Generation of entangled states for many multilevel atoms in a thermal cavity and ions in thermal motion

Shi-Biao Zheng*

Department of Electronic Science and Applied Physics, Fuzhou University, Fuzhou 350002, People's Republic of China

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We propose a scheme for generating entangled states for two or more multilevel atoms in a thermal cavity. The photon-number-dependent parts in the effective Hamiltonian are canceled with the assistance of a strong classical field. Thus the scheme is insensitive to both the cavity decay and the thermal field. The scheme does not require individual addressing of the atoms in the cavity. The scheme can also be used to generate entangled states for many hot multilevel ions.

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Entanglement of two or more particles is not only of significance for test of quantum mechanics against local hidden theory [1–3], but also useful in quantum cryptography [4] and quantum teleportation [5]. Most of research in quantum nonlocality and quantum information is based on entanglement of two-level particles. Entangled states for two-level particles have been observed for photons [6–8], atoms in cavity QED [9–11], and ions in a trap [12–14].

Recently, it has been shown that violations of local realism by two entangled N-dimensional systems are stronger than those for two qubits [15]. The Greenberger-Horne-Zeiliner paradox has also been extended to many N-dimensional systems [16]. Furthermore, it has been shown that quantum cryptography based on entangled qutrits is more secure than that based on entangled qubits [17]. High-dimensional entanglement for photons has been observed [18–21]. However, there have been no reports on the realization of entanglement for multilevel massive particles. Recently, Zou *et al.* [22] have proposed a scheme for the generation of entangled states for two three-level atoms in cavity QED using nonresonant interaction of two atoms with a cavity [23]. The scheme is insensitive to cavity decay. The main drawback of the scheme is that it requires individual addressing of the atoms when both atoms are still in the cavity, which is experimentally problematic.

In this paper we propose a scheme for generating entangled states for many multilevel atoms in cavity QED and ions in a trap. In cavity QED, our scheme does not require individual addressing of the atoms in the cavity. Another distinct feature of the present scheme is that the photon-number-dependent parts in the effective Hamiltonian are canceled with the assistance of a strong classical driving field. Due to this feature the scheme is insensitive to both the cavity decay and thermal field. For the trapped ions, our scheme is insensitive to the thermal motion.

We consider N identical ladder-type three-level atoms simultaneously interacting with a single-mode cavity field and driven by a classical field. The atomic states are denoted by $|g\rangle$, $|e\rangle$, and $|i\rangle$. The transition frequency between the states $|e\rangle$ and $|i\rangle$ is highly detuned from the cavity frequency, and thus the state $|i\rangle$ is not affected during the atom-cavity interaction. The Hamiltonian (assuming $\hbar=1$) [24,25] is

$$H=\omega_0\sum_{j=1}^{N} S_{z,j}+\omega_a a^\dagger a+\sum_{j=1}^{N}[g(a^\dagger S_j^- + aS_j^+) + \Omega(S_j^+ e^{-i\omega t}+S_j^- e^{i\omega t})], \tag{1}$$

where $S_j^+=|e_j\rangle\langle g_j|$, $S_j^-=|g_j\rangle\langle e_j|$, $S_{z,j}=\frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, with $|e_j\rangle$ and $|g_j\rangle$ $(j=1,2)$ being the excited and ground states of the jth atom, $a^\dagger$ and a are the creation and annihilation operators for the cavity mode, g is the atom-cavity coupling strength, and Ω is the Rabi frequency of the classical field. We assume that $\omega_0=\omega$. Then the interaction Hamiltonian, in the interaction picture, is

$$H_i=\sum_{j=1}^{N}[g(e^{-i\delta t}a^\dagger S_j^- + e^{i\delta t}aS_j^+)+\Omega(S_j^+ + S_j^-)], \tag{2}$$

where δ is the detuning between the atomic transition frequency ω_0 and cavity frequency ω. We define the new atomic basis

$$|+_j\rangle=\frac{1}{\sqrt{2}}(|g_j\rangle+|e_j\rangle), \quad |-_j\rangle=\frac{1}{\sqrt{2}}(|g_j\rangle-|e_j\rangle). \tag{3}$$

Then we can rewrite H_i as

$$H_i=\sum_{j=1}^{N}[ge^{-i\delta t}a^\dagger(\sigma_{z,j}+\tfrac{1}{2}\sigma_j^+ - \tfrac{1}{2}\sigma_j^-) + e^{i\delta t}a(\sigma_{z,j}+\tfrac{1}{2}\sigma_j^- - \tfrac{1}{2}\sigma_j^+)+2\Omega\sigma_{z,j}], \tag{4}$$

where $\sigma_{z,j}=\frac{1}{2}(|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j|)$, $\sigma_j^+=|+_j\rangle\langle -_j|$, and $\sigma_j^-=|-_j\rangle\langle +_j|$.

The time evolution of this system is decided by Schrödinger's equation:

$$i[d|\psi(t)\rangle/dt]=H_i|\psi(t)\rangle. \tag{5}$$

We perform the unitary transformation

$$|\psi(t)\rangle=e^{-iH_0t}|\psi'(t)\rangle, \tag{6}$$

with

$$H_0=2\Omega\sum_{j=1}^{N}\sigma_{z,j}. \tag{7}$$

*Email address: sbzheng@pub5.fz.fj.cn

Then we obtain

$$i[d|\psi'(t)\rangle/dt]=H_i'|\psi'(t)\rangle, \tag{8}$$

where

$$H_i'=\sum_{j=1,2}[ge^{-i\delta t}a^\dagger(\sigma_{z,j}+\tfrac{1}{2}\sigma_j^+e^{2i\Omega t}-\tfrac{1}{2}\sigma_j^-e^{-2i\Omega t}) + e^{i\delta t}a(\sigma_{z,j}+\tfrac{1}{2}\sigma_j^-e^{-2i\Omega t}-\tfrac{1}{2}\sigma_j^+e^{2i\Omega t})]. \tag{9}$$

Assuming that $2\Omega \gg \delta, g$, we can neglect the terms oscillating fast. Then H_i' reduces to

$$H_i'=\sum_{j=1}^{N} g(e^{-i\delta t}a^\dagger+e^{i\delta t}a)\sigma_{z,j} =\frac{1}{2}\sum_{j=1}^{N} g(e^{-i\delta t}a^\dagger+e^{i\delta t}a)(S_j^++S_j^-). \tag{10}$$

In the case $\delta \gg g/2$, there is no energy exchange between the atomic system and the cavity. The resonant transitions are $|e_jg_kn\rangle \leftrightarrow |g_je_kn\rangle$ and $|e_je_kn\rangle \leftrightarrow |g_jg_kn\rangle$. The transition $|e_jg_kn\rangle \leftrightarrow |g_je_kn\rangle$ is mediated by $|g_jg_kn\pm 1\rangle$ and $|e_je_kn\pm 1\rangle$. The contributions of $|g_jg_kn\pm 1\rangle$ are equal to those of $|e_je_kn\pm 1\rangle$. The corresponding Rabi frequency is given by

$$2\frac{\langle e_jg_kn|H_i'|g_jg_kn+1\rangle\langle g_jg_kn+1|H_i'|g_je_kn\rangle}{\delta} +2\frac{\langle e_jg_kn|H_i'|g_jg_kn-1\rangle\langle g_jg_kn-1|H_i'|g_je_kn\rangle}{-\delta}=\frac{g^2}{2\delta}. \tag{11}$$

Since the transition paths interfere destructively, the Rabi frequency is independent of the photon number of the cavity mode. The destructive interference of transition amplitudes was first proposed for trapped ions [26,27]. The Rabi frequency for $|e_je_kn\rangle \leftrightarrow |g_jg_kn\rangle$, mediated by $|e_jg_kn\pm 1\rangle$ and $|g_je_kn\pm 1\rangle$, is also equal to $g^2/(2\delta)$. The Stark shift for the state $|e_j\rangle$ is

$$\frac{\langle e_jn|H_i'|g_jn+1\rangle\langle g_jn+1|H_i'|e_jn\rangle}{\delta} +\frac{\langle e_jn|H_i'|g_jn-1\rangle\langle g_jn-1|H_i'|e_jn\rangle}{-\delta}=\frac{g^2}{4\delta}. \tag{12}$$

The Stark shift for $|g_j\rangle$ is also $g^2/(4\delta)$. The strong classical field induces the terms $g(e^{-i\delta t}a^+S_j^++e^{i\delta t}aS_j^-)$, which result in the photon-number-dependent Stark shifts negative to those induced by $g(e^{-i\delta t}a^+S_j^-+e^{i\delta t}aS_j^+)$. Thus the photon-number-dependent Stark shifts are also canceled. Then the effective Hamiltonian is given by

$$H_e=\lambda\left[\frac{1}{2}\sum_{j=1}^{N}(|e_j\rangle\langle e_j|+|g_j\rangle\langle g_j|) +\sum_{j,k=1}^{N}(S_j^+S_k^++S_j^+S_k^-+\text{H.c.})\right],\quad j\neq k, \tag{13}$$

where $\lambda=g^2/2\delta$. The distinct feature of the effective Hamiltonian is that it is independent of the photon number of the cavity field. Without the strong classical field, the Stark shift terms are proportional to the photon number, and the terms $S_j^+S_k^++\text{H.c.}$ do not exist. The evolution operator of the system is given by

$$U(t)=e^{-iH_0t}e^{-iH_et}. \tag{14}$$

We note that the atomic state evolution operator $U(t)$ is independent of the cavity field state, allowing it to be in a thermal state.

We first consider the case where $N=2$ and assume that the two atoms are initially in the state $|g_1\rangle|g_2\rangle$. After an interaction time t_1 the state of the system is

$$\begin{aligned}|g_1\rangle|g_2\rangle\to e^{-i\lambda t_1}\{&\cos(\lambda t_1)[\cos\Omega t_1|g_1\rangle-i\sin\Omega t_1|e_1\rangle]\\ &\times[\cos\Omega t_1|g_2\rangle-i\sin\Omega t_1|e_2\rangle]-i\sin(\lambda t_1)\\ &\times[\cos\Omega t_1|e_1\rangle-i\sin\Omega t_1|g_1\rangle][\cos\Omega t_1|e_2\rangle\\ &-i\sin\Omega t_1|g_2\rangle]\}.\end{aligned} \tag{15}$$

We choose the interaction time t_1 and Rabi frequency Ω appropriately so that $\sin(\lambda t_1)=1/\sqrt{3}$ and $\Omega t_1=k\pi$, with k being an integer. Then we have

$$|g_1\rangle|g_2\rangle\to e^{-i\lambda t_1}\left\{\sqrt{2\tfrac{2}{3}}|g_1\rangle|g_2\rangle-i\frac{1}{\sqrt{3}}|e_1\rangle|e_2\rangle\right\}. \tag{16}$$

Now we switch off the classical field tuned to the $|g\rangle\to|e\rangle$, and switch on another classical field tuned to the $|e\rangle\to|f\rangle$. Choosing the Rabi frequency and interaction time appropriately so that the atoms undergo the transitions $|e\rangle\to|f\rangle$. We here assume that this classical field is sufficiently strong, and thus the interaction time is so short that the dispersive atom-cavity interaction can be neglected during the application of this classical field. This leads to

$$e^{-i\lambda t_1}\{\sqrt{2\tfrac{2}{3}}|g_1\rangle|g_2\rangle-i(1/\sqrt{3})|f_1\rangle|f_2\rangle\}. \tag{17}$$

Then we again switch on the classical field tuned to the $|g\rangle\to|e\rangle$, and switch off the field tuned to the $|e\rangle\to|f\rangle$. The Hamiltonian is again given by Eq. (13). After another interaction time t_2, we obtain

$$\begin{aligned}e^{-i\lambda(t_1+t_2)}\sqrt{\tfrac{2}{3}}\{&\cos(\lambda t_2)[\cos\Omega' t_2|g_1\rangle-i\sin\Omega' t_2|e_1\rangle]\\ &\times[\cos\Omega' t_2|g_2\rangle-i\sin\Omega' t_2|e_2\rangle]-i\sin(\lambda t_2)\\ &\times[\cos\Omega' t_2|e_1\rangle-i\sin\Omega' t_2|g_1\rangle][\cos\Omega' t_2|e_2\rangle\\ &-i\sin\Omega' t_2|g_2\rangle]\}-ie^{-i\lambda t_1}(1/\sqrt{3})|f_1\rangle|f_2\rangle,\end{aligned} \tag{18}$$

where Ω' is the Rabi frequency of the classical field during the interaction time t_2. We choose the interaction time t_2 and Rabi frequency Ω' appropriately so that $\lambda t_2=\pi/4$ and $\Omega' t_2=2k'\pi$, with k' being an integer. Then we have

$$e^{-i\lambda t_1}\sqrt{\tfrac{1}{3}}\{e^{-i\lambda t_2}|g_1\rangle|g_2\rangle-ie^{-i\lambda t_2}|e_1\rangle|e_2\rangle-i|f_1\rangle|f_2\rangle\}. \tag{19}$$

This is a maximally entangled state for the two three-level atoms. We here do not require individual addressing of the atoms when they are in the cavity. Furthermore, our scheme is not only insensitive to the cavity decay but also insensitive to the thermal photons. The thermal field gradually builds up during the operations [10]. Thus, our scheme is important in view of experiment.

We now turn to the problem of generating entanglement for three or more three-level atoms with a thermal cavity. The effective Hamiltonian H_e can also be rewritten as

$$H_e=2\lambda S_x^2, \tag{20}$$

where

$$S_x=\frac{1}{2}\sum_{j=1}^{N}(S_j^{+}+S_j^{-}). \tag{21}$$

We assume that the atoms are initially in the state $|g_1g_2\cdots g_N\rangle$. Using the representation of the operator S_z, the atomic states $|g_1g_2\cdots g_N\rangle$ and $|e_1e_2\cdots e_N\rangle$ can be expressed as $|N/2,-N/2\rangle$ and $|N/2,N/2\rangle$, respectively. On the other hand, such states can be expanded in terms of the eigenstates of S_x [25,26,28],

$$|N/2,-N/2\rangle=\sum_{M=-N/2}^{N/2}C_M|N/2,M\rangle_x, \tag{22}$$

$$|N/2,N/2\rangle=\sum_{M=-N/2}^{N/2}C_M(-1)^{N/2-M}|N/2,M\rangle_x. \tag{23}$$

Thus, the evolution of the system is

$$\sum_{M=-N/2}^{N/2}C_M e^{-2i(\Omega M+\lambda M^2)t}|N/2,M\rangle_x. \tag{24}$$

When N is even, M is an integer. With the choice $\lambda t=\pi/4$ and $\Omega t=n\pi$, we obtain

$$\begin{aligned}\frac{1}{\sqrt{2}}\sum_{M=-N/2}^{N/2}C_M[e^{-i\pi/4}+e^{i\pi/4}(-1)^M]|N/2,M\rangle_x\\ =(1/\sqrt{2})(e^{-i\pi/4}|g_1g_2\cdots g_N\rangle\\ +e^{i\pi/4}(-1)^{N/2}|e_1e_2\cdots e_N\rangle).\end{aligned} \tag{25}$$

On the other hand, for the case where N is odd we choose $\lambda t=\pi/4$ and $\Omega t=(2n+3/4)\pi$. Then we obtain

$$\begin{aligned}(1/\sqrt{2})\,e^{i(7/8)\pi}[e^{-i\pi/4}|g_1g_2\cdots g_N\rangle\\ +e^{i\pi/4}(-1)^{(1+N)/2}|g_1g_2\cdots g_N\rangle].\end{aligned} \tag{26}$$

By this way we obtain a multiatom Greenberger-Horne-Zeilinger state [2].

We here assume that N is even. After the state of Eq. (26) is prepared, we switch off the classical field tuned to $|e\rangle\to|g\rangle$ and perform the transformation $|e\rangle\to|f\rangle$. Then we have

$$\frac{1}{\sqrt{2}}(e^{-i\pi/4}|g_1g_2\cdots g_N\rangle+e^{i\pi/4}(-1)^{N/2}|f_1f_2\cdots f_N\rangle). \tag{27}$$

Then we again switch on the classical field tuned to $|e\rangle\to|g\rangle$. After another interaction time t we obtain an entangled state for the N three-level atoms,

$$\begin{aligned}\tfrac{1}{2}\,e^{-i\pi/4}(e^{-i\pi/4}|g_1g_2\cdots g_N\rangle+e^{i\pi/4}(-1)^{N/2}|e_1e_2\cdots e_N\rangle)\\ +(1/\sqrt{2})\,e^{i\pi/4}(-1)^{N/2}|f_1f_2\cdots f_N\rangle.\end{aligned} \tag{28}$$

After the N atoms exit the cavity, we can prepare $N-1$ atoms into a maximally entangled state via manipulating the Nth atom. We first perform the transformations

$$\begin{aligned}|g_N\rangle&\to(1/\sqrt{2})\,|g_N\rangle+(1/\sqrt{10})\,|e_N\rangle-\sqrt{\tfrac{2}{5}}|f_N\rangle,\\ |e_N\rangle&\to-(1/\sqrt{2})\,|g_N\rangle+(1/\sqrt{10})\,|e_N\rangle-\sqrt{\tfrac{2}{5}}|f_N\rangle,\\ |f_N\rangle&\to(2/\sqrt{5})\,|e_N\rangle+\sqrt{\tfrac{1}{5}}|f_N\rangle.\end{aligned} \tag{29}$$

Then we detect the state of the Nth atom. The detection of the state $|f_N\rangle$ collapses the $N-1$ atoms onto the maximally entangled state,

$$\begin{aligned}(1/\sqrt{3})\,(e^{-i\pi/2}|g_1g_2\cdots g_{N-1}\rangle+(-1)^{N/2}|e_1e_2\cdots e_{N-1}\rangle\\ -e^{i\pi/4}(-1)^{N/2}|f_1f_2\cdots f_{N-1}\rangle.\end{aligned} \tag{30}$$

The probability of success is 0.3.

We note we can generate a maximally entangled state for N four-level atoms determinately. The fourth level is $|h\rangle$. After the atoms are prepared in the state of Eq. (28), we perform the transformations $|g\rangle\leftrightarrow|f\rangle$ and $|e\rangle\leftrightarrow|h\rangle$. This leads to

$$\begin{aligned}(1/\sqrt{2})\,e^{i\pi/4}(-1)^{N/2}|g_1g_2\cdots g_N\rangle+\tfrac{1}{2}\,(e^{-i\pi/2}|f_1f_2\cdots f_N\rangle\\ +(-1)^{N/2}|h_1h_2\cdots h_N\rangle).\end{aligned} \tag{31}$$

Then we again switch on the classical field tuned to $|e\rangle\to|g\rangle$. After another interaction time t, we obtain an entangled state for the N four-level atoms,

$$\begin{aligned}\tfrac{1}{2}\,[(-1)^{N/2}|g_1g_2\cdots g_N\rangle+e^{i\pi/2}|e_1e_2\cdots e_N\rangle\\ +e^{-i\pi/2}|f_1f_2\cdots f_N\rangle+(-1)^{N/2}|h_1h_2\cdots h_N\rangle].\end{aligned} \tag{32}$$

We note that the idea can also be used to the ion trap system. We consider that N ions are confined in a linear trap.

Then we simultaneously excite the ions with two lasers of frequencies $\omega_0+\nu+\delta$ and $\omega_0-\nu-\delta$, where ω_0 is the frequency of the transition $|e\rangle\rightarrow|g\rangle$ and ν is the frequency of the one collective vibrational mode. Suppose δ is much smaller than ν, and thus we can neglect other vibrational modes. In this case the Hamiltonian for the system is given [26,27] by

$$\hat{H}=\nu\hat{a}^{\dagger}\hat{a}+\omega_0\sum_{j=1}^{N}\hat{S}_{z,j}+\Bigg\{\Omega e^{-i\phi}\sum_{j=1}^{N}\hat{S}_j^{+}e^{i\eta(\hat{a}^{\dagger}+\hat{a})}\times[e^{-i(\omega_0+\nu+\delta)t}+e^{-i(\omega_0-\nu-\delta)t}]+\text{H.c.}\Bigg\}, \qquad (33)$$

where $\hat{a}^{\dagger}$ and $\hat{a}$ are the creation and annihilation operators for the collective vibrational mode, and $\eta=k/\sqrt{2\nu M}$ is the Lamb-Dicke parameter, with k being the wave vector along the trap axis and M the mass of the ion collection. We here have assumed that the lasers have the same Rabi frequency Ω, phase ϕ, and same wave vector k. Furthermore, we consider the resolved sideband regime, where the vibrational frequency ν is much larger than other characteristic frequencies of the problem. In this case we discard the rapidly oscillating terms and obtain the Hamiltonian in the interaction picture,

$$\hat{H}=\Omega e^{-\eta^2/2}e^{-i\phi}\sum_{j=1}^{N}\hat{S}_j^{+}\sum_{j=0}^{\infty}\frac{(i\eta)^{2j+1}}{j!(j+1)!}\times[\hat{a}^{+(j+1)}\hat{a}^{j}e^{-i\delta t}+\hat{a}^{\dagger j}\hat{a}^{j+1}e^{i\delta t}]+\text{H.c.} \qquad (34)$$

In the Lamb-Dicke regime (i.e., $\eta\sqrt{n+1}\ll 1$), with n being the phonon number, the Hamiltonian of Eq. (34) can be approximated by the expansion to the first order in η,

$$\hat{H}=i\eta\Omega e^{-i\phi}\sum_{j=1}^{N}\hat{S}_j^{+}(\hat{a}^{\dagger}e^{-i\delta t}+\hat{a}e^{i\delta t})+\text{H.c.} \qquad (35)$$

When $\delta\gg\eta\Omega$ and $\phi=\pi/2$, the effective Hamiltonian has the same form as Eq. (13), with $\lambda=2\Omega^2\eta^2/\delta$. For the case $N=2$ we focus the two lasers on the ions for a time $\arcsin(1/\sqrt{3})/\lambda$, then perform the transformation $|e\rangle\rightarrow|f\rangle$ with $|f\rangle$ being another internal state, followed by the application of the two above-mentioned lasers for a time $\pi/(4\lambda)$. The two ions are prepared in the state of Eq. (19). Using the procedure similar to that for cavity QED we can also generate entangled states for many multilevel ions. The effective Hamiltonian does not involve the external degree of freedom and thus the scheme is insensitive to the external state, allowing it to be in a thermal state. For the generation of the states of Eqs. (19), (28), and (32), we do not require individual addressing of the ions.

In conclusion, we have proposed a scheme for generating entangled states for two or more multilevel particles in both cavity QED and ion trap. In cavity QED, our scheme does not require individual addressing of atoms in the cavity. In cavity QED the scheme is insensitive to both cavity decay and thermal field, which is of importance from the experimental point of view. In ion trap, our scheme is insensitive to the thermal motion. Based on the experiments reported in Refs. [10,13,14], our scheme is realizable with techniques presently available.

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