Remote information concentration by a Greenberger-Horne-Zeilinger state and by a bound entangled state

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We compare remote quantum information concentration by a Greenberger-Horne-Zeilinger (GHZ) state with an unlockable bound entangled state. We find that in view of communication security the bound entangled state works better than the GHZ state.

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Quantum entanglement, a peculiar feature of quantum formalism, has been at the center of quantum information processing. For example, the Einstein-Podolsky-Rosen state has manifested highly counterintuitive effects such as quantum teleportation [1], quantum dense coding [2], quantum cryptography [3]. In practice, one usually deals with noisy entanglement described by mixed state of a composite system. All the noisy entanglement of two-qubit system can be distilled to the singlet form [4]. But, for the mixed entangled states of more-than-two-qubit system, there are two qualitatively different kinds of entanglement: free entanglement which can always be distilled and bound one which cannot be brought to the singlet form only by local quantum operations and classical communication (LOCC) [5]. In the recent years, the bound entangled state of multipartite system is extensively studied because it arouses a deeper understanding of the entanglement and the nonlocality of quantum states [6–9].

At first glance, the bound entanglement seems to be useless alone for quantum information work such as reliable transmission of quantum state via teleportation [5,10]. But, recent researches have shown that the bound entanglement can be activated and used to process quantum information [11–14]. And more recently, Murao and Vedral have presented in Ref. [15] a surprising fact that a single copy of a bound entangled state [16] can perform effectively remote information concentration, which cannot be achieved by classically correlated state. In the process of remote information concentration [15], they considered that the quantum information of a single qubit was previously distributed to three spatially separated qubits by a *symmetric* telecloning procedure [17], and now has to be remotely concentrated back to a single qubit by an unlockable bound entangled state [16] with LOCC. In this paper, we show that the same task can also be achieved by a four-particle Greenberger-Horne-Zeilinger (GHZ) state. But, importantly, when we investigate the more general case where the quantum information of the single qubit was previously distributed by an *asymmetric* telecloning procedure [18], we find that the bound entangled state does the work of remote information concentration better than the GHZ state in view of communication security. In the following, we discuss the issue in detail.

Remote information concentration begins with a situation where three separate parties Alice, Bob, and Charlie hold three qubits A, B, and C, respectively. For generality, the three qubits are in a cloning state $|\psi\rangle_{ABC}$ after the asymmetric telecloning procedure [17,18],

$$|\psi\rangle_{ABC}=\alpha|\phi_0\rangle_{ABC}+\beta|\phi_1\rangle_{ABC}, \tag{1}$$

where α and β are complex numbers and satisfy $|\alpha|^2+|\beta|^2=1$. The states $|\phi_0\rangle_{ABC}$ and $|\phi_1\rangle_{ABC}$ are defined as

$$|\phi_0\rangle_{ABC}=\frac{1}{\sqrt{N}}(|0\rangle_A|0\rangle_B|0\rangle_C+p|1\rangle_A|0\rangle_B|1\rangle_C+q|1\rangle_A|1\rangle_B|0\rangle_C),$$

$$|\phi_1\rangle_{ABC}=\frac{1}{\sqrt{N}}(|1\rangle_A|1\rangle_B|1\rangle_C+p|0\rangle_A|1\rangle_B|0\rangle_C+q|0\rangle_A|0\rangle_B|1\rangle_C), \tag{2}$$

where $q=1-p,p>q,N$ is a normalization factor given by $N=1+p^2+q^2$ (when $p=q=\frac{1}{2}$, it reduces to the case considered in Ref. [15]). That is, the information of an unknown state $|\chi\rangle=\alpha|0\rangle+\beta|1\rangle$ is diluted to a composite system consisting of qubits A, B, and C by the asymmetric telecloning procedure [17,18]. Now our task is to concentrate the information back to a single qubit without the collective operations among the three qubits. The schematic picture of the remote information concentration is illustrated in Fig. 1.

The first protocol achieving the task is to share in advance a four-particle GHZ state

$$|\varphi\rangle_{DEFG}=\frac{1}{\sqrt{2}}(|0\rangle_D|0\rangle_E|0\rangle_F|0\rangle_G+|1\rangle_D|1\rangle_E|1\rangle_F|1\rangle_G) \tag{3}$$

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among four parties Alice, Bob, Charlie, and David, who hold the qubits E, F, G, and D, respectively. Then, three parties Alice, Bob, and Charlie perform the Bell-state measurements (BSMs) on the respective pairs of qubits $A+E$, $B+F$, and $C+G$, and inform David of the results of the measurements. Each of the three BSMs gives one of four possible outputs $\{|\Phi^i\rangle_{(i=0,1,2,3)}\}$, where $|\Phi^i\rangle_{(i=0,1,2,3)}$ represent the four Bell states $|\Phi^{0,1}\rangle=(|00\rangle\pm|11\rangle)/\sqrt{2}$, $|\Phi^{2,3}\rangle=(|01\rangle\pm|01\rangle)/\sqrt{2}$. And each of the four possible outputs is associated with a Pauli operator in the set $\{\sigma^i_{(i=0,1,2,3)}\}$, where $\sigma_0=I,\sigma_1=\sigma_z,\sigma_2=\sigma_x$, and $\sigma_3=\sigma_y$. Finally, according to their results of the measurements, David rotates his qubit D correctly and obtains the state $|\chi\rangle$.

The process of remote information concentration by the GHZ state can be understood in a formula way. We write the states $|\psi\rangle_{ABC}$ and $|\varphi\rangle_{DEFG}$ together:

$$\begin{aligned}|\psi\rangle_{ABC}|\varphi\rangle_{DEFG}&=(\alpha|\phi_0\rangle_{ABC}+\beta|\phi_1\rangle_{ABC})\otimes\frac{1}{\sqrt{2}}(|0\rangle_D|0\rangle_E|0\rangle_F|0\rangle_G+|1\rangle_D|1\rangle_E|1\rangle_F|1\rangle_G)\\
&=\frac{1}{\sqrt{2N}}\{\alpha(|00\rangle_{AE}|00\rangle_{BF}|00\rangle_{CG}|0\rangle_D+p|10\rangle_{AE}|00\rangle_{BF}|10\rangle_{CG}|0\rangle_D+q|10\rangle_{AE}|10\rangle_{BF}|00\rangle_{CG}|0\rangle_D\\
&\quad+|01\rangle_{AE}|01\rangle_{BF}|01\rangle_{CG}|1\rangle_D+p|11\rangle_{AE}|01\rangle_{BF}|11\rangle_{CG}|1\rangle_D+q|11\rangle_{AE}|11\rangle_{BF}|01\rangle_{CG}|1\rangle_D)\\
&\quad+\beta(|10\rangle_{AE}|10\rangle_{BF}|10\rangle_{CG}|0\rangle_D+q|00\rangle_{AE}|00\rangle_{BF}|10\rangle_{CG}|0\rangle_D+p|00\rangle_{AE}|10\rangle_{BF}|00\rangle_{CG}|0\rangle_D\\
&\quad+|11\rangle_{AE}|11\rangle_{BF}|11\rangle_{CG}|1\rangle_D+q|01\rangle_{AE}|01\rangle_{BF}|11\rangle_{CG}|1\rangle_D+p|01\rangle_{AE}|11\rangle_{BF}|01\rangle_{CG}|1\rangle_D)\}.\end{aligned}\tag{4}$$

A lengthy but straightforward calculation shows that when the combination of the results of the three BSMs is in the set $\{\Phi^0_{AE}\Phi^0_{BF}\Phi^0_{CG}$, $\Phi^0_{AE}\Phi^1_{BF}\Phi^1_{CG}$, $\Phi^1_{AE}\Phi^0_{BF}\Phi^1_{CG}$, $\Phi^1_{AE}\Phi^1_{BF}\Phi^0_{CG}$, $\Phi^2_{AE}\Phi^0_{BF}\Phi^2_{CG}$, $\Phi^3_{AE}\Phi^0_{BF}\Phi^3_{CG}$, $\Phi^3_{AE}\Phi^1_{BF}\Phi^2_{CG}$, $\Phi^2_{AE}\Phi^1_{BF}\Phi^3_{CG}$, $\Phi^2_{AE}\Phi^2_{BF}\Phi^0_{CG}$, $\Phi^3_{AE}\Phi^3_{BF}\Phi^0_{CG}$, $\Phi^2_{AE}\Phi^3_{BF}\Phi^1_{CG}$, $\Phi^3_{AE}\Phi^2_{BF}\Phi^1_{CG}\}$, David chooses the operator σ^0 on the qubit D in order to get the state $|\chi\rangle$. In other words, if σ^l_{AE}, σ^j_{BF}, and σ^k_{CG}, where $l,j,k=0,1,2,3$, denote the Pauli operators pertaining to the results of three BSMs on the pairs of qubits $A+E$, $B+F$, and $C+G$, respectively, and if the product of σ^l_{AE}, σ^j_{BF}, and σ^k_{CG} is σ^0 up to a global phase factor, the operator σ^0 is chosen to perform on the qubit D. The same rule is suited to the other combining results of the three BSMs. That is, if up to global phase factor, $\sigma^i=\sigma^l_{AE}\sigma^j_{BF}\sigma^k_{CG}$, David performs the operator σ^i on his qubit D to retrieve the state $|\chi\rangle$. So the protocol achieves the task: $|\psi\rangle_{ABC}\rightarrow|\chi\rangle$ with certainty by presharing a four-particle GHZ state.

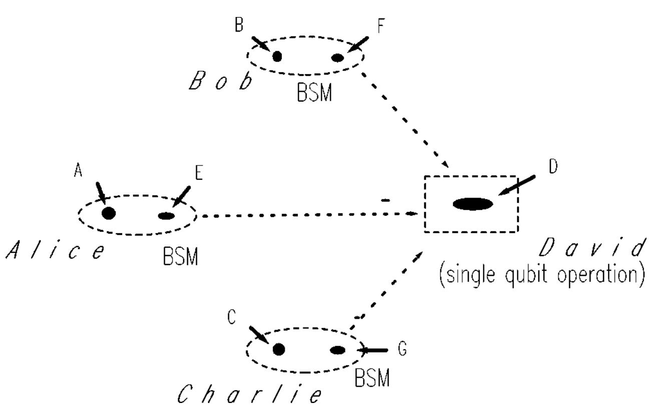

FIG. 1. The schematic picture for remote information concentration.

It is worth noting that the distribution probability of the combining results reveals some information of the state $|\psi\rangle_{ABC}$. For instance, while we get $\Phi^0_{AE}\Phi^0_{BF}\Phi^0_{CG}$ with the probability of $1/16N$, we obtain $\Phi^2_{AE}\Phi^0_{BF}\Phi^2_{CG}$ with the one of $p^2/16N$, $\Phi^2_{AE}\Phi^2_{BF}\Phi^0_{CG}$ with $q^2/16N$, and $\Phi^0_{AE}\Phi^2_{BF}\Phi^2_{CG}$ with zero. When using Eq. (3) to perform remote information concentration with $p=q=\frac{1}{2}$, i.e., the case discussed in Ref. [15], $\Phi^0_{AE}\Phi^0_{BF}\Phi^0_{CG}$ occurs with the probability of $\frac{1}{24}$, $\Phi^2_{AE}\Phi^0_{BF}\Phi^2_{CG}$ with $\frac{1}{96}$, $\Phi^2_{AE}\Phi^2_{BF}\Phi^0_{CG}$ with $\frac{1}{96}$, and $\Phi^0_{AE}\Phi^2_{BF}\Phi^2_{CG}$ with zero.

Then, we inspect the achievement of remote information concentration by an unlockable bound entangled state and LOCC. Instead of the previous four-particle GHZ state $|\varphi\rangle_{DEFG}$, a four-particle unlockable bound entangled state [16]

$$\rho^{ub}_{DEFG}=\frac{1}{4}\sum_{i=0}^{3}|\Phi^i\rangle_{DE}\langle\Phi^i|\otimes|\Phi^i\rangle_{FG}\langle\Phi^i|,\tag{5}$$

where $|\Phi^i\rangle$ are defined as before, is preshared among Alice, Bob, Charlie, and David. In the same way as above, the qubit E is sent to Alice, the qubit F to Bob, the qubit G to Charlie, and the qubit D to David. No joint operations among qubits belonging to different parties are allowed. The three parties Alice, Bob, and Charlie perform the Bell-state measurements on their respective pairs of qubits in hand. Likewise, each of them obtains one of the possible outcomes $\{|\Phi^i\rangle_{(i=0,1,2,3)}\}$ which has a corresponding Pauli operator in the set $\{\sigma^i_{(i=0,1,2,3)}\}$, and communicates the result with David, respectively. According to the product of the three Pauli operators pertaining to the results of the three BSMs, David determines an appropriate Pauli operator σ^i on his qubit D to retrieve the state $|\chi\rangle$. Finally, David gets a qubit D in the state $|\chi\rangle$ with certainty.

The process can also be expressed in the same formular way as before. While the direct calculation shows that there is something different from the above protocol with the GHZ state. The distribution of the combining results in the protocol with the bound entangled state does not reveal any information about the state $|\psi\rangle_{ABC}$. Each of all combining results of the three BSMs happens with the equal probability of $\frac{1}{64}$. For example, the combining results $\Phi^0_{AE}\Phi^0_{BF}\Phi^0_{CG}$, $\Phi^2_{AE}\Phi^0_{BF}\Phi^2_{CG}$, $\Phi^0_{AE}\Phi^2_{BF}\Phi^2_{CG}$, and $\Phi^2_{AE}\Phi^2_{BF}\Phi^0_{CG}$ come out with the equal probability of $\frac{1}{64}$. Because the probability of distribution of the combining results is independent of the parameters p, q, and N, all combining results occur with the same probability of $\frac{1}{64}$ in the process of remote information concentration discussed in Ref. [15]. So, allowing for secure communication, the unlockable bound entangled state is more suitable for remote information concentration than the GHZ state.

Hence, by employing an unlockable bound entangled state, the same task of remote information concentration is achieved as by a GHZ state. But from the view of communication security, the bound entangled state is more useful than the GHZ state.

In summary, it is shown that the task of remote information concentration can be achieved by a GHZ state and by a copy of a bound entangled state. This demonstrates that in a sense, a single copy of a bound entangled state is as useful in quantum information processing as a GHZ state. But, importantly, in view of communication security, the bound entangled state does the work of remote information concentration better than the GHZ state. Because in the protocol of using the bound entangled state, the distribution of the combining results of the three local BSMs does not reveal any information about the concentrated state $|\psi\rangle_{ABC}$, while that does so in the protocol of using a GHZ state. The reason of this difference is open. And the consideration for the difference is helpful for deeper understanding the relation between the free entanglement and the bound one, and quantum entanglement itself. We hope that our work will stimulate more research into the nature of entanglement and its applications in quantum communication.

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