Backaction-induced spin-squeezed states in a detuned quantum-nondemolition measurement

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We propose a scheme for producing entangled spin-squeezed states of an atomic ensemble inside an optical cavity by backaction of a detuned quantum-nondemolition (QND) measurement. By illuminating the atoms with bichromatic light, an interaction Hamiltonian of the cross-Kerr effect between the cavity and atoms is generated to implement QND measurements. The feedback effect is obtained through mixing the phase and amplitude quadratures of the cavity field, due to the detuning of the optical cavity. Therefore the continuous nondemolition measurements are fed back to correct the quantum state of the atomic sample such that unconditional spin squeezing is produced without requiring the use of any external electronics.

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I. INTRODUCTION

Squeezed-spin systems [1] of atoms and ions have attracted considerable attention in recent years due to the potential for practical applications, such as in the fields of quantum information [2,3] and high-precision spectroscopy [4,5]. Spin squeezing is related to the fundamental concept of entanglement and specifically represents many-particle entanglement [6,7]. The squeezed-spin state is generated via quantum-state transfer between nonclassical light and an atomic ensemble [8,9]. This method has recently produced weakly squeezed states [10]. In analogy with nonlinear optics, another proposal involves the collisional interactions in a Bose-Einstein condensate (BEC). These represent a nonlinearity which will dynamically generate spin squeezing in the trapped state [6,11,12] and also any out-coupled beams [13,14]. There are also schemes for direct coupling to the entangled state through intermediate states such as collective motional modes for ions [15] or molecular states for atoms [16]. A related proposal is the photodissociation of molecular condensates [17,18] in analogy with the down-conversion process in quantum optics. There has also been the suggestion that spin squeezing may be produced in dilute optical lattices [19,20], and experimental evidence that the ground state of a BEC confined in an optical lattice can be produced in an atom-number squeezed state [21].

Production of spin-squeezed states via quantum nondemolition (QND) detection has also been considered [22] and spin-noise reduction using this method has been experimentally observed [23]. QND measurement is also utilized in a proposal for the entanglement of two macroscopic atomic samples [2], which has recently been achieved experimentally [24]. These schemes represent conditional squeezing of the atomic ensembles. Achieving deterministic spin squeezing and entanglement of two macroscopic atomic samples via quantum feedback have been proposed [25,26]. These involve external electronics controlling the amplitude modulation of a radio-frequency magnetic field, to correct the quantum state of the samples. The possibility of producing self-spin squeezing by having a large number of atoms in a bad cavity was already proposed [27]. More efficient schemes to produce self-spin squeezing have been proposed [28,29], in which the fundamental ideas and the results are similar but the energy levels are different.

In this work, we present a scheme to produce entangled spin-squeezed states inside an optical cavity by the backaction of a detuned QND measurement, in analogy with optical squeezing in Ref. [30]. The bichromatic auxiliary lasers, illuminating the atoms, combine with the cavity fields to drive Raman transitions. A cross-Kerr effective interaction is generated for the QND measurement. A feedback effect is obtained through mixing the phase and amplitude quadratures of the cavity field, due to detuning of the optical cavity. Therefore the continuous nondemolition measurements are fed back to correct the quantum state of the sample such that unconditional spin squeezing is produced without requiring the use of external electronics. Although the energy levels and the results in Ref. [29] are similar to ours, the mechanism employed in that paper is very different from what is proposed here and we give a simple and clear physical model for generating spin-squeezed states.

II. MODEL

The energy levels of the atoms and the laser couplings of the current scheme are depicted in Fig. 1. We consider a Λ-type three-level atom with two stable ground states $|a\rangle$ and $|b\rangle$, with an energy difference ω_{ab} and an excited state $|e\rangle$ with energy difference ω_{ae} to the ground state $|a\rangle$. The state $|a\rangle$ is coupled to the excited state $|e\rangle$ by a strong classical laser with a resonant Rabi frequency Ω_1 and a frequency ω_1 which is detuned from the excited state by Δ_1. Similarly the $|b\rangle$ state is also coupled to the excited state by a second detuned laser with resonant Rabi frequency Ω_2 and a frequency ω_2 which is detuned from the excited state by Δ_2. The two frequencies of the lasers are chosen such that their difference is exactly twice the energy splitting between the two ground states $\omega_1-\omega_2=2\omega_{ab}$ $(\Delta_2-\Delta_1=\omega_{ab})$. A quantized field $\hat{c}$ with frequency ω_q in an optical cavity

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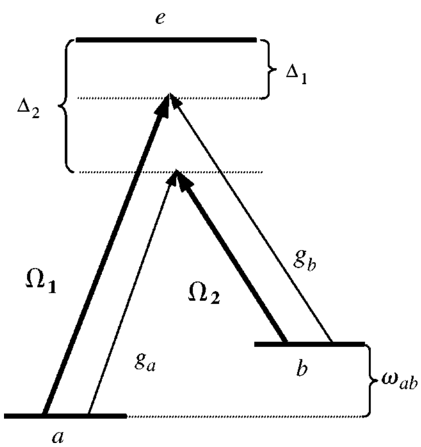

FIG. 1. Energy levels and couplings.

couples both states $|a\rangle$ and $|b\rangle$ to the excited state $|e\rangle$ via coupling constants g_a and g_b. Thus, two Raman interactions are activated: one comes from the quantized field and the classical field Ω_1 both detuned from excited state $|e\rangle$ by Δ_1; the other comes from the quantized field and the classical field Ω_2 detuned by Δ_2. The cavity-resonance frequency closest to the quantized-field frequency is ω_c. The cavity detuning for the quantized field is $\sigma=\omega_q-\omega_c$.

The three-level system is described using collective operators for N atoms of the ensemble: the populations of levels $|e\rangle$, $|a\rangle$, and $|b\rangle$,

$$\hat{\Pi}_e=\sum_{k=1}^{N}|e\rangle_{kk}\langle e|,\quad \hat{\Pi}_a=\sum_{k=1}^{N}|a\rangle_{kk}\langle a|,\quad \hat{\Pi}_b=\sum_{k=1}^{N}|b\rangle_{kk}\langle b|, \tag{1}$$

the components of the optical dipoles

$$\hat{P}_1=\sum_{k=1}^{N}|a\rangle_{kk}\langle e|,\qquad \hat{P}_2=\sum_{k=1}^{N}|b\rangle_{kk}\langle e|, \tag{2}$$

and operators associated with the coherence between levels $|a\rangle$ and $|b\rangle$

$$\hat{S}_+=\sum_{k=1}^{N}|b\rangle_{kk}\langle a|,\qquad \hat{S}_-=\sum_{k=1}^{N}|a\rangle_{kk}\langle b|. \tag{3}$$

If we assume all fields to be propagating in the same direction, the experimental situation is described by the Hamiltonian

$$\hat{H}=\hbar\omega_q\hat{c}^\dagger\hat{c}+\hbar\omega_{ae}\hat{\Pi}_e+\hbar\omega_{ab}\hat{\Pi}_b+\hbar[(\Omega_1e^{-i\omega_1t}+g_a\hat{c}e^{-i\omega_ct})\hat{P}_1^\dagger+(\Omega_2e^{-i\omega_2t}+g_b\hat{c}e^{-i\omega_ct})\hat{P}_2^\dagger+\text{H.c.}]. \tag{4}$$

We now adiabatically eliminate the excited state of the atoms by assuming that the population of that state is negligible, the detunings of the light fields from the atomic-transition frequency to be very large, and that the atomic spontaneous emission can be neglected. The corresponding Hamiltonian then takes the following form in a frame rotating at the laser frequency:

$$\begin{aligned}\hat{H}=&\hbar\sigma\hat{c}^\dagger\hat{c}-\hbar\left(\frac{|\Omega_1|^2}{\Delta_1}+\frac{|g_a|^2\hat{c}^\dagger\hat{c}}{\Delta_2}\right)\hat{\Pi}_a\\&-\hbar\left(\frac{|\Omega_2|^2}{\Delta_2}+\frac{|g_b|^2\hat{c}^\dagger\hat{c}}{\Delta_1}\right)\hat{\Pi}_b\\&-\hbar\left(\frac{\Omega_2g_a^*}{\Delta_2}\hat{S}_-\hat{c}^\dagger+\frac{\Omega_2^*g_a}{\Delta_2}\hat{S}_+\hat{c}\right)\\&-\hbar\left(\frac{\Omega_1g_b}{\Delta_1}\hat{S}_-\hat{c}+\frac{\Omega_1^*g_b^*}{\Delta_1}\hat{S}_+\hat{c}^\dagger\right).\end{aligned} \tag{5}$$

The second and third terms in Eq. (5) represent the ac-Stark shifts of the ground states. The first part of the shifts containing the classical fields Ω_1 and Ω_2 can be compensated if we make a change in the frequency of the fields. The second part containing the quantum field $\hat{c}$ is much smaller than the first and we shall neglect this part. After the excited state is eliminated adiabatically, the collective properties of the N atoms are conveniently described by two stable ground states with pseudo-angular-momentum operators defined by

$$\hat{S}_z=\frac{1}{2}\sum_{k=1}^{N}(|a\rangle_{kk}\langle a|-|b\rangle_{kk}\langle b|), \tag{6}$$

$$\hat{S}_x=\frac{1}{2}(\hat{S}_++\hat{S}_-),$$

$$\hat{S}_y=\frac{1}{2i}(\hat{S}_+-\hat{S}_-).$$

If we assume that the initial state, where all atoms are in the $|a\rangle$ state, is an eigenstate of the $\hat{S}_z$ with eigenvalue $S_z=N/2$. The Heisenberg evolution equations of the system operators are given by

$$\dot{\hat{c}}=(i\sigma-k)\hat{c}-i\left(\frac{\Omega_2g_a^*}{\Delta_2}\hat{S}_-+\frac{\Omega_1^*g_b^*}{\Delta_1}\hat{S}_+\right)+\sqrt{2k}\hat{c}_{\text{in}}, \tag{7}$$

$$\dot{\hat{S}}_-=-\Gamma\hat{S}_--i\left(\frac{\Omega_2^*g_a}{\Delta_2}\hat{c}\hat{S}_z+\frac{\Omega_1^*g_b^*}{\Delta_1}\hat{c}^\dagger\hat{S}_z\right)+\sqrt{2\Gamma}\hat{F}_{S_-},$$

where k is the decay rate of the field in the cavity, Γ is the decay rate of two ground states, and the operators $\hat{c}_{\text{in}}$ and $\hat{F}_{S_-}$ correspond to the coupling of the field and atoms with their respective baths. We assume that g_a, g_b are real and $\Omega_1=|\Omega_1|e^{i\theta_1}$, $\Omega_2=|\Omega_2|e^{i\theta_2}$. If we choose the strength of the two Raman processes to be identical $|2\Omega_2g_a/\Delta_2|=|2\Omega_1g_b/\Delta_1|=\Theta$, and $\theta_2=-\theta_1=\pi/2$, the corresponding evolution equations of the system operators are then given by

$$\dot{\hat{c}}=(i\sigma-k)\hat{c}+\Theta\hat{S}_x+\sqrt{2k}\hat{c}_{\text{in}}, \tag{8}$$

$$\dot{\hat{S}}_- = -\Gamma \hat{S}_- - \frac{\Theta}{2}\hat{S}_z(\hat{c}-\hat{c}^\dagger) + \sqrt{2\Gamma}\hat{F}_{S_-}.$$

If $\hat{S}_z$ keeps $S_z \approx N/2$ constantly, then $\hat{S}_x$ and $\hat{S}_y$ may be replaced by the canonical conjugate position and momentum operators

$$\hat{J}_x = \frac{2\hat{S}_x}{\sqrt{N}}, \quad \hat{J}_y = \frac{2\hat{S}_y}{\sqrt{N}}. \tag{9}$$

The quadratures of the cavity field and atomic sample corresponding to Eq. (8) then evolve as

$$\dot{\hat{X}}_c = -\sigma \hat{Y}_c - k\hat{X}_c + 2\chi \hat{J}_x + \sqrt{2k}\hat{X}_{c_{\rm in}}, \tag{10}$$

$$\dot{\hat{Y}}_c = \sigma \hat{X}_c - k\hat{Y}_c + \sqrt{2k}\hat{Y}_{c_{\rm in}},$$

$$\dot{\hat{J}}_x = -\Gamma \hat{J}_x + \sqrt{2\Gamma}\hat{F}_{J_x},$$

$$\dot{\hat{J}}_y = -\Gamma \hat{J}_y - 2\chi \hat{Y}_c + \sqrt{2\Gamma}\hat{F}_{J_y},$$

where $\chi = (\sqrt{N}/2)\Theta$, $\hat{X}_c = \hat{c} + \hat{c}^\dagger$, and $\hat{Y}_c = -i(\hat{c} - \hat{c}^\dagger)$. Equation (10) is the main result of this paper. This kind of interaction configuration has been used in optical squeezing in Ref. [30].

III. QND MEASUREMENT WITH $\sigma = 0$

When the cavity detuning is zero, $\sigma = 0$, the effective interaction Hamiltonian between the cavity and atomic sample has the simple form

$$\hat{H}_{\rm eff} = \hbar \chi \hat{Y}_c \hat{J}_x. \tag{11}$$

This just is the interaction Hamiltonian of the cross-Kerr effect [31]. The important feature of this Hamiltonian is that the amplitude quadrature $\hat{X}_c$ of cavity field picks up information about the amplitude quadrature of the spin $\hat{J}_x$, while the latter is left unchanged. The Hamiltonian of Eq. (11) is identical to that of an off-resonant interaction between a laser field and an atomic ensemble [2,3,23]. Spin squeezing and entanglement of two macroscopic atomic samples have been produced experimentally by the QND measurements with this Hamiltonian [23,24]. Protocols for quantum communication between atomic ensembles have also been proposed, including quantum teleportation and quantum swapping [2,3]. However, these schemes represent conditional squeezing of the atomic ensembles. A scheme was proposed to achieve unconditional squeezing via quantum feedback [25]. The results of a QND measurement, which conditionally squeeze the motion, are used to drive the system into the desired, deterministic, squeezed atomic spin state. This involves amplitude modulation of a radio-frequency magnetic field, where the feedback strength varied in time.

IV. BACKACTION-INDUCED SPIN-SQUEEZED STATE

Let us now detune the quantized field from the cavity frequency $\sigma \neq 0$ [see Eq. (10)]. Equation (10) still includes the important features of QND that the amplitude quadrature $\hat{X}_c$ of the cavity field picks up information about the amplitude quadrature of the atomic sample $\hat{J}_x$, while the latter is left unchanged. As a result of QND measurement, the excess noise on the phase quadrature of the spin is entirely due to the phase quadrature $\hat{Y}_c$ of the quantized field, and it appears as the backaction of the measurement. Due to the detuning of the quantized cavity field, the amplitude and phase quadratures of the quantized field will be mixed, transferring information about $\hat{J}_x$ to the phase quadrature $\hat{J}_y$ of the spin. A consequence of this effect is that a mixed quadrature component of the spin (a combination of $\hat{J}_x$ and $\hat{J}_y$) will be squeezed, as can be easily checked from the above expressions. Assuming $\chi \gg k$, we adiabatically eliminate the cavity field and Eq. (10) now becomes

$$\dot{\hat{J}}_x = -\Gamma \hat{J}_x + \sqrt{2\Gamma}\hat{F}_{J_x}, \tag{12}$$

$$\dot{\hat{J}}_y = -\Gamma \hat{J}_y - 4\chi' \hat{J}_x + \sqrt{2\Gamma}\hat{F}_{J_y},$$

where $\chi' = \chi^2/\sigma$. The effective interaction Hamiltonian corresponding to Eq. (12) is

$$\hat{H}_{\rm eff} = \hbar \chi' \hat{J}_x^2. \tag{13}$$

The squeezing arising from this Hamiltonian can be calculated analytically [1]. Starting from an initial state where all atoms are in the $|a\rangle$ state, squeezing by a factor of $\xi^2 = \min_\phi \langle \Delta^2 \hat{J}_\phi \rangle \approx N^{-2/3}$ [$\hat{J}_\phi = \cos(\phi)\hat{J}_x + \sin(\phi)\hat{J}_y$] is produced (in the limit $N \gg 1$). This is a significant noise reduction if a large number of atoms is present. With the realistic parameters: the cavity decay rate $k/(2\pi) = 5$ MHz, cavity detuning $\sigma/(2\pi) = 20$ MHz, the number of atoms $N = 10^6$, cavity coupling parameter $g_a = g_b = (2\pi)100$ kHz, two Raman coupling strength $\Theta/(2\pi) = 10$ kHz, atomic decay rate $\Gamma/(2\pi) = 5$ MHz, we are able to produce squeezing by approximately an order of magnitude from Eq. (10) after a very short interaction time. Since the Kerr coefficient χ depends on the number of atoms, the time it takes to produce a spin-squeezed state of many atoms is very short for a large number of atoms. The different decoherence mechanisms therefore have less time to affect the preparation of the squeezed states.

V. CONCLUSION

In conclusion, we have described a scheme for producing deterministic spin-squeezed states of an atomic ensemble inside an optical cavity by backaction of a detuned quantum-nondemolition measurement. This scheme has the advantage over previous schemes involving QND measurement in that it produces unconditional, or deterministic, squeezing without external electronics feedback.

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