Polarization studies on the radiative recombination of highly charged bare ions

Andrey Surzhykov* and Stephan Fritzsche
Universität Kassel, Heinrich-Plett-Strasse 40, D-34132 Kassel, Germany

Thomas Stöhlker and Stanislav Tachenov
Gesellschaft für Schwerionenforschung (GSI), D-64291 Darmstadt, Germany

(Received 29 April 2003; published 27 August 2003)

The polarization of the emitted photons is studied for the radiative recombination of free electrons into the bound states of bare, highly charged ions. We apply density matrix theory in order to investigate how the photon polarization is affected if the incident electrons are themselves spin polarized. For *K*-shell electron capture, for instance, the linear polarization of the light, which is measured out of the reaction plane, is defined by the degree of polarization of the electrons and may be used as a tool for studying the polarization properties of the electron targets and/or the projectile ions. Detailed computations of the Stokes parameters of x-ray emission following the radiative recombination of bare uranium ions U^{92+} are carried out for a wide range of projectile energies and for different polarization states of the incident electrons.

DOI: 10.1103/PhysRevA.68.022710 PACS number(s): 34.80.Lx, 34.80.Nz, 34.70.+e, 32.80.Fb

I. INTRODUCTION

With the recent experimental advances in heavy-ion accelerators and ion storage rings, more possibilities arise to study ion-electron and ion-atom collisions. For the relativistic collisions of highly charged ions with low-*Z* target atoms (or free electrons), for instance, a number of case studies on radiative electron capture, *K*-shell Coulomb excitation and ionization of projectiles, electron bremsstrahlung, and even correlated two-electron capture have proceeded in recent years at the GSI storage ring in Darmstadt [1]. So far, however, most of these experiments have dealt with target atoms (or electrons) and ion beams that are both unpolarized. A wide range of qualitatively different *polarization* studies will be opened up by using spin-polarized projectile ions or/and target atoms. Such experiments are very likely to be carried out at the future GSI facilities which will be installed within the next ten years.

Polarization collision experiments, however, require an effective tool for diagnostics the polarization properties of the beam as well as of the target atoms (or electrons). It is necessary, therefore, to find a *probe* process whose characteristics are sensitive to the polarization states of the collision system. One such probe process, which we suggest from the theoretical viewpoint, is the capture of a free (or quasifree) electron into a bound state of the projectile ion with the simultaneous emission of a photon which carries away the excess energy and momentum. This capture process, denoted *radiative recombination*, has been intensively studied during recent years in the relativistic collisions of high-*Z* projectile ions with low-*Z* target atoms (or free electrons). A series of experiments, for instance, has been carried out at the GSI storage ring [1,2] in order to explore the total and angle-differential recombination cross sections, which were found to be in good agreement with theoretical predictions based on relativistic Dirac theory [3–5]. However, neither the total recombination cross section nor the angular distribution of the emitted photons was found to be (much) dependent on the polarization of the ion beam or atomic target and therefore they cannot be used for polarization studies.

In contrast to the total and angle-differential cross sections, the polarization of the emitted photons may appear very sensitive to the particle polarization. A similar effect, for instance, has long been known for the atomic photoeffect [6,7], where the spin polarization of the emitted electron is strongly affected by the polarization of the incident photon. Since the photoeffect is the time-inverse process of radiative recombination, we can expect that measurements of the polarization of the recombination photons will provide us with information on the spin polarization of the target electrons (atoms) or ion beam. In fact, such measurements are possible nowadays for the *linear* polarization of x-ray photons due to the recent improvements in position sensitive polarization detectors. In the last year, for instance, measurements of the linear polarization of *K*-shell recombination photons have been carried out for electron capture into bare uranium ions U^{92+}.

In this paper, we study the linear polarization of the photons that are emitted due to the capture of free polarized electrons into bound states of bare, high-*Z* ions. For such investigations of the angular distribution and polarization properties of the emitted radiation, density matrix theory has been found to be the appropriate framework in order to accompany the system through the collision process [8]. Since, however, the concept of density matrix theory has been presented elsewhere in a number of places [8–10], we may restrict ourselves to rather a short outline of the basic relations within the two following sections. Starting from the basic representation of the density matrix, we first derive the explicit expressions for the Stokes parameters of the recombination photons and simplify them by using the parity properties of the levels involved. In Sec. II D, moreover, we introduce a (so-called) *polarization ellipse*, which helps us to discuss and better understand the linear polarization of the emitted x-ray radiation. This representation in terms of an ellipse also shows explicitly how the polarization of the x

*Electronic address: surz@physik.uni-kassel.de

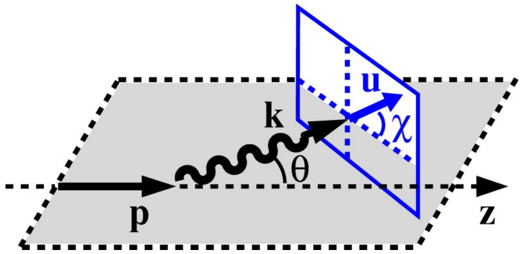

FIG. 1. The unit vector $\mathbf{u}(\chi)$ of the linear polarization is defined in the plane that is perpendicular to the photon momentum $\mathbf{k}$, and is characterized by an angle χ with respect to the reaction plane.

rays is affected if the incident electrons are also polarized. In Sec. III, we describe a series of computations that were carried out for the linear polarization of the emitted photons following the capture of an electron into the K shell of bare uranium (projectile) ions U^{92+}. As seen from this computation, polarization of the incident electrons generally leads also to a rotation of the polarization vector of the light *out of the reaction plane*. A summary of this important result and its implication for future experiments is finally given in Sec. IV.

II. BASIC FORMULAS

A. Polarization vector of the photon

For the radiative recombination of free electrons into bare, high-Z ions, several case studies are known today, which are based on Dirac's equation [4,11–13]. In such a relativistic treatment of the electronic capture, Dirac-Coulomb wave functions are usually applied throughout the computations, both for the incident (free) electron with well defined asymptotic momentum $\mathbf{p}$ and spin projection m_s as well as for the final *bound* state $|n_b j_b \mu_b\rangle$ of the electron. In addition, the emitted—or recombination—photon is typically described in terms of a plane wave with wave vector $\mathbf{k}$ ($k = \omega/c$) and with a polarization that points perpendicular to $\mathbf{k}$ along some unit vector $\mathbf{u}$. The wave vector $\mathbf{k}$ and the electron momentum $\mathbf{p}$ span the reaction plane in the experiment. Of course, the (polarization) vector $\mathbf{u}$ can always be rewritten in terms of any two (linearly independent) basis vectors, such as the *circular-polarization* vectors $\mathbf{u}_{\pm 1}$, which are perpendicular to the wave vector $\mathbf{k}$ and which for $\mathbf{u}_{+1}$ and $\mathbf{u}_{-1}$ refer to right- and left-circularly polarized photons [9], respectively. In such a basis, the unit vector for the linear polarization of the emitted x rays can be written as

$$\mathbf{u}(\chi) = \frac{1}{\sqrt{2}}(e^{-i\chi}\mathbf{u}_{+1} + e^{i\chi}\mathbf{u}_{-1}), \tag{1}$$

where χ is the angle between $\mathbf{u}(\chi)$ and the reaction plane (see Fig. 1).

B. Density matrix approach

While the definition (1) of the polarization vector $\mathbf{u}$ is appropriate to describe the linear polarization of photons in a pure polarization state, it is not sufficient if several photons with different polarization states are emitted in the course of a capture or collision process. If, for example, we consider a photon beam in some mixed state, the polarization of the photons is then better described in terms of the spin-density matrix. Since the photon (with spin $S=1$) has only two allowed spin (or helicity) states $|\mathbf{k}\lambda\rangle$, $\lambda = \pm 1$, the spin-density matrix of the photon is a 2×2 matrix and, hence, can be parametrized by the three (real) *Stokes parameters* [8,9]

$$\langle \mathbf{k}\lambda|\hat{\rho}_\gamma|\mathbf{k}\lambda'\rangle = \frac{1}{2}\begin{pmatrix} 1+P_3 & P_1 - iP_2 \\ P_1 + iP_2 & 1-P_3 \end{pmatrix}. \tag{2}$$

In fact, these parameters are often utilized in experiments in order to characterize the degree of polarization of the emitted light; while the Stokes parameter P_3 reflects the degree of circular polarization, the two parameters P_1 and P_2 together denote the (degree and direction of the) linear polarization of the light in the plane perpendicular to the photon momentum $\mathbf{k}$. Experimentally, these Stokes parameters are determined simply by measuring the intensities of the light I_χ, linearly polarized at different angles χ with respect to the reaction plane. For instance, the parameter P_1 is given by the intensity ratio

$$P_1 = \frac{I_0 - I_{90}}{I_0 + I_{90}}, \tag{3}$$

while the parameter P_2 is obtained from a very similar ratio at angles $\chi = 45°$ and $\chi = 135°$, respectively (see Fig. 1):

$$P_2 = \frac{I_{45} - I_{135}}{I_{45} + I_{135}}. \tag{4}$$

As seen from Eq. (2), the three Stokes parameters can obviously also be expressed in terms of the matrix elements of the photon spin-density matrix. For electron capture into a bound state $|n_b j_b \mu_b\rangle$ of a (subsequently hydrogenlike) projectile ion, an expression for these matrix elements was derived previously [11]:

$$\langle \mathbf{k}\lambda|\hat{\rho}_\gamma|\mathbf{k}\lambda'\rangle = \sum_{\nu\mu} D^{\nu}_{0\mu}(0,\theta,0) \sum_{L\pi L'\pi'} \beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda'), \tag{5}$$

where θ denotes the angle of the photons with respect to the momentum $\mathbf{p}$ of the (incoming) electrons (see Fig. 1). The angular parameters $\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda')$ refer to the contributions of the different multipoles of the radiation field to the polarization state of the emitted photons and can be written as

$$\begin{aligned}\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda') = \sum_{m_s\mu_b} & i^{L'+\pi'-L-\pi}(-1)^{m_s-\mu_b}[L,L']^{1/2} \\ & \times \langle \mathbf{p}m_s|\hat{\rho}_e|\mathbf{p}m_s\rangle \lambda^{\pi}\lambda'^{\pi'}\langle L'\lambda' L -\lambda|\nu\mu\rangle \\ & \times \langle L' m_s - \mu_b L \mu_b - m_s|\nu 0\rangle \\ & \times \langle \mathbf{p}m_s|\boldsymbol{\alpha}\mathbf{A}^{\pi'}_{L' m_s-\mu_b}|\kappa_b\mu_b\rangle \\ & \times \langle \mathbf{p}m_s|\boldsymbol{\alpha}\mathbf{A}^{\pi}_{L m_s-\mu_b}|\kappa_b\mu_b\rangle^*,\end{aligned} \tag{6}$$

where $[L]=2L+1$ and $\langle \mathbf{p}m_s|\boldsymbol{\alpha}\mathbf{A}^{\pi}_{LM}|\kappa_b\mu_b\rangle$ denotes the matrix element for either the electric ($\pi=1$) or magnetic ($\pi=0$) multipole *free-bound* transition of the electron. The explicit separation of the transition amplitudes into their electric and magnetic components, as displayed in Eqs. (5) and (6), will help us later in simplifying the expressions for the Stokes parameters. Of course, the angular coefficient (6) still depends on the (initially prepared) spin-density matrix $\langle \mathbf{p}m_s|\hat{\rho}_e|\mathbf{p}m_s\rangle$, i.e., on the polarization state of the incident electrons.

The transition matrix elements in the last two lines of expression (6) contain the wave function $|\mathbf{p}m_s\rangle$ of a free electron with a definite asymptotic momentum. For further simplification of the spin-density matrix (5), it is therefore necessary to decompose this continuum wave into partial waves $|E\kappa jm_s\rangle$, in order to apply later the standard techniques from the theory of angular momentum. As discussed previously [11–13], however, special care has to be taken about the choice of the quantization axis since this directly influences the particular form of the partial wave decomposition. Using, for example, the electron momentum $\mathbf{p}$ as the quantization axis, the full expansion of the continuum wave function is given by [4]

$$|\mathbf{p}m_s\rangle=\sum_{\kappa} i^l e^{i\Delta_\kappa}\sqrt{4\pi(2l+1)}\langle l01/2m_s|jm_s\rangle|E\kappa jm_s\rangle, \tag{7}$$

where the summation runs over all partial waves $\kappa=\pm1, \pm2, \ldots$, i.e., along all values of (Dirac's) angular momentum quantum number $\kappa=\pm(j+1/2)$ for $l=j\pm1/2$. In this notation, the (nonrelativistic orbital) momentum l now represents the parity of the partial waves $|E\kappa jm_s\rangle$, and Δ_κ is the Coulomb phase shift.

Using the decomposition (7) of the continuum wave function together with the Wigner-Eckart theorem [8], the angular parameters (6) can be rewritten in the form

$$\begin{aligned}
&\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda')\\
&\quad=\sum_{\kappa\kappa'} i^{L'+\pi'-L-\pi} i^{l-l'} e^{i(\Delta_\kappa-\Delta'_\kappa)}[L,L',l,l']^{1/2}\\
&\quad\times\begin{Bmatrix} L & L' & \nu\\ j' & j & j_b\end{Bmatrix}\lambda^{\pi}\lambda'^{\pi'}\langle L'\lambda' L-\lambda|\nu\mu\rangle\\
&\quad\times\langle E\kappa' j'||\boldsymbol{\alpha}\mathbf{A}^{\pi'}_{L'}||n_b j_b\rangle\langle E\kappa j||\boldsymbol{\alpha}\mathbf{A}^{\pi}_{L}||n_b j_b\rangle^{*} C^{\nu}_{\kappa\kappa'},
\end{aligned} \tag{8}$$

where the polarization properties of the incident electron now occur only in the coefficient

$$\begin{aligned}
C^{\nu}_{\kappa\kappa'}=\sum_{m_s} &(-1)^{-m_s}\langle \mathbf{p}m_s|\hat{\rho}_e|\mathbf{p}m_s\rangle\langle l01/2m_s|jm_s\rangle\\
&\times\langle l'01/2m_s|j'm_s\rangle\langle j'-m_s jm_s|\nu 0\rangle.
\end{aligned} \tag{9}$$

Therefore, making use of these last two expressions, the evaluation of the spin-density matrix can be traced back to just the computation of the reduced matrix elements $\langle E\kappa j||\boldsymbol{\alpha}\mathbf{A}^{\pi}_{L}||n_b j_b\rangle$ which describe the interaction of an electron with the radiation field for a (standard) free-bound transition. The computation of these matrix elements within the framework of Dirac theory was discussed elsewhere at several places in the past [5,13].

C. Symmetry properties of the Stokes parameters

The decomposition of the continuum wave functions in Eqs. (8) and (9) helps analyze the symmetry properties of the two Stokes parameters P_1 and P_2 and, hence, of the linear polarization of the emitted light. As seen from the expression (8), for instance, the helicity quantum numbers λ and λ', which characterize the different partial waves of the outgoing photon, appear only in the phase and in the single Clebsch-Gordan coefficient $\langle L'\lambda' L-\lambda|\nu\mu\rangle$. From the symmetry properties of the Clebsch-Gordan coefficients, it therefore follows immediately that the $\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda')$ angular coefficients must also obey the symmetry

$$\beta^{\nu\mu}_{L\pi L'\pi'}(-1,+1)=(-1)^f\beta^{\nu-\mu}_{L\pi L'\pi'}(+1,-1), \tag{10}$$

where the proper phase is given by $f=L+\pi+L'+\pi'-\nu$. The symmetry of the angular coefficients enables one, in turn, to express the two Stokes parameters P_1 and P_2 in a simpler form:

$$\begin{aligned}
P_1&=\frac{\langle \mathbf{k}+1|\hat{\rho}_\gamma|\mathbf{k}-1\rangle+\langle \mathbf{k}-1|\hat{\rho}_\gamma|\mathbf{k}+1\rangle}{\langle \mathbf{k}+1|\hat{\rho}_\gamma|\mathbf{k}+1\rangle+\langle \mathbf{k}-1|\hat{\rho}_\gamma|\mathbf{k}-1\rangle}\\
&=\frac{\sum_\nu D^{\nu}_{02}(0,\theta,0)\sum_{L\pi L'\pi'}\beta^{\nu2}_{L\pi L'\pi'}(-1,1)[1+(-1)^f]}{2\sum_\nu P_\nu(\cos\theta)\sum_{L\pi L'\pi'}\beta^{\nu0}_{L\pi L'\pi'}(+1,+1)}
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
P_2&=-i\frac{\langle \mathbf{k}-1|\hat{\rho}_\gamma|\mathbf{k}+1\rangle-\langle \mathbf{k}+1|\hat{\rho}_\gamma|\mathbf{k}-1\rangle}{\langle \mathbf{k}+1|\hat{\rho}_\gamma|\mathbf{k}+1\rangle+\langle \mathbf{k}-1|\hat{\rho}_\gamma|\mathbf{k}-1\rangle}\\
&=-i\frac{\sum_\nu D^{\nu}_{02}(0,\theta,0)\sum_{L\pi L'\pi'}\beta^{\nu2}_{L\pi L'\pi'}(-1,1)[1-(-1)^f]}{2\sum_\nu P_\nu(\cos\theta)\sum_{L\pi L'\pi'}\beta^{\nu0}_{L\pi L'\pi'}(+1,+1)},
\end{aligned} \tag{12}$$

which, however, still includes a summation over all the possible multipoles in the electron-photon interaction. Owing to parity conservation in the interaction of the electron with the radiation field, of course, not all of these multipoles will contribute in practice to the polarization properties of the photons, as is reflected above by the phase factor $(-1)^f \equiv (-1)^{L+\pi+L'+\pi'-\nu} = \pm 1$. Therefore, in order to understand the effects of parity conservation, we shall return to expression (8) for the angular parameters $\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda')$ and analyze it in some more detail.

In expression (8), of course, the parity selection rules apply to both of the reduced matrix elements and do require that the parities of the (electric and magnetic) multipole fields must be equal to (-1) times the product of the parities that are associated with the bound state and the (outgoing) partial wave, respectively,

$$(-1)^{L+\pi} = -\pi_{n_b j_b}\pi_{\kappa j} = (-1)^{l_b+l+1},$$

$$(-1)^{L'+\pi'} = -\pi_{n_b j_b}\pi_{\kappa' j'} = (-1)^{l_b+l'+1}, \tag{13}$$

which immediately leads to the relation

$$(-1)^{L+\pi+L'+\pi'-\nu} = (-1)^{l+l'-\nu}. \tag{14}$$

However, before we continue with the discussion of the Stokes parameters, let us first reconsider the coefficient (9), i.e., that part of the $\beta^{\nu\mu}_{L\pi L'\pi'}$ parameter which depends explicitly on the electron density matrix $\langle \mathbf{p} m_s | \hat{\rho}_e | \mathbf{p} m_s \rangle$ of the incident electrons. Since, in the relativistic theory, the projection of the electron spin has a sharp value only along the electron momentum, the quantization axis (z axis) is chosen parallel to $\mathbf{p}$. For spin-1/2 particles, moreover, a single parameter $-1 \leqslant \mathcal{P} \leqslant 1$ is of course sufficient to describe the polarization of the electrons and hence can be used to express the *electron* spin-density matrix

$$\langle \mathbf{p} m_s | \hat{\rho}_e | \mathbf{p} m_s \rangle = \frac{1}{2}(I + \mathcal{P}\sigma_z) = \frac{1}{2}\begin{pmatrix} 1+\mathcal{P} & 0 \\ 0 & 1-\mathcal{P} \end{pmatrix} \tag{15}$$

in terms of the unit matrix I and the Pauli matrix σ_z. In this parametrization of the initial spin-density matrix, obviously, a degree of polarization $\mathcal{P}=0$ corresponds to a beam of *unpolarized* electrons, while $\mathcal{P}=\pm 1$ refers to a *completely polarized* electron beam with spin projections $m_s = \pm 1/2$.

We are now prepared to study the influence of an initially polarized electron beam on the angular and Stokes parameters. By inserting expression (15) into Eq. (9), we first see that the coefficient $C^{\nu}_{\kappa\kappa'}$ can be decomposed into an ''unpolarized'' and a ''polarized'' component

$$C^{\nu}_{\kappa\kappa'} = C^{\nu}_{\kappa\kappa'}(\mathrm{unpol}) + \mathcal{P} C^{\nu}_{\kappa\kappa'}(\mathrm{unpol}), \tag{16}$$

which, due to the parity rules, behave quite differently under a (sign) change in the spin state of the electron (in either its initial or final state). Taking into account the properties of the Clebsch-Gordan coefficients in Eq. (9), we find that these two parts obey the symmetry relations

$$C^{\nu}_{\kappa\kappa'}(\mathrm{unpol}) = (-1)^{l+l'-\nu} C^{\nu}_{\kappa\kappa'}(\mathrm{unpol}),$$

$$C^{\nu}_{\kappa\kappa'}(\mathrm{pol}) = (-1)^{l+l'-\nu+1} C^{\nu}_{\kappa\kappa'}(\mathrm{pol}). \tag{17}$$

A similar decomposition as found for the $C^{\nu}_{\kappa\kappa'}$ coefficients applies of course also to the $\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda')$ angular parameters in Eq. (8):

$$\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda') = \beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda';\mathrm{unpol}) + \mathcal{P}\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda';\mathrm{pol}), \tag{18}$$

where, using Eqs. (14) and (17), the corresponding ''unpolarized'' and ''polarized'' parts satisfy the two symmetry relations

$$\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda';\mathrm{unpol}) = (-1)^{L+\pi+L'+\pi'-\nu}\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda';\mathrm{unpol}),$$

$$\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda';\mathrm{pol}) = (-1)^{L+\pi+L'+\pi'-\nu+1}\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda';\mathrm{pol}). \tag{19}$$

That is, while the unpolarized part of the $\beta^{\nu\mu}_{L\pi L'\pi'}$ parameter is always *zero* if the phase f is odd, the same is true for the polarized part for even f. Making use of this property of the angular parameter (18), we can now simplify the expressions (11) and (12) for the Stokes parameters to

$$P_1 = \frac{\sum_{\nu} D^{\nu}_{02}(0,\theta,0) \sum_{L\pi L'\pi'} \beta^{\nu 2}_{L\pi L'\pi'}(-1,1;\mathrm{unpol})}{\sum_{\nu} P_{\nu}(\cos\theta) \sum_{L\pi L'\pi'} \beta^{\nu 0}_{L\pi L'\pi'}(+1,+1)}, \tag{20}$$

$$P_2 = -i\mathcal{P}\frac{\sum_{\nu} D^{\nu}_{02}(0,\theta,0) \sum_{L\pi L'\pi'} \beta^{\nu 2}_{L\pi L'\pi'}(-1,1;\mathrm{pol})}{\sum_{\nu} P_{\nu}(\cos\theta) \sum_{L\pi L'\pi'} \beta^{\nu 0}_{L\pi L'\pi'}(+1,+1)}, \tag{21}$$

which shows us immediately that only the P_2 parameter depends on the polarization $\mathcal{P}$ of the incident electrons and that this parameter is simply proportional to $\mathcal{P}$. Therefore, the Stokes parameter P_2 vanishes identically if the electrons are initially unpolarized and hence can be used as a very valuable tool for studying the polarization of the incident electron (and/or ion) beam. A measurement of the Stokes parameter P_1, in contrast, will not be affected by the polarization of the incoming electrons and depends only on the nuclear charge Z, the projectile energy, and the geometry in the setup of the photon detectors [11,12].

D. Polarization ellipse of the photons

The two Stokes parameters P_1 and P_2 specify the linear polarization of the radiation completely, i.e., both the *degree* of the polarization as well as its *direction* in the plane perpendicular to the photon momentum $\mathbf{k}$. Instead of the Stokes parameters, however, we may represent the linear polarization of the emitted x rays also in terms of a polarization

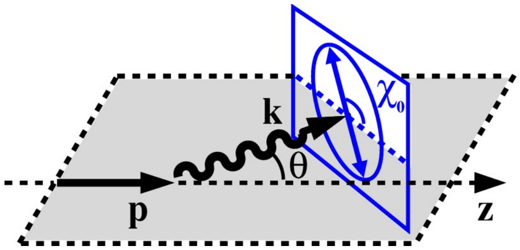

FIG. 2. Definition of the polarization ellipse; its principal axis is characterized by χ_0, the angle with respect to the reaction plane in the given measurement.

ellipse which is defined in this plane (perpendicular to **k**). In such a representation, the degree of linear polarization

$$P_L = \sqrt{P_1^2 + P_2^2} \tag{22}$$

is characterized by the relative length of the principal axis (of the ellipse), and the direction by its angle χ_0 with respect to the reaction plane. Figure 2 shows the concept of the polarization ellipse and how χ_0 is defined; when expressed in terms of the Stokes parameters, this angle is given by the two ratios [8]

$$\cos 2\chi_0 = \frac{P_1}{P_L}, \quad \sin 2\chi_0 = \frac{P_2}{P_L}, \tag{23}$$

and can be used to interpret the measurements. While, obviously, an angle $\chi_0 = 0$ or $\chi_0 = \pi/2$ corresponds to a linear polarization of the x rays within or perpendicular to the reaction plane (and with degree $P_L = |P_1|$), any contribution from a nonzero P_2 parameter will rotate the polarization vector (i.e., $\chi_0 \neq 0$ and $\chi_0 \neq \pi/2$). Recalling, moreover, the linear dependence of $P_2 \sim \mathcal{P}$ on the polarization of the incident electrons, we can therefore conclude that any polarization vector that is not in the reaction plane or perpendicular to it reflects a polarization of the (incident) electrons.

For the case of a polarized electron target (and for unpolarized ions), we can express the angle χ_0 of the polarization ellipse also directly in terms of the polarization $\mathcal{P}$ and the $\beta^{\nu\mu}_{L\pi L'\pi'}(\lambda,\lambda')$ angular parameters:

$$\cos 2\chi_0 = \frac{\mathrm{sign}(P_1)}{\sqrt{1+\mathcal{P}^2\mathcal{R}^2}}, \tag{24}$$

$$\sin 2\chi_0 = \frac{\mathrm{sign}(P_2)|\mathcal{P}|\mathcal{R}}{\sqrt{1+\mathcal{P}^2\mathcal{R}^2}}, \tag{25}$$

where

$$\mathcal{R} = \left| \frac{i\sum_{\nu} D^{\nu}_{02}(0,\theta,0) \sum_{L\pi L'\pi'} \beta^{\nu 2}_{L\pi L'\pi'}(-1,1;\mathrm{pol})}{\sum_{\nu} D^{\nu}_{02}(0,\theta,0) \sum_{L\pi L'\pi'} \beta^{\nu 2}_{L\pi L'\pi'}(-1,1;\mathrm{unpol})} \right| \tag{26}$$

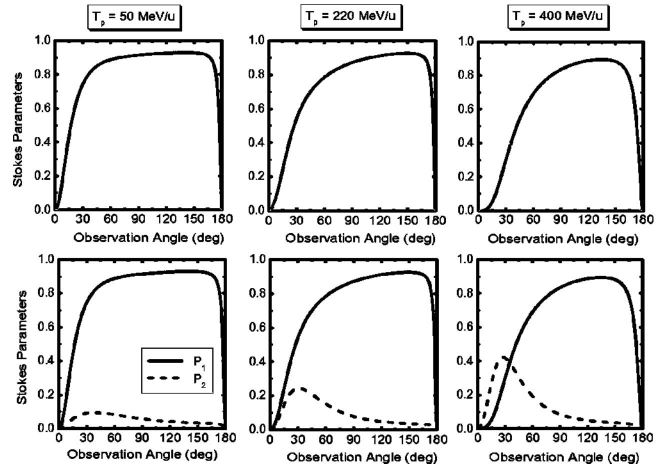

FIG. 3. The Stokes parameters P_1 and P_2 of the x-ray photons that are emitted in electron capture into the K shell of bare uranium ions. The Stokes parameters are shown for the capture of unpolarized (top panels) and completely polarized (bottom panels) electrons. Calculations are presented in the laboratory frame (i.e., the rest frame of the electron target).

denotes some ratio of the "polarized" and "unpolarized" components of the $\beta^{\nu\mu}_{L\pi L'\pi'}$ parameters. In experiments with highly charged ions, it is this representation of the angle χ_0 which, along with theoretical data, may help determine immediately the degree of polarization of the incident electrons without any need to measure the linear polarization in detail.

III. RESULTS AND DISCUSSION

Measurements on the linear polarization of x-ray radiation following the capture of electrons into highly charged ions are no longer impractical today. For the K-shell recombination of bare uranium ions U^{92+}, for example, experiments on the polarization of the photons were carried out at the GSI storage ring in Darmstadt during the last year. These studies on the x-ray polarization became possible owing to the use of position sensitive germanium detectors. These detectors enable one to obtain information not only on the degree of x-ray polarization but also concerning its direction within the detector plane. They may be used therefore for studying the polarization of electron (or atom) targets or even the polarization properties of ion beams at storage rings in the future.

In the following, we analyze the linear polarization of the photons that are emitted in the radiative recombination of bare uranium ions with energies in the range $50 \leqslant T_p \leqslant 400$ MeV/u. Detailed calculations have been carried out, in particular, for electron capture into the K shell of U^{92+} projectiles. To explore the effects of a polarized electron target on the (linear) polarization of the recombination photons, two cases are considered: the capture of (i) unpolarized and (ii) completely polarized electrons. For these two cases, Fig. 3 displays the Stokes parameters as a function of the observation angle θ of the recombination photons. In the upper panels of this figure, the P_1 parameter for the capture of unpolarized electrons ($\mathcal{P}=0$) is shown; it is positive and quite large for most angles apart from the forward and back-

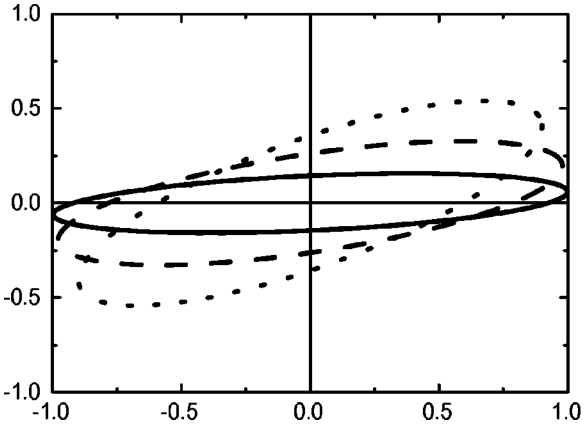

FIG. 4. Rotation of the polarization ellipses of the recombination photons, calculated for the three projectile energies $T_p = 50$ MeV/u (—), 220 MeV/u (– –), and 400 MeV/u (- - -) at the photon emission angle $\theta = 30°$.

ward directions of emission. As seen from Eq. (21), the Stokes parameter P_2 must vanish identically in the case of unpolarized electrons; since, moreover, $P_1 > 0$ for projectile energies $T_p \leqslant 400$ MeV/u, the principal axis of the polarization ellipse always lies within the reaction plane, $\chi_0 = 0$, for all angles of observation of the recombination photons and for unpolarized electrons.

A rather different situation arises in the second case (ii) for the capture of completely polarized electrons ($\mathcal{P} = 1$) as shown in the lower panels of Fig. 3. Here, a nonvanishing Stokes parameter P_2 appears, which peaks at around $\theta = 30°$ and becomes larger for increasing projectile energies, while the P_1 parameter remains unaffected by the polarization of the electron target. As mentioned above, a nonzero value of P_2 also leads to a rotation of the polarization ellipse out of the reaction plane. This rotation is seen in Fig. 4, which displays the polarization ellipses of the recombination photons at the observation angle $\theta = 30°$, calculated for the three projectile energies $T_p = 50$, 220 and 400 MeV/u. According to the increase of the Stokes parameter P_2 at this angle, the (rotation) angle χ_0 of the polarization ellipse increases from 3.5° for $T_p = 50$ MeV/u to almost 30° for $T_p = 400$ MeV/u. As seen from Figs. 3 and 4, therefore, the effects of the target polarization become apparently more pronounced if the projectile energy is increased.

So far, we have analyzed the linear polarization of the recombination photons for the two limiting cases of either unpolarized or completely polarized electrons. As discussed above, these two cases can be easily distinguished by the polarization ellipse whose principal axis must always lie within or perpendicular to the reaction plane for the capture of unpolarized electrons. As seen from Eqs. (24)–(26), however, observation of the rotation angle χ_0 may provide information on both the degree as well as the direction of the electron polarization $\mathcal{P}(-1 \leqslant \mathcal{P} \leqslant +1)$ and hence can be used for studying the spin polarization of the electrons and atomic targets, respectively. Figure 5 displays the rotation angle χ_0 of the polarization ellipse for various degrees of the electron polarization $\mathcal{P}$, following the capture of electrons into the K shell of bare uranium ions at a projectile energy

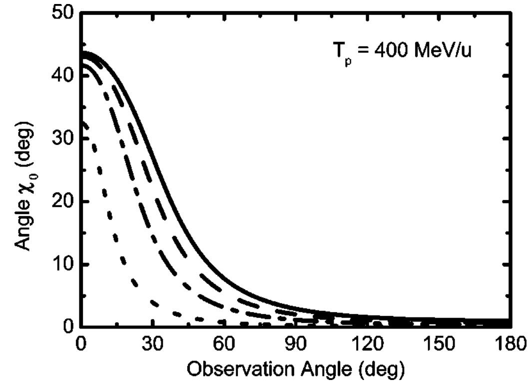

FIG. 5. Rotation angle χ_0 of the polarization ellipse in dependence on the observation angle of the recombination photons. The angle χ_0 is calculated for the capture of longitudinally polarized electrons into the K shell of a bare uranium projectile U^{92+} and is shown for four different degrees of the electron polarization: $\mathcal{P} = 1.0$ (—), $\mathcal{P} = 0.7$ (– –), $\mathcal{P} = 0.4$ (– - –), and $\mathcal{P} = 0.1$ (- - -).

$T_p = 400$ MeV/u. In this figure, the rotation angle χ_0 is shown as a function of the observation angle θ of the recombination photons (in the laboratory frame, i.e., the rest frame of the electron target); for a given energy of the projectiles, apparently, the maximal rotation of the polarization ellipse arises in the forward direction for the emission of recombination photons. Note, however, that χ_0 is not defined at the emission angles $\theta = 0$ and $\theta = 180°$, because photon emission in either the forward or backward direction does not break the *axial* symmetry for the collision system. At these two angles, therefore, the linear polarization of the light must always be zero (see Fig. 3). For the same reason also, all polarization measurements at angles near $\theta = 0$ will become difficult as the degree of linear polarization $P_L = \sqrt{P_1^2 + P_2^2} \leqslant 0.1$ in this range. For larger emission angles, however, the (degree of) linear polarization increases and may become as large as $P_L \approx 0.5$ for emission angles around $\theta = 30°$. At these angles, the effect from the polarization of the incident electrons is still quite sizable and leads, for $\theta = 30°$ and $T_p = 400$ MeV/u, to a decrease of the rotation angle χ_0 from 27.4° for the capture of completely polarized electrons to 4.0° if the polarization of the incident electrons is $\mathcal{P} = 0.1$.

IV. SUMMARY AND OUTLOOK

Density matrix theory has been applied for studying the polarization of the emitted photons following the radiative recombination of bare, high-Z ions. In our theoretical analysis, emphasis was placed particularly on the two questions of (i) how the polarization of the incident electrons affects the linear polarization of the recombination photons and (ii) how this polarization of the electrons (or of any atomic target) can be observed by experiment. As seen from these investigations, the linear polarization of the recombination photons may serve as a valuable tool for "measuring" the polarization properties of the electrons: While the capture of unpolarized electrons always leads to x-ray photons that are polarized within or perpendicular to the reaction plane, a rotation of the polarization ellipse occurs for polarized electrons. Calculations of this (linear) effect have been carried out especially for the capture of longitudinally polarized electrons into the K shell of bare uranium projectiles U^{92+}.

For the sake of simplicity, here we considered the case of polarized electrons, while the ion beam has been assumed unpolarized throughout the analysis. Owing to the symmetry of the collision system ''ion plus electron,'' however, similar effects on the polarization of the recombination photons as found for a polarized electron target can also be expected if the ion beam is polarized. Of course, for a nuclear spin $I > 1/2$, an enlarged parametrization of the ion density matrix will be required [cf. Eq. (15)]. Investigations along these lines are currently under way and will provide, together with proper measurements of the photon polarization, a method of determining the polarization of (heavy-)ion beams—up to the present a rather unresolved problem in the physics at storage rings.

ACKNOWLEDGMENTS

We gratefully acknowledge Dr. Sepp for helpful discussions. This work has been supported by the GSI Project No. KS-FRT.

[1] Th. Stöhlker, Phys. Scr. **T80**, 165 (1999).
[2] Th. Stöhlker, C. Kozhuharov, P. Mokler, A. Warczak, F. Bosch, H. Geissel, R. Moshammer, C. Scheidenberger, J. Eichler, A. Ichihara, T. Shirai, Z. Stachura, and P. Rymuza, Phys. Rev. A **51**, 2098 (1995).
[3] R. Pratt, A. Ron, and H. Tseng, Rev. Mod. Phys. **45**, 273 (1973).
[4] J. Eichler and W.E. Meyerhof, *Relativistic Atomic Collisions* (Academic, San Diego, 1995).
[5] J. Eichler, A. Ichihara, and T. Shirai, Phys. Rev. A **58**, 2128 (1998).
[6] R. Pratt, R. Levee, R. Pexton, and W. Aron, Phys. Rev. **134**, A916 (1964).
[7] J. Scofield, Phys. Rev. A **40**, 3054 (1989).
[8] V.V. Balashov, A.N. Grum-Grzhimailo, and N.M. Kabachnik, *Polarization and Correlation Phenomena in Atomic Collisions* (Kluwer Academic/Plenum, New York, 2000).
[9] K. Blum, *Density Matrix Theory and Applications* (Plenum, New York, 1981).
[10] U. Fano and G. Racah, *Irreducible Tensorial Sets* (Academic, New York, 1959).
[11] A. Surzhykov, S. Fritzsche, and Th. Stöhlker, Phys. Lett. A **289**, 213 (2001).
[12] J. Eichler and A. Ichihara, Phys. Rev. A **65**, 052716 (2002).
[13] A. Surzhykov, S. Fritzsche, and Th. Stöhlker, J. Phys. B **35**, 3713 (2002).