $|1\rangle$ . From Eq. (15) we can know that the effective scattering length to describe the atom-photon collisions is proportional to the ratio of the square of the dipole interaction and the two-photon detuning. One can manipulate signs of the effective scattering length by changing signs of the two-photon detuning. Therefore, atom-photon quantum entanglement is produced by atom-photon and atom-atom collisional interactions described by the second and third terms in the effective Hamiltonian (16), respectively, and one can control or manipulate atom-photon entanglement by varying the dipole interaction strength between the probe laser and the BEC and the two-photon detuning to create desired atom-photon entangled states.

It should be mentioned that in most textbook introductions to EIT [38], both probe and coupling lasers are treated classically, the decay of the excited state is included in the dynamics of the internal state through a set of density-matrix equations of the atomic system, the decay promotes the trapping of the atom into a dark state. Contrary to the usual treatment of EIT, the probe laser is quantized in this paper. A limitation of our present treatment is that we have ignored the decay rates of various levels. However, this ignoring of the decay rates is a good approximation for the adiabatic EIT that we study in the present paper, since the adiabatic EIT is insensitive to any possible decay of the top level [39].

### III. ATOM-PHOTON ENTANGLED STATES

In this section we investigate generation of various atom-photon entangled states when the condensate in internal states  $|1\rangle$  and the probe laser are initially in nonentangled coherent states and superposition states of coherent states.

We assume that the probe laser and the condensate are initially uncorrelated and in the product coherent state  $|\alpha, \beta\rangle \equiv |\alpha\rangle \otimes |\beta\rangle$ . With the laser fields turned on at  $t=0$ , then at time  $t>0$  the state of the system becomes

$$|\Phi(\tau)\rangle = \sum_{n,m=0}^{\infty} \exp\{-it[2\omega'_1 m + 4\omega'_1 n m + \lambda_1 m(m-1)]\} \times \exp\left[-\frac{1}{2}(|\alpha|^2 + |\beta|^2)\right] \frac{\alpha^n \beta^m}{\sqrt{n!m!}} |n, m\rangle. \quad (19)$$

In what follows we shall see that starting with state (19) different atom-photon entangled states can be obtained at different times through adjusting various interaction strengths and the two-photon detuning. In order to do this, we rewrite state (19) as the following simple form:

$$|\Phi(\tau)\rangle = e^{-1/2(|\alpha|^2 + |\beta|^2)} \sum_{n,m=0}^{\infty} e^{i\tau\theta_{n,m}} \frac{\alpha^n \beta^m}{\sqrt{n!m!}} |n, m\rangle, \quad (20)$$

where we have used a scaled time  $\tau = \lambda_1 t$  and a running frequency

$$\theta_{n,m} = (1+K)m + 2Kn m - m^2, \quad (21)$$

where we have introduced a real effective interaction parameter  $K$  defined by

$$K = \frac{2|g_1|^2}{\lambda_1 \Delta}, \quad (22)$$

which describes an effective interaction induced by three adjustable parameters: the dipole interaction strength  $g_1$ , the interatomic interaction strength  $\lambda_1$ , and the two-photon detuning  $\Delta$ . The effective interaction parameter  $K$  can take positive or negative values depending on the signs of  $\lambda_1$  and  $\Delta$ . For an atomic BEC with a positive  $s$ -wave scattering length,  $K$  is positive (negative) when the two-photon detuning  $\Delta$  is positive (negative). From Eqs. (20) and (21) we can see that the time evolution characteristic of the system under our consideration is completely determined by the effective interaction parameter.

We note that the wave function of system (20) is a two-mode extension of generalized coherent states [40–42]. These generalized coherent states differ from a conventional Glauber coherent state by an extra phase factor appearing in the decomposition of such states into a superposition of Fock states. They can always be represented as a continuous sum of coherent states. And under appropriate periodic conditions, they can reduce to discrete superpositions of coherent states. It should be kept in mind that we use to create continuous-variable-type entangled states what we expect.



For the two-mode case under our consideration, we can express state (20) as the following integral form:

$$|\Phi(\tau)\rangle = \int_0^{2\pi} \frac{d\phi_1}{2\pi} \int_0^{2\pi} \frac{d\phi_3}{2\pi} f(\phi_1, \phi_3) |\alpha e^{i\phi_1}, \beta e^{i\phi_3}\rangle, \quad (23)$$

where the phase function is given by

$$f(\phi_1, \phi_3) = \exp[i(\tau\theta_{n,m} - n\phi_1 - m\phi_3)]. \quad (24)$$

Equation (24) indicates that the state  $|\Phi(\tau)\rangle$  is a continuous superposition state of two-mode product coherent states. Note that entangled states that we expect are generally superposition states of two-mode product coherent states.

From Eqs. (21)–(24) we can see that the values of the real parameter  $K$  may seriously affect the form of the wave function (23). Of particular interest is a situation where  $K$  may take values of nonzero integers. In this case, making use of Eq. (21), from Eq. (20) we can see that  $|\Phi(\tau + 2\pi)\rangle = |\Phi(\tau)\rangle$ , which implies that the time evolution of the wave function (20) is a periodic evolution with respect to scaled time  $\tau$  with the period  $2\pi$ . On the other hand, suppose that the scaled time  $\tau$  takes its values in the following manner:

$$\tau = \frac{M}{N} 2\pi, \quad (25)$$

where  $M$  and  $N$  are mutually prime integers, then we can find

$$\exp\left[i2\pi\frac{M}{N}\theta_{n+N, m+N}\right] = \exp\left[i2\pi\frac{M}{N}\theta_{n, m}\right], \quad (26)$$

which means that the exponential phase function in state (24)  $\exp[2\pi i(M/N)\theta_{n,m}]$  is a periodic function with respect to both  $n$  and  $m$  with the same period  $N$ . If  $\tau$  takes its values according to Eq. (25), as a fraction of the period, then the wave function (23) becomes a discrete superposition state of product coherent states which can be expressed as follows:

$$\left|\Phi\left(\tau = \frac{M}{N} 2\pi\right)\right\rangle = \sum_{r=1}^N \sum_{s=1}^N c_{rs} |\alpha e^{i\varphi_r}, \beta e^{i\varphi_s}\rangle, \quad (27)$$

where the two running phases are defined by

$$\varphi_r = \frac{2\pi}{N} r, \quad \varphi_s = \frac{2\pi}{N} s \quad (r, s = 1, 2, \dots, N). \quad (28)$$

From Eqs. (20) and (27) we can find the following equation to determine the coefficients  $c_{rs}$ :

$$\sum_{r,s=1}^N c_{rs} \exp\left\{\frac{2\pi i}{N}[nr + ms - M\theta_{n,m}]\right\} = 1. \quad (29)$$

Carrying out summations over  $n$  and  $m$  in both sides of the above equation from 1 to  $N$ , and taking into account the normalization condition  $\sum_{r,s=1}^N c_{rs} c_{rs}^* = 1$ , we find the coefficients  $c_{rs}$  to be given by

$$c_{rs} = \frac{1}{N^2} \sum_{n,m=1}^N \exp\left\{-\frac{2\pi i}{N}[nr + ms - M\theta_{n,m}]\right\}. \quad (30)$$

It is straightforward to see that the discrete superposition state (27) is generally an entangled coherent state with  $N^2$  independent product coherent states. As a specific example of creating continuous-variable-type entangled states, in what follows we discuss generation of entangled states for the case of  $K = -1$ .

*Two-state entangled states.* Assume that the probe laser and the condensate are initially in coherent states  $|\alpha\rangle$  and  $|\beta\rangle$ , respectively. When  $K = -1$ ,  $N = 4$ , and  $M = 1$ , from Eqs. (21) and (30) we find that nonzero  $c$  coefficients are

$$c_{22} = -c_{24} = ic_{42} = ic_{44} = \frac{i}{2}, \quad (31)$$

which results in the following unnormalized two-state entangled state:

$$\left|\Phi\left(\tau = \frac{\pi}{2}\right)\right\rangle = \frac{1}{2} [|\beta_+|\alpha\rangle \otimes |\beta\rangle_+ - i|\beta_-|\alpha\rangle \otimes |\beta\rangle_-], \quad (32)$$

where  $|\beta\rangle_{\pm}$  are two normalized atomic Schrödinger cat states, i.e., even and odd coherent quantum superposition states defined by

$$|\beta\rangle_{\pm} = \frac{1}{\beta_{\pm}} (|\beta\rangle \pm |-\beta\rangle), \quad \beta_{\pm} = \sqrt{2(1 \pm e^{-2|\beta|^2})}. \quad (33)$$

Hence, state (32) is an entangled state between two photon-coherent states and two atomic Schrödinger cat states where one member is photons in the probe laser the other is atoms in the condensate.

The degree of quantum entanglement of the two-state entangled states (32) can be measured in terms of the concurrence [43,44] which is generally defined for discrete-variable entangled states to be [44]

$$\mathcal{C} = |\langle \Psi | \sigma_y \otimes \sigma_y | \Psi^* \rangle|, \quad (34)$$

where  $|\Psi^*\rangle$  is the complex conjugate of  $|\Psi\rangle$ . The concurrence equals one for a maximally entangled state.

In order to calculate the concurrence of continuous-variables-type entangled states such as Eq. (32), we consider a general biparticle entangled state

$$|\Psi\rangle = \mu |\eta\rangle \otimes |\gamma\rangle + \nu |\xi\rangle \otimes |\delta\rangle, \quad (35)$$

where  $|\eta\rangle$  and  $|\xi\rangle$  are normalized states of subsystem 1 and  $|\gamma\rangle$  and  $|\delta\rangle$  are normalized states of subsystem 2 with complex  $\mu$  and  $\nu$ . After normalization, the biparticle state  $|\Psi\rangle$  can be expressed as

$$|\Psi\rangle = \frac{1}{N} [\mu |\eta\rangle \otimes |\gamma\rangle + \nu |\xi\rangle \otimes |\delta\rangle], \quad (36)$$

where the normalization constant is given by

$$N^2 = |\mu|^2 + |\nu|^2 + 2\text{Re}(\mu^* \nu p_1 p_2^*),$$

$$p_1 = \langle \eta | \xi \rangle, \quad p_2 = \langle \delta | \gamma \rangle. \quad (37)$$

Assume that  $|\eta\rangle$  and  $|\xi\rangle$  ( $|\gamma\rangle$  and  $|\delta\rangle$ ) are linearly independent and span a 2-dim subspace of the Hilbert space. Then one can choose a discrete orthogonal basis  $\{|0\rangle_i, |1\rangle_i\}$  as in Ref. [45]

$$|0\rangle_1 = |\eta\rangle, \quad |1\rangle_1 = \frac{1}{\sqrt{1-|p_1|^2}}(|\xi\rangle - p_1|\eta\rangle),$$

$$|0\rangle_2 = |\delta\rangle, \quad |1\rangle_2 = \frac{1}{\sqrt{1-|p_2|^2}}(|\gamma\rangle - p_2|\delta\rangle). \quad (38)$$

By using the above discrete orthogonal basis, one can express the biparticle state  $|\Psi\rangle$  as the following two-qubit entangled state:

$$|\Psi\rangle = \frac{1}{N} [(\mu p_2 + \nu p_1)|00\rangle + \mu \sqrt{1-|p_2|^2}|01\rangle + \nu \sqrt{1-|p_1|^2}|10\rangle], \quad (39)$$

which can be written in terms of a Schmidt decomposition [45] as

$$|\Psi\rangle = c_+|++\rangle + c_-|--\rangle, \quad (40)$$

where  $|+\rangle_i$  and  $|-\rangle_i$  are the orthonormal eigenvectors of the reduced density operators for state (39) and  $c_\pm = \sqrt{\lambda_\pm}$  are the square roots of the corresponding eigenvalues given by

$$\lambda_\pm = \frac{1}{2} \pm \frac{1}{2} \left[ 1 - \frac{4|\mu\nu|}{N^2} \sqrt{(1-|p_1|^2)(1-|p_2|^2)} \right]^{1/2}. \quad (41)$$

Making use of the Schmidt decomposition (40) and Eq. (41), from Eq. (34) it is easy to find the concurrence of the entangled state (35) to be [46]

$$\mathcal{C} = 2c_+c_- = \frac{2|\mu||\nu|}{N^2} \sqrt{(1-|p_1|^2)(1-|p_2|^2)}. \quad (42)$$

For the two-state entangled state (32), from Eq. (42) we find the corresponding concurrence

$$\mathcal{C} = \sqrt{[1 - \exp(-4|\alpha|^2)][1 - \exp(-4|\beta|^2)]}, \quad (43)$$

which indicates that the concurrence of the two-state entangled state (32) is dependent on the values of the initial-state parameters of the condensate and the probe laser. From Eq. (43) we can also see that one can manipulate quantum entanglement of the two-state entangled states (32) by varying the intensity of the probe laser and the initial number of atoms in the condensate. In particular, in the parameter regime in which one can adiabatically eliminate the top level state and the second ground internal state, i.e.,  $(g_1/g_2)^2 \ll 1$ , the stronger the the intensity of the probe laser is or/and

the more the number of atoms in the condensate are, the larger the amount of entanglement of state (32).

It is interesting to note that state (32) can also be expressed as an entangled state of two atomic coherent states with two photon Schrödinger cat states,

$$\left| \Phi \left( \tau = \frac{\pi}{2} \right) \right\rangle = \frac{1}{2} [|\alpha\rangle_- \otimes |\beta\rangle + |\alpha\rangle_+ \otimes |-\beta\rangle], \quad (44)$$

where  $|\alpha\rangle_\pm$  are two normalized photon Schrödinger cat states defined by

$$|\alpha\rangle_\pm = \frac{1}{\sqrt{2}} (|\alpha\rangle \pm i|-\alpha\rangle). \quad (45)$$

Although states (32) and (44) are two different decompositions of the same state of the system under our consideration, they have the same concurrence, which means that their degrees of entanglement are the same.

In order to obtain atom-photon entangled coherent states, we assume that the probe laser is initially in a coherent photon cat state while the condensate is initially in a coherent state. Namely, the initial state of the atom-photon system is

$$|\Phi(\tau=0)\rangle = \frac{1}{\sqrt{2}} (|\alpha\rangle + i|-\alpha\rangle) \otimes |\beta\rangle. \quad (46)$$

When  $K = -1$ ,  $N = 4$ , and  $M = 1$ , a  $\pi/2$  evolution drives the system to the following entangled coherent state:

$$\left| \Phi \left( \tau = \frac{\pi}{2} \right) \right\rangle = \frac{1}{\sqrt{2}} (|\alpha\rangle \otimes |\beta\rangle + i|-\alpha\rangle \otimes |-\beta\rangle), \quad (47)$$

where we have used Eq. (32). The entangled state (47) is a coherent superposition state of two distinct pairs of correlated coherent states of the probe laser and the atomic condensate. It is a two-mode extension of a Yurke-Stoler state [47]. This superposition state can be interpreted in the spirit of Schrödinger's gedanken experiment, where the different coherent states replace the states of the cat being dead and alive.

Similarly, when the initial state of the atom-photon system is

$$|\Phi(\tau=0)\rangle = \frac{1}{\sqrt{2}} (|\alpha\rangle - i|-\alpha\rangle) \otimes |\beta\rangle, \quad (48)$$

if  $K = -1$ ,  $N = 4$ , and  $M = 1$ , then a  $\pi/2$  evolution drives the system to the following entangled coherent state:

$$\left| \Phi \left( \tau = \frac{\pi}{2} \right) \right\rangle = \frac{1}{\sqrt{2}} (|\alpha\rangle \otimes |-\beta\rangle - i|-\alpha\rangle \otimes |\beta\rangle). \quad (49)$$

The two two-state entangled coherent states (47) and (49) have the same degree of quantum entanglement indicated by the concurrence given by

$$\mathcal{C} = \sqrt{[1 - \exp(-4|\alpha|^2)][1 - \exp(-4|\beta|^2)]}, \quad (50)$$

which implies that the amount of entanglement of the two two-state entangled coherent states (47) and (49) increases with  $\alpha$  and  $\beta$  in the regime where the conditions of the adiabatic elimination of the related levels are satisfied.

*Three-state entangled states.* In order to obtain photon-atom three-state entangled states, we consider a situation where the probe laser and the condensate are initially in coherent states  $|\alpha\rangle$  and  $|\beta\rangle$ , respectively, while  $K = -1$ ,  $N = 3$ , and  $M = 1$ . In this case, from Eqs. (20) and (30) we find that nonzero  $c$  coefficients are given by

$$\begin{aligned} c_{11} = c_{22} = c_{13}^* = c_{23}^* &= -\frac{1}{3}e^{-i\pi/3}, \\ c_{12} = c_{21} = c_{31} = c_{32} = c_{33} &= \frac{1}{3}, \end{aligned} \quad (51)$$

which results in the following three-state entangled state:

$$\begin{aligned} \left| \Phi \left( \tau = \frac{2\pi}{3} \right) \right\rangle &= \frac{1}{3} [ |\alpha\rangle_1 \otimes |\beta\rangle_1 + |\alpha\rangle_2 \otimes |\beta\rangle_2 + e^{-i\pi/3} |\alpha\rangle_3 \\ &\otimes |\beta\rangle_3 ], \end{aligned} \quad (52)$$

where we have discarded the common phase factor  $\exp(i\pi/3)$  on the right-hand side of the above equation and three photon-coherent states and three atomic macroscopic quantum superposition (MQS) states are defined by

$$\begin{aligned} |\alpha\rangle_k &= |(-1)^k \alpha e^{-ik\pi/3}\rangle \quad (k=1,2,3) \\ |\beta\rangle_1 &= e^{-i\pi/3} |-\beta e^{i\pi/3}\rangle + e^{i\pi/3} |-\beta e^{-i\pi/3}\rangle - |\beta\rangle, \\ |\beta\rangle_2 &= e^{i\pi/3} |-\beta e^{i\pi/3}\rangle + e^{-i\pi/3} |-\beta e^{-i\pi/3}\rangle - |\beta\rangle, \\ |\beta\rangle_3 &= |-\beta e^{i\pi/3}\rangle + |-\beta e^{-i\pi/3}\rangle + |\beta\rangle. \end{aligned} \quad (53)$$

Hence, state (52) is an entangled state between three photon-coherent states and three atomic MQS states.

*Four-state entangled states.* Suppose that the probe laser and the condensate are initially in coherent states  $|\alpha\rangle$  and  $|\beta\rangle$ , respectively. We consider a  $\pi/4$  evolution of the system,  $K = -1$ ,  $N = 8$ , and  $M = 1$ . From Eqs. (20) and (30) we find that nonzero  $c$  coefficients are given by

$$\begin{aligned} c_{22} = -c_{24}^* = -c_{26} = c_{28}^* = -c_{62} = -c_{64}^* = c_{66} = c_{68}^* \\ = \frac{1}{4} \exp\left(i\frac{\pi}{4}\right), \\ c_{42} = -c_{44} = c_{46} = -c_{48} = c_{82} = c_{84} = c_{86} = c_{88} = \frac{1}{4}, \end{aligned} \quad (54)$$

which result in the following four-state entangled state

$$\begin{aligned} \left| \Phi \left( \tau = \frac{\pi}{4} \right) \right\rangle &= \frac{1}{4} [ e^{i\pi/4} |i\alpha\rangle_- \otimes |i\beta\rangle_- + e^{-i\pi/4} |i\alpha\rangle_+ \otimes |i\beta\rangle_+ \\ &+ |\alpha\rangle_+ \otimes |i\beta\rangle_+ + |\alpha\rangle_- \otimes |i\beta\rangle_+ ], \end{aligned} \quad (55)$$

where  $|\gamma\rangle_{\pm} = |\gamma\rangle \pm |-\gamma\rangle$  with  $\gamma = \alpha, \beta, i\alpha$ , and  $i\beta$  being unnormalized Schrödinger cat states. Hence, state (55) is an entangled state between two pairs of photon Schrödinger cat states and two pairs of atom Schrödinger cat states.

It is worthwhile to mention that besides their importance on quantum entanglement for the above two-state, three-state, and four-state entangled states, we note that all of them are superposition states of a macroscopic number of atoms with a macroscopic number of photons. In this sense, we have also given a scheme for generating atom-photon Schrödinger cat states. The problem of creating a macroscopic superposition of atoms and photons raises an interest itself, because rather than two states of a given object, the atom-photon system is a seemingly impossible macroscopic superposition of two different objects. This is beyond the usual macroscopic superposition of two states of a given object; the atom-photon system actually leads to a more counterintuitive situation since atoms and photons are different objects.

#### IV. CONCLUDING REMARKS

We have presented a scheme for the generation of atom-photon entangled states of continuous-variable-type pure states in the atomic BEC which exploits quantum interference, or EIT, in three-level atoms. We have shown how to generate multistate entangled coherent states when the atom-photon system is initially in an uncorrelated product coherent state. As examples, the generation of two-state, three-state, and four-state entangled states has been investigated explicitly. We have created not only atom-photon entangled coherent states, but also entangled states between photon (atom)-coherent states and atom (photon)-cat states, and between photon-cat and atom-cat states. All of these entangled states are MQS states of the atom-photon system. They may be regarded as an extension of the usual Schrödinger cat states of one given object to two different objects (atoms and photons).

In the proposed scheme we use the atomic BEC with three internal states which are coupled with a weak quantized probe laser and a strong classical coupling laser. When the BEC is in the EIT, since there is no atomic population at the upper and intermediate levels, the atomic field operators at these levels may be adiabatically eliminated. The system then becomes an effective two-mode system in which one member is the probe laser (photons), the other is the condensate (atoms) in the internal state  $|1\rangle$ . In this process the (probe) laser-atom dipole interaction is converted as an atom-photon effective collisional interaction. The strength of the atom-photon collisional interaction is determined by the dipole interaction and the two-photon detuning. Atom-photon entanglement is produced by interatomic and *atom-photon effective collisional interactions*. This mechanism to create quantum entanglement is different from that in our previous analysis of generating atom-atom continuous-variable-type entangled states in a three-level  $\Lambda$ -shaped BEC system [48]. Atom-atom entanglement in Ref. [48] is produced by interatomic collisional interaction and *an interatomic effective tunnelling interaction* generated by adiabatically eliminating the atomic field operators at the upper level, and no EIT is required. In the present scheme the technical requirements involve two lasers with a well-controlled frequency difference, a controllable dipole interaction be-

tween the probe laser and BEC, and a manipulable scattering length for atoms in the internal state  $|1\rangle$ . These are met by most existing EIT and BEC experiments. We hope that the proposed method for generating atom-photon continuous-variable-type entangled states can find its applications in quantum information processing and studies of fundamental problems of quantum mechanics. It is possible to apply the approach to create entangled states in the present paper to study quantum entanglement between two distant condensates and quantum teleportation of quantum states of BEC's, however, it is outside the scope of the present paper and will be discussed elsewhere.

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