One-mirror random laser

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(Received 16 January 2003; published 29 August 2003)

A one-mirror random laser is described. The necessary feedback is from random scattering inside a disordered medium and a reflective mirror together. With such a half-random and half-conventional cavity, the lasing threshold can be reduced dramatically, especially for systems in the localized regime, where the lasing threshold is otherwise very difficult to reach by optical pumping. The threshold decrease is due to better overlap between the pumped region and lasing modes, as well as the different eigenmode structure of the half-close system. An Anderson-type lattice Hamiltonian model was used to investigate the decay rate distributions in one-dimensional systems open at one end and at both ends. Analysis on the small decay rate tail showed an enhancement of localization in the former kind of systems.

DOI: 10.1103/PhysRevA.68.025803 PACS number(s): 42.55.Zz, 42.60.By, 71.55.Jv

A random laser is a laser where the feedback is not from end mirrors as in conventional lasers but from random scattering inside the medium. More than 30 years ago, Letokhov [1] predicted theoretically the emergence of a lasing instability in a disordered system where transport of light intensity can be described by a diffusion formalism. In the past decade, laser like emission was discovered experimentally in several different systems [2–8], such as powders of rare-earth-doped or stoichiometric materials, Ti:sapphire powders, dye with Ti_2O_3 scatterers, and ZnO powders. There are actually two kinds of essentially different phenomena in random media with gain: amplified spontaneous emission (ASE) and random lasing. Cao and Zhao [3] proved the existence of random lasing experimentally in 1999. These two phenomena are sometimes referred to as random lasers with incoherent and coherent feedback, respectively [8].

Although up to now most experimental discoveries of random lasing were not in the regime of localization, lasing in the localized situation is of most interest. It is expected to be most efficient for lasing because of photon trapping [9,10]. There exist at least two experimental studies where strong localization of light has been found [11,12]. But in practice, it is difficult to achieve random lasing in the localized regime. Most studies use optical pumping to obtain gain in media. Lasing light as well as pumping light is possible to be within or close to the localized regime. Thus, pumping light can only penetrate into the sample by a depth scale of ξ_p, which is the localization length for pumping light. So the gain is located within a thin surface layer of the sample. Therefore, lasing modes, which have significant amount of overlap with the gain region, will have a large loss as well. The lasing threshold would be very high.

Of course, one can achieve deeper distribution of gain by using light of much shorter wavelength that is far from the localization regime. Even electron beam pumping is a choice for some applications, which had been seen in literature indeed [13]. According to the authors it was the only publication that demonstrated very low threshold by continuous-wave operation, which is one of the expectations in the strongly localized regime. But for the large difference of quantum energy between pumping and lasing particles, a thermal problem is expected.

In this Brief Report, we describe a one-mirror random laser with the aim of reducing the lasing threshold and achieving lasing in random media in the localized regime by optical pumping. First, we demonstrate how higher overlap between lasing modes and pump distribution can be guaranteed and contributes to reducing the threshold. After that, contribution from the mirror symmetry of the system is investigated by numerical simulations, which compare small decay rate distributions of systems open at one end and both ends. It should be noted that there are publications on the effect of feedback from an external mirror on lasing properties [14,15]. But the systems they studied were in the diffusive regime. Moreover, the mirror mainly acted as a source of external extra feedback, whereas in this study the importance of the mirror is so high that the laser ‘‘cavities’’ are formed by random media and the mirror together. Besides, it may be worth noting that the influence of inhomogeneous pumping had been included in the study of random laser in diffusion media [5], but the method of two-sided pumping obviously does not work for localized media.

In the localized regime, the mode wave functions have the asymptotic behavior [16]

$$\psi(r)=f(r)^{-r/\xi}, \tag{1}$$

where $f(r)$ is a randomly varying function, ξ is the localization length. This also means the transmission decreases exponentially instead of being linear with the thickness of a sample. Therefore, as mentioned above, pump light can penetrate only by a depth scale of ξ_p, which is comparable to one wavelength. For the same reason, loss of the localized mode depends exponentially on the distance from the ‘‘center of mode’’ to the medium-air interface. On the other hand, gain for a specific mode is proportional to the overlap between the mode and gain distribution [17]. When the gain for a certain mode reaches a value that equals the loss of the mode, lasing instability occurs, which is the lasing threshold. Unfortunately, those modes, which have larger overlap with the gain region, also have higher loss because they are located near the medium-air interface. So the lasing threshold is difficult to approach.

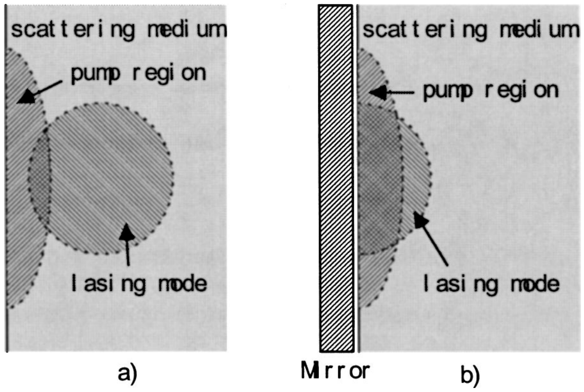

FIG. 1. Schematic illustrations of the concept of a one-mirror random laser. (a) System without mirror; (b) system with mirror. In the presence of a mirror, better overlap between pump and lasing modes can be achieved.

The setup we propose places a mirror close to one surface of the sample, as shown in Fig. 1. The mirror is highly transmissive for pump light but highly reflective for lasing light. Because of the high reflectivity of the mirror, the localized modes, which are "centered" at the sample-mirror interface, can have very low loss that mainly depends on the reflectivity of the mirror. Simultaneously, high overlap between those modes and the pumped region is guaranteed. So one can expect a large decrease in the lasing threshold. To understand the design is quite straightforward. We investigate a one-dimensional system as an example here for the integrity of the study, and compare the lasing threshold of the system with and without mirror.

The gain distribution $g(x)$ is proportional to the pump distribution, therefore,

$$g(x)=G_0A(x)e^{-x/\xi_p}. \tag{2}$$

G_0 is a structure-independent parameter, which represents gain and is proportional to pump power. $A(x)$ is a randomly varying function. ξ_p is the localization length of pump light, which is due to scattering as well as absorption [18]. According to Eq. (1) the intensity pattern of the localized mode follows

$$I(x,x_0)=B(x,x_0)e^{-|x-x_0|/\xi_l}, \tag{3}$$

where $B(x,x_0)$ is a randomly varying function. ξ_l is the localization length of lasing light. x_0 represents the distance from "mode center" to the left surface. Losses of modes can be described as

$$\delta_n \sim e^{-x_0/\xi_l}+e^{-(L-x_0)/\xi_l}+\delta_{0n}, \tag{4}$$

where L is the width of the sample and δ_{0n} is the mode loss due to absorption inside the sample. With the presence of mirror, modes, which center at the surface of sample ($x_0=0$), are of interest because of high overlap with the gain distribution. Losses of these δ_m, are

$$\delta_m \sim e^{-L/\xi_l}+\delta_{0m}, \tag{5}$$

δ_{0m} are contributions from transmissivity of the mirror and absorption in the medium. At the threshold, gain equals the loss, one has

$$\int_0^L g(x)I(x,x_0)dx=\int_0^L G_0A(x)B(x,x_0)e^{-x/\xi_p-|x-x_0|/\xi_l}dx$$
$$=G_0C\int_0^\infty e^{-x/\xi_p-|x-x_0|/\xi_l}dx=\delta. \tag{6}$$

The integration is extended to ∞ because $L \gg \xi_p,\ \xi_l$. The product of two randomly varying functions $A(x)$ and $B(x,x_0)$ is arbitrarily replaced by constant C for our objective is a qualitative analysis. δ is total loss of the mode.

One can then calculate the threshold gain. Provided that $\xi_l=\xi_p=\xi$, which will not lose features that we are interested in, one can get a simple formula. For the system with a mirror, the threshold gain G_{0mt} is evaluated to be

$$G_{0mt}\sim \frac{2}{C_m\xi}(e^{-L/\xi}+\delta_{0m}). \tag{7}$$

For the system without a mirror, the threshold gain of the mode centered at x_0, G'_{0nt}, is

$$G'_{0nt}\sim \frac{2}{C_n(\xi+2x_0)}(1+e^{-(L-2x_0)/\xi}+\delta_{0n}e^{x_0/\xi}). \tag{8}$$

C_m and C_n are constants for the cases with and without a mirror, respectively. One may not assume $x_0=L/2$. However, the lasing threshold for the system G_{0nt} is the minimum of G'_{0nt}, which cannot give an analytical form. In the limit of δ_{0m} and δ_{0n} equal zero and $L \gg \xi$, one can obtain

$$\frac{G_{0mt}}{G_{0nt}}\sim \frac{L}{\xi}e^{-L/\xi}, \tag{9}$$

where all constants are omitted. So a nearly exponential decrease of threshold is expected by using a mirror. For a general value of parameters, Fig. 2 gives out numerical results for comparison, where ξ is set as 1 μm and L is 1 mm. It is shown that the decrease of threshold would be weakened when δ_{0m} and δ_{0n} increase, but is still a large amount.

Up to now, threshold reduction due to better overlap between pump and lasing modes is demonstrated. In the following, we present some results of our numerical simulations on the decay rate distributions of systems open at one end and at both ends. For random laser study, the small part of decay rate distribution, which determines the laser threshold, is interesting. In these simulations, pump inhomogeneity was no longer considered.

The numerical model we used was identical to that in Ref. [19]. The system was described by an Anderson-type lattice Hamiltonian. We considered a one-dimensional system ($N=1$) for simplicity. The out-coupling strength k was set to be 1, hence modeling ideal coupling at the center of energy band. Based on the investigations in Ref. [19], only eigen-

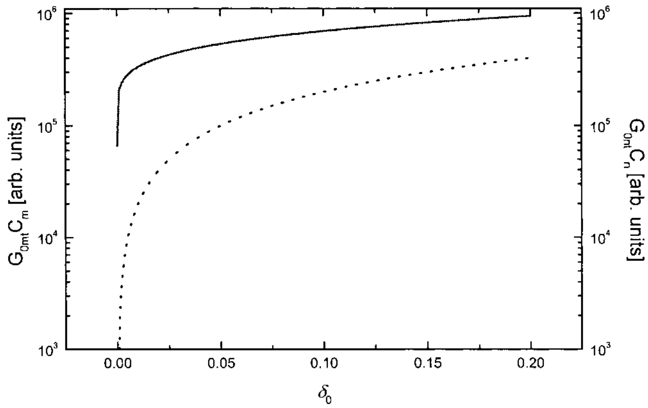

FIG. 2. Comparison of the threshold gain with and without mirror. Dashed line is for the system with mirror, whereas the solid line is for the system without mirror. C_m and C_n are two constants, which have comparable values. δ_0 stands for δ_{0m} and δ_{0n} for systems with and without mirror. It can be seen that the decrease of threshold would be weakened when δ_0 increase, but is still a large amount.

values whose real parts were in window $[-0.1; 0.1]$ were included in the analysis. Disorder was modeled by assigning random values to potential $P(x)$. Those random values were assumed to be uniformly distributed in the interval $[-w;w]$, and w was set to be 0.5 throughout our calculations.

The numerical work was done with MATLAB. The function *eigs* was used to find the eigenvalues close to zero. By setting options we guaranteed that all eigenvalues whose absolute values were smaller than a were solved. a was selected to be around 0.15. After that, eigenvalues with real part in $[-0.1; 0.1]$ were picked up for further analysis. One will find that this procedure leads to an artificial error because eigenvalues whose absolute values are larger than a but have their real part in window $[-0.1; 0.1]$ are not included. But this only affects large decay rates; the influence is negligible for a small decay rate tail in which we are interested.

Figure 3 shows some typical decay rate distributions in

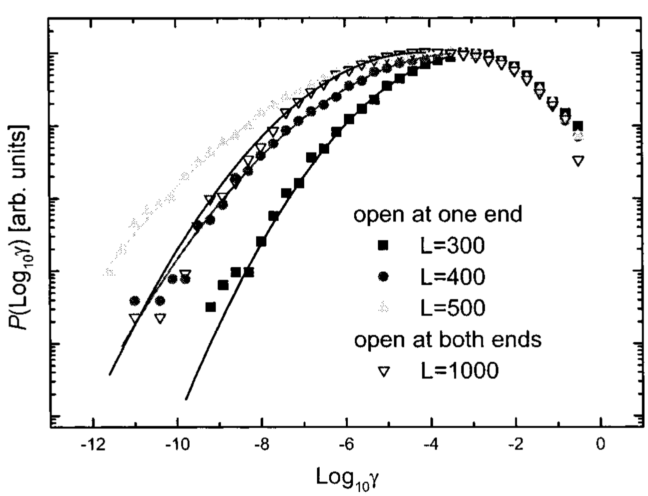

FIG. 3. Typical decay rate distributions in the localized regime and fit of left wings to the log-normal function.

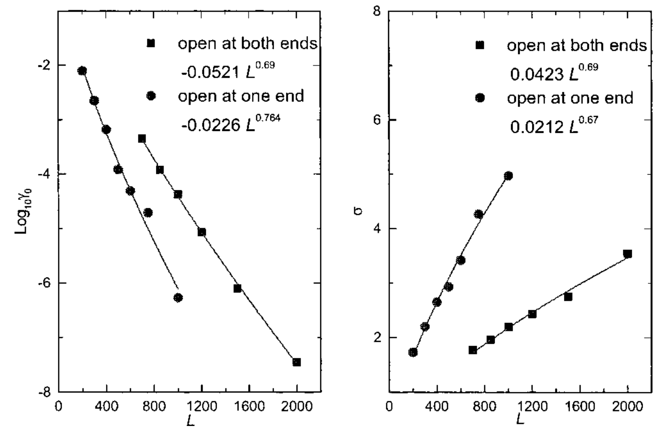

FIG. 4. Left: fits of distribution peak positions $\log_{10}\gamma_0$ to the power law of sample length L. Right: fits of distribution width σ to the power law of L.

the localized regime, which are scaled to the same peak value. The right wings follow a power law as discussed in Refs. [20,21]. The rapid drop at the right end is an artifact as mentioned above. It seems that a system of length 1000 that is open at both ends is comparable to that of length 400, instead of 500, that is open at one end, as far as the small decay rate tail is concerned. This means an enhancement of localization for those modes of small decay rates in systems open at one end. It would be interesting to seek a possible mapping relation between these two kinds of systems.

It is generally accepted that the distribution of the decay rates γ (for small γ) in the localized regime is log-normal,

$$P(\gamma)\propto\exp\left(-\frac{(\log_{10}\gamma-\log_{10}\gamma_0)^2}{\sigma^2}\right). \tag{10}$$

Figure 3 also shows log-normal fits to the left wings of distributions. By fitting we got peak positions $\log_{10}\gamma_0$ and widths σ for a series of systems with different lengths. The obtained data were then fitted to power-law dependence to length, $C_1L^{C_1}$, which is presented in Fig. 4. We got (C_1,C_2) values of (0.0423, 0.69) and (0.0212, 0.67) for σ in systems open at one end and both ends, respectively, which is strikingly consistent with the formula with $\frac{2}{3}$ power law proposed in Ref. [19]. Assuming all other parameters except localization length ξ are identical for both kinds of systems, we would like to propose a simple numerical relationship between localization lengths ξ_1, ξ_2 of systems open at one end and both ends, respectively,

$$\xi_1\approx 2^{-3/2}\xi_2. \tag{11}$$

So systems open at one end do not simply correspond to systems open at both ends whose length is two times longer. The localization length here should be understood in the viewpoint of dwell time instead of transmission, since for one-end systems it is not possible to calculate the transmission rate. The enhancement of localization is due to coherence nature of the problem. As for the microscopic mechanism of this enhancement we would like to suggest the role of localized modes near the closed end. However, within the content of this report, mapping the relation between these two kinds of systems cannot be deduced yet.

We have shown the effect of the mirror on reducing the lasing threshold in the localized regime. But it is worth noting that the effect is not limited to this regime. For weaker random scattering media, a lower but still large amount of reduction of the lasing threshold can be expected. For a real three-dimensional system, it may suffer from incident-angle-dependent reflectivity of common coating. This can be partly overcome by combining the total reflection effect at sample surface since materials with high refractive index are always used for stronger scattering.

In summary, a random laser with one mirror is proposed for reducing the lasing threshold and achieving lasing in disordered media in the localized regime. The feedback is from random scattering inside the medium as well as the mirror. The origins of the threshold reduction are twofold: guarantee of better overlap between the pumped region and lasing modes, and different eigenmode structure of the half-close system. Numerical simulations based on an Anderson-type lattice Hamiltonian model showed an enhancement of localization in such systems open at one end over those open at both ends. We proposed a numerical relation between localization lengths of these two kinds of systems. But a complete comparison of these two kinds of systems cannot be deduced in this report yet. Such half-random lasers should have much scientific and practical interest.

Yan Feng thanks Michael Patra of the Laboratory for Computational Engineering, Helsinki University of Technology, Finland, for discussions.

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