Teleportation of coherent-state superpositions within a network

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(Received 3 February 2003; published 28 August 2003)

We propose a protocol for teleporting an unknown coherent-state superposition within a network consisting of 2^N parties with N an arbitrary positive integer. We show explicitly that for moderate and high intensity fields the probability of success is 50%, i.e. the same as in the case of $N=1$.

DOI: 10.1103/PhysRevA.68.022321 PACS number(s): 03.67.Hk, 03.65.Ud, 03.65.Ta

The prototype scenario of quantum teleportation by Bennett *et al*. [1] is based on a dual usage of both classical and quantum channels to transfer an unknown quantum state between two remote parties without sending the state itself. Inspired by that thrilling idea, a subsequent number of protocols have been proposed and some of them have been experimentally realized to teleport pure photonic [2] as well as atomic [3] states, entangled states [4], and states with continuous variables [5]. Coherent-state superpositions can also be teleported employing entangled coherent states [6] as a quantum channel. Superpositions of coherent states not only play a fundamental role in understanding the transfer of the ‘‘indeterminacy'' of a microscopic system to the ‘‘uncertainty'' of a macroscopic one, but they can also be used as logical qubit encoding for the correction of spontaneous emission errors [7]. For example, an exotic form of qubit can be made of two kinds of coherent-state superpositions: one is called even and the other is called odd coherent state. The advantage of utilizing such coherent-state superpositions rests in their distinguishability by parity providing an easy way via a proper circuit to detect and correct bit-flip errors if any [8].

Because of the reasons mentioned above teleporting coherent-state superpositions proves necessary in its own right. Recently, teleportation of states such as

$$|\Psi\rangle \propto a|\alpha\rangle + b|-\alpha\rangle \tag{1}$$

or

$$|\Phi\rangle \propto a|\alpha\rangle_1|\alpha\rangle_2 + b|-\alpha\rangle_1|-\alpha\rangle_2, \ldots, \tag{2}$$

with $|\alpha\rangle$ a coherent state and a, b unknown complex coefficients, have been investigated by van Enk and Hirota [9] and Wang [10], respectively. However, both the publications have confined to teleporting between two parties only. As a practical fact, real (quantum) information needs to be communicated among as many parties as possible. This is a network problem which has been studied for clonning machines [11] as well as for teleporting schemes [12,13]. Note that in Ref. [12] a discrete two-state system was treated among three parties, whereas in Ref. [13] a general network was considered for teleporting quadratures of a continuous field. In this work we study teleportation of states of form (1) within a network consisting of more than two parties. The network we are interested in is a symmetric network, any party of which has an equal right. That is, teleportation should be equally possible between any pair of parties within the network. The question is that how many parties may be involved in the game? The answer depends on kinds of quantum channel entangling all the participants. Because of the structure of the state to be teleported, state (1), and the network symmetry requirement, one could think of using a multipartite entangled coherent state of the form [14]

$$|\Psi\rangle_{12\cdots M} = A_{12\cdots M}(|\alpha\rangle_1|\alpha\rangle_2\cdots|\alpha\rangle_M - |-\alpha\rangle_1|-\alpha\rangle_2\cdots|-\alpha\rangle_M), \tag{3}$$

with $A_{12\cdots M}$ the normalization coefficient, as a quantum channel to perform the teleportation within a network of any number M of parties. Our purpose is twofold: first produce appropriate multipartite entangled states for the quantum channel and then use them to teleport the state of interest within the network.

Let us now describe a technique to produce state (3) for $M=2^N$ with $N=1,2,3,\ldots$. Denote the phase shifter by $\hat{P}_j(\theta)=\exp(-i\theta a_j^\dagger a_j)$ and the beam splitter by $\hat{B}_{i,j}(\theta)=\exp[i\theta(a_i^\dagger a_j + a_j^\dagger a_i)]$ where $a_j^\dagger$ (a_j) is the bosonic creation (annihilation) operator for the state of mode j. It is not difficult to check that the action of the modified beam splitter $\hat{\mathcal{B}}_{i,j}\equiv\hat{P}_j(\pi/2)\hat{B}_{i,j}(\pi/4)\hat{P}_j(\pi/2)$ reads

$$\hat{\mathcal{B}}_{i,j}|\alpha\rangle_i|\beta\rangle_j = \left|\frac{\alpha+\beta}{\sqrt{2}}\right\rangle_i \left|\frac{\alpha-\beta}{\sqrt{2}}\right\rangle_j. \tag{4}$$

By means of transformation (4), a 2^N-mode state $|\alpha\rangle_1|\alpha\rangle_2\cdots|\alpha\rangle_{2^N}\equiv\Pi_{p=1}^{2^N}|\alpha\rangle_p$ can be generated by 2^{N-1} modified beam splitters from the 2^{N-1}-mode state $|\sqrt{2}\alpha\rangle_1|\sqrt{2}\alpha\rangle_2\cdots|\sqrt{2}\alpha\rangle_{2^{N-1}}\equiv\Pi_{l=1}^{2^{N-1}}|\sqrt{2}\alpha\rangle_l$ and 2^{N-1} vacua in the following way:

$$\prod_{p=1}^{2^N}|\alpha\rangle_p = \prod_{q=1}^{2^{N-1}}\hat{\mathcal{B}}_{q,q+2^{N-1}}\prod_{l=1}^{2^{N-1}}|\sqrt{2}\alpha\rangle_l\prod_{m=1}^{2^{N-1}}|0\rangle_{m+2^{N-1}}. \tag{5}$$

Applying Eq. (5) to its right-hand side and continuing this process until we obtain the formula

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$$\prod_{p=1}^{2^N}|\alpha\rangle_p=\prod_{Q=N}^{1}\left(\prod_{q=1}^{2^{Q-1}}\hat{\mathcal{B}}_{q,q+2^{Q-1}}\right)|2^{N/2}\alpha\rangle_1\prod_{m=2}^{2^N}|0\rangle_m. \tag{6}$$

It is clear from Eq. (6) that state (3) with $M=2^N$ can be produced by applying a suitable sequence of modified beam splitters to the direct product of the state $A_{12\cdots 2^N}(|2^{N/2}\alpha\rangle_1-|-2^{N/2}\alpha\rangle_1)$ and the vacua $|0\rangle_2|0\rangle_3\cdots|0\rangle_{2^N}$. Namely,

$$|\Psi\rangle_{12\cdots 2^N}=A_{12\cdots 2^N}\prod_{Q=N}^{1}\left(\prod_{q=1}^{2^{Q-1}}\hat{\mathcal{B}}_{q,q+2^{Q-1}}\right)(|2^{N/2}\alpha\rangle_1-|-2^{N/2}\alpha\rangle_1)\prod_{m=2}^{2^N}|0\rangle_m$$
$$=A_{12\cdots 2^N}(|\alpha\rangle_1|\alpha\rangle_2\cdots|\alpha\rangle_{2^N}-|-\alpha\rangle_1|-\alpha\rangle_2\cdots|-\alpha\rangle_{2^N}), \tag{7}$$

with the normalization coefficient given by

$$A_{12\cdots 2^N}=[2\{1-\exp(-2^{N+1}|\alpha|^2)\}]^{-1/2}. \tag{8}$$

Since coherent states are nonorthogonal to each other, state (7) is not maximally entangled in general. The concurrence [15] between one system, say, system 1, and the remaining systems, say, systems 2, 3, ..., 2^N, can be calculated as follows. The 2^N-mode system is separated into two parts: part X is system 1 and part Y includes all the remaining systems 2, 3, ..., 2^N. Each part is treated as linearly independent with respect to α and $-\alpha$ spanning a two-dimensional subspace of the Hilbert space. According to the Gram-Schmidt theorem, we can always build in each subspace an orthonormal basis $\{|0\rangle_i,|1\rangle_i\}$, $i=X,Y$, which is determined by

$$|0\rangle_X=|\alpha\rangle_1, \tag{9}$$

$$|1\rangle_X=\frac{|-\alpha\rangle_1-{}_1\langle\alpha|-\alpha\rangle_1|\alpha\rangle_1}{\sqrt{1-({}_1\langle\alpha|-\alpha\rangle_1)^2}}, \tag{10}$$

$$|0\rangle_Y=|\alpha\rangle_2|\alpha\rangle_3\cdots|\alpha\rangle_{2^N}, \tag{11}$$

$$|1\rangle_Y=\frac{|-\alpha\rangle_2\cdots|-\alpha\rangle_{2^N}-{}_{2^N}\langle\alpha|\cdots{}_2\langle\alpha|-\alpha\rangle_2\cdots|-\alpha\rangle_{2^N}|\alpha\rangle_2\cdots|\alpha\rangle_{2^N}}{\sqrt{1-({}_{2^N}\langle\alpha|\cdots{}_2\langle\alpha|-\alpha\rangle_2\cdots|-\alpha\rangle_{2^N})^2}}. \tag{12}$$

In terms of the orthonormal bases $\{|0\rangle_{X,Y},|1\rangle_{X,Y}\}$ defined by Eqs. (9)–(12), our state (7) is expressed as

$$|\Psi\rangle_{12\cdots 2^N}=a_{00}|0\rangle_X|0\rangle_Y+a_{01}|0\rangle_X|1\rangle_Y+a_{10}|1\rangle_X|0\rangle_Y+a_{11}|1\rangle_X|1\rangle_Y, \tag{13}$$

where

$$a_{00}=A_{12\cdots 2^N}(1-Z^{2^N}), \tag{14}$$

$$a_{01}=-A_{12\cdots 2^N}Z\sqrt{(1-Z^{2(2^N-1)})}, \tag{15}$$

$$a_{10}=-A_{12\cdots 2^N}Z^{2^N-1}\sqrt{(1-Z^2)}, \tag{16}$$

$$a_{11}=-A_{12\cdots 2^N}\sqrt{(1-Z^2)(1-Z^{2(2^N-1)})}, \tag{17}$$

with

$$Z={}_j\langle\alpha|-\alpha\rangle_j=\exp(-2|\alpha|^2),\quad \forall j. \tag{18}$$

The concurrence $C_{1(23\cdots 2^N)}$ is then given by

$$C_{1(23\cdots 2^N)}=2|a_{00}a_{11}-a_{01}a_{10}|=\frac{\sqrt{[1-\exp(-4|\alpha|^2)]\{1-\exp[-4(2^N-1)|\alpha|^2]\}}}{1-\exp(-2^{N+1}|\alpha|^2)}, \tag{19}$$

which varies from $\sqrt{2^N-1}/2^{N-1}$ in the limit $|\alpha|\to 0$ to 1 in the limit $|\alpha|\to\infty$ (see Fig. 1). The limiting value $C_{1(23\cdots 2^N)}=\sqrt{2^N-1}/2^{N-1}$ at $|\alpha|=0$ agrees with the equality [16]

$$C_{12}^2+C_{13}^2+\cdots+C_{12^N}^2=C_{1(23\cdots 2^N)}^2, \tag{20}$$

as can be expected, since when $|\alpha|\to 0$ the state $|\Psi\rangle_{12\cdots 2^N}$ reduces to the W_{2^N} state [17] for which $C_{12}=C_{13}=\cdots=C_{12^N}=1/2^{N-1}$. However, the nonmaximal entanglement of state $|\Psi\rangle_{12\cdots 2^N}$ does not prevent having a perfect teleportation with a finite probability as we shall show in what follows.

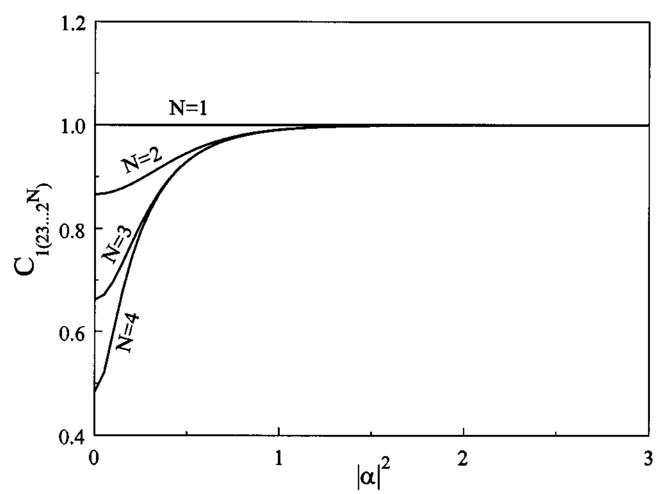

FIG. 1. The concurrence $C_{1(23\cdots 2^N)}$ between system 1 and systems 2, 3, ..., 2^N in the 2^N-partite entangled coherent state $|\Psi\rangle_{12\cdots 2^N}$, Eq. (7), as a function of $|\alpha|^2$ for $N=1$, 2, 3, and 4.

First we deal with $N=2$, i.e. a network consisting of four spacelike separate parties: Alice, Bob, Clair, and David. The four parties will share a 4-partite entangled state serving as a quantum channel of the form

$$|\Psi\rangle_{1234}=A_{1234}(|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4 -|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4), \tag{21}$$

with the normalization coefficient given by

$$A_{1234}=[2\{1-\exp(-8|\alpha|^2)\}]^{-1/2}. \tag{22}$$

Let us rewrite the state to be teleported in a more precise form as

$$|\Psi\rangle_0=A_0(a|\alpha\rangle_0+b|-\alpha\rangle_0), \tag{23}$$

with the normalization coefficient given by

$$A_0=[|a|^2+|b|^2+2\,\mathrm{Re}(a^*b)\exp(-2|\alpha|^2)]^{-1/2}. \tag{24}$$

Without loss of generality, we suppose that Alice possesses the state $|\Psi\rangle_0$ and her task is to teleport it to David. For that purpose, system 1 is sent to Alice, system 2 to Bob, system 3 to Clair, and system 4 to David. The entire system of the state to be teleported, state (23), and the 4-partite entangled coherent state of the quantum channel, state (21), is their direct product

$$\begin{aligned}|\Phi\rangle_{01234}&=|\Psi\rangle_0|\Psi\rangle_{1234}\\ &=A_0A_{1234}(a|\alpha\rangle_0|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4\\ &\quad-a|\alpha\rangle_0|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4\\ &\quad+b|-\alpha\rangle_0|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4\\ &\quad-b|-\alpha\rangle_0|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4).\end{aligned} \tag{25}$$

Under the action of $\hat{\mathcal{B}}_{0,1}$ on the two modes at Alice's station the initial state (25) is transformed into a new state $|\Theta\rangle_{01234}$ of the form

$$\begin{aligned}|\Theta\rangle_{01234}=A_0A_{1234}(&a|\sqrt{2}\alpha\rangle_0|0\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4\\ &-a|0\rangle_0|\sqrt{2}\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4\\ &+b|0\rangle_0|-\sqrt{2}\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4\\ &-b|-\sqrt{2}\alpha\rangle_0|0\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4).\end{aligned} \tag{26}$$

Then Alice needs detecting the photon numbers of mode 0 and mode 1 by two detectors D_0 and D_1 at her station, while Bob and Clair should carry out the local number measurement of mode 2 and mode 3 by their detectors D_2 and D_3 (see Fig. 2). Let the measurement outcomes of Alice be n_0 and n_1, whereas n_2 (n_3) photons are counted by Bob's (Clair's) detector. It can be realized from Eq. (26) that there are only two possibilities: case 1 with $n_0=0$, $n_1>0$, and case 2 with $n_0>0$, $n_1=0$. If case 1 happens then the state at David's station collapses into

$$|\Psi'\rangle_4=A_0[(-1)^{n_2+n_3}a|-\alpha\rangle_4-(-1)^{n_1}b|\alpha\rangle_4]. \tag{27}$$

Clearly, if Alice, Bob, and Clair send to David their measurement outcomes via a public (classical) channel, and $n_1+n_2+n_3$ is odd, then after obtaining the classical information David will apply the operator $\hat{P}_4(\pi)$ to Eq. (27) to get the state $|\Psi\rangle_4=\hat{P}_4(\pi)|\Psi'\rangle_4$ which coincides with $|\Psi\rangle_0$ (up to a global unimportant phase constant sometimes). The probability of success in this situation is given by

$$\Pi'_4=|{}_1\langle n_1|{}_2\langle n_2|{}_3\langle n_3|\Theta\rangle_{01234}|^2=A_{1234}^2\sum_{n_1=1;n_2,n_3=0;n_1+n_2+n_3:odd}^{\infty}|\mathcal{N}_{n_1}(\sqrt{2}\alpha)\mathcal{N}_{n_2}(\alpha)\mathcal{N}_{n_3}(\alpha)|^2, \tag{28}$$

where

$$\mathcal{N}_n(\beta)={}_j\langle n|\beta\rangle_j=\frac{\beta^n}{\sqrt{n!}}\exp\left(\frac{-|\beta|^2}{2}\right), \tag{29}$$

independent of mode j.

Alternatively, if case 2 occurs then the state at David's station collapses into

$$|\Psi''\rangle_4=A_0[a|\alpha\rangle_4-(-1)^{n_0+n_2+n_3}b|-\alpha\rangle_4]. \tag{30}$$

Transparently, if $n_0+n_2+n_3$ is odd, then nothing should be done by David because in this situation state (30) is an exact replica of $|\Psi\rangle_0$. It is easy to verify that the success probability $\Pi''_4=|{}_0\langle n_0|{}_2\langle n_2|{}_3\langle n_3|\Theta\rangle_{01234}|^2$ in case 2 is precisely equal to that in case 1, i.e., $\Pi''_4=\Pi'_4$. Therefore, the total probability of successful teleportation is explicitly given by

$$\Pi_4=2\Pi'_4=\frac{1}{2}[1-\mathrm{csch}(4|\alpha|^2)\sinh(2|\alpha|^2)], \tag{31}$$

which tends to 1/4 in the limit $|\alpha|\to0$ and to 1/2 in the limit $|\alpha|\to\infty$, as illustrated in Fig. 3.

It is straightforward to generalize the above obtained result to the case of an arbitrary N. The quantum channel is now served by the 2^N-partite entangled coherent state (7). Without loss of generality, we suppose that party 1 possesses the state $|\Psi\rangle_0$ and the task is to teleport $|\Psi\rangle_0$ to party 2^N. For that purpose, system 1 is sent to party 1, system 2 to party 2, etc., and system 2^N sent to party 2^N. The entire

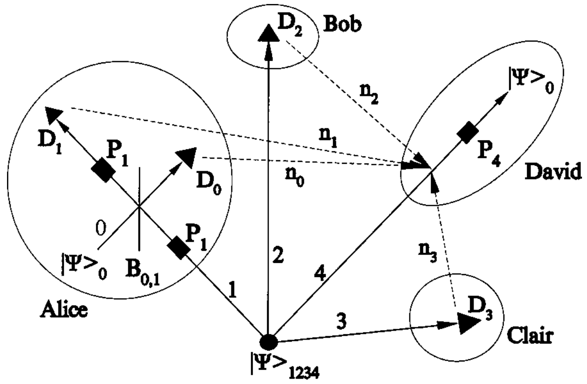

FIG. 2. The scheme for teleporting the superposition state $|\Psi\rangle_0$, Eq. (23), from Alice to David within a network of four parties. P_j, phase shifters; $B_{0,1}$, nonabsorbing beam splitter; D_j, detectors counting photon numbers; n_j, the measurement outcomes; and the dashed lines, classical communication channels.

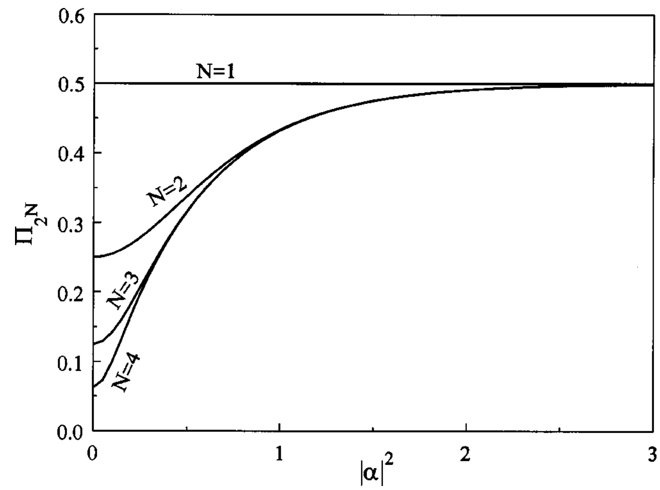

FIG. 3. The probability of successful teleportation Π_{2^N}, Eq. (37), as a function of $|\alpha|^2$ for $N=1$, 2, 3, and 4.

system of the state to be teleported, state (23), and the 2^N-partite entangled coherent state of the quantum channel, state (7), is

$$\begin{aligned}|\Phi\rangle_{012\cdots2^N}=&|\Psi\rangle_0|\Psi\rangle_{12\cdots2^N}\\ =&A_0A_{12\cdots2^N}(a|\alpha\rangle_0|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3\cdots|\alpha\rangle_{2^N}\\ &-a|\alpha\rangle_0|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3\cdots|-\alpha\rangle_{2^N}\\ &+b|-\alpha\rangle_0|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3\cdots|\alpha\rangle_{2^N}-b|-\alpha\rangle_0\\ &\times|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3\cdots|-\alpha\rangle_{2^N}). \end{aligned}\tag{32}$$

At party 1 station a sequence of local operators $\hat{P}_1(\pi/2)\hat{B}_{0,1}(\pi/4)\hat{P}_1(\pi/2)$ is performed on system 0 and system 1 transforming state (32) into state $|\Theta\rangle_{012\cdots2^N}$ of the form

$$\begin{aligned}|\Theta\rangle_{012\cdots2^N}=&A_0A_{12\cdots2^N}(a|\sqrt{2}\alpha\rangle_0|0\rangle_1|\alpha\rangle_2|\alpha\rangle_3\cdots|\alpha\rangle_{2^N}\\ &-a|0\rangle_0|\sqrt{2}\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3\cdots|-\alpha\rangle_{2^N}\\ &+b|0\rangle_0|-\sqrt{2}\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3\cdots|\alpha\rangle_{2^N}-b|\\ &-\sqrt{2}\alpha\rangle_0|0\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3\cdots|-\alpha\rangle_{2^N}). \end{aligned}\tag{33}$$

To fulfill the task mentioned above party 1 needs counting the photon numbers of mode 0 and mode 1 by two local detectors D_0 and D_1, while parties 2, 3, ..., 2^N-1 should, respectively, carry out the local number measurement of mode 2, mode 3, ..., mode 2^N-1 by local detectors D_2, D_3, ..., D_{2^N-1}. Let the measurement outcomes at party 1 station be n_0 and n_1, whereas n_2, n_3, ..., n_{2^N-1} photons are detected at the stations of party 2, party 3, ..., party 2^N-1, respectively. Again, the structure of state $|\Theta\rangle_{012\cdots2^N}$, Eq. (33), allows only two possibilities as in the case of $N=2$. Let us recall that the two cases are case 1 with $n_0=0$, $n_1>0$, and case 2 with $n_0>0$, $n_1=0$. If case 1 happens then the state at party 2^N station collapses into

$$\begin{aligned}|\Psi'\rangle_{2^N}=&A_0[(-1)^{n_2+n_3+\cdots+n_{2^N-1}}a|-\alpha\rangle_{2^N}\\ &-(-1)^{n_1}b|\alpha\rangle_{2^N}]. \end{aligned}\tag{34}$$

Clearly, if all the parties who performed the number measurement send their outcomes to party 2^N via a public (classical) channel and the outcomes are such that $n_1+n_2+\cdots+n_{2^N-1}$ is odd, then after obtaining the classical information party 2^N will apply the operator $\hat{P}_{2^N}(\pi)$ to Eq. (34) to get the state $|\Psi\rangle_{2^N}=\hat{P}_{2^N}(\pi)|\Psi'\rangle_{2^N}$ which is nothing else but the desired state $|\Psi\rangle_0$ (up to a global unimportant phase constant sometimes). The probability of success in this situation is equal to

$$\begin{aligned}\Pi'_{2^N}=&|{}_1\langle n_1|{}_2\langle n_2|\ldots{}_{2^N-1}\langle n_{2^N-1}|\Theta\rangle_{012\cdots2^N}|^2\\ =&A^2_{12\cdots2^N}\sum_{n_1=1;n_2,n_3,\ldots,n_{2^N-1}=0;n_1+\cdots+n_{2^N-1}:odd}^{\infty}|\mathcal{N}_{n_1}(\sqrt{2}\alpha)\mathcal{N}_{n_2}(\alpha)\cdots\mathcal{N}_{n_{2^N-1}}(\alpha)|^2. \end{aligned}\tag{35}$$

Alternatively, if case 2 occurs then the state at party 2^N station collapses into

$$|\Psi''\rangle_{2^N}=A_0[a|\alpha\rangle_{2^N}-(-1)^{n_0+n_2+n_3+\cdots+n_{2^N-1}}b|-\alpha\rangle_{2^N}]. \tag{36}$$

Transparently, if $n_0+n_2+n_3+\cdots+n_{2^N-1}$ is odd, then, by doing nothing, party 2^N gets an exact replica of the desired state $|\Psi\rangle_0$. In this case 2 the success probability $\Pi''_{2^N} = |_0\langle n_0|_2\langle n_2|\cdots{}_{2^N-1}\langle n_{2^N-1}|\Theta\rangle_{012\ldots 2^N}|^2$ is also precisely equal to that in case 1, i.e., $\Pi''_{2^N}=\Pi'_{2^N}$. Hence, the total probability of successful teleportation can be calculated by the formula

$$\Pi_{2^N}=2\Pi'_{2^N}=\frac{1}{2}\{1-\mathrm{csch}(2^N|\alpha|^2)\sinh[2(2^{N-1}-1)|\alpha|^2]\}. \tag{37}$$

Formula (37) holds true also for $N=1$ for which $\Pi_2=1/2$, independent of α, recovering the result reported in Ref. [9]. For any $N>1$, the probability of perfect teleportation tends to $1/2^N$ in the limit $|\alpha|\to 0$ and to 1/2 in the limit $|\alpha|\to\infty$ (see Fig. 3). A property worth emphasizing is that in fact Π_{2^N} for any $N>1$ saturates to 1/2 already starting from $|\alpha|^2 = 3$. So, for moderate and high intensity fields (say, $|\alpha|^2 \geqslant 3$) the success of teleportation can be considered α independent and is equal to 1/2 as in the case of $N=1$. In this sense, for fields containing at least three photons on average, i.e., for $|\alpha|^2\geqslant 3$, the teleportation can be regarded as N independent as well.

As is evident from above, the success of the proposed teleportation is determined by parity of the counted photon numbers rather than by the numbers themselves. The detectors thus must be highly parity sensitive. For high intensity fields distinguishing the number parity may be confused. To get rid of such confusion an additional technique, if necessary, can be applied in the following way. The Fock state $|n_j\rangle_j$ appearing after the measurement of mode j by detector D_j is coupled to a two-level atom via a dispersive interaction [18] governed by the interaction Hamiltonian $H_{int}=fa_j^\dagger a_j\sigma_x$ where f is the coupling strength and $\sigma_x = |g\rangle\langle e|+|e\rangle\langle g|$ with $|g\rangle$ ($|e\rangle$) the atomic ground (excited) state. Let the atom be initially prepared either in the excited or in the ground state. After a finite interaction time τ the Fock state of the field remains unchanged but the atomic state becomes a superposition like this

$$\begin{Bmatrix}|e\rangle\\|g\rangle\end{Bmatrix}\to\cos(f\tau n_j)\begin{Bmatrix}|g\rangle\\|e\rangle\end{Bmatrix}-i\sin(f\tau n_j)\begin{Bmatrix}|e\rangle\\|g\rangle\end{Bmatrix}. \tag{38}$$

If the interaction time is chosen such that $\tau f=\pi/2$, then observing a state-flipping (e.g., $|g\rangle\to|e\rangle$ or $|e\rangle\to|g\rangle$) means that the number n_j is odd, whereas no-flipping event signals an even n_j, no matter how many n_j is. This thus provides a convenient way to infer the parity of the detected photon number by just monitoring the atomic state at $\tau=0.5\pi/f$.

In summary, we have dealt with a symmetric network problem for teleporting an unknown coherent-state superposition from a party to any one of the other parties. We have found out that for low intensity fields ($|\alpha|^2<3$) the probability of successful teleportation decreases with decreasing $|\alpha|$ and with increasing size of the network: $\Pi_{2^N}\propto 2^{-N}$ in the limit of vanishing $|\alpha|$. However, for fields containing at least three photons on average the teleportation turns out to be faithful with a success probability equal to 1/2, independent of the number of participating parties as well as of the field intensity. Of course, no more than one party can get an exact replica of the teleported state because the parties 1, 2, ..., 2^N-1 counted the photon number and, by doing so, they destroyed the state at their locations. This accords with the no-clonning theorem. Also, no party is left jobless. All are involved in a global cooperation. Without the action of any of the parties the teleportation can never be completed since the measurement outcomes of all the 2^N-1 parties should arrive at the party 2^N station before that latter party is able to infer the teleported state. This takes some time for the party 2^N to collect all the necessary data by means of classical channels guaranteeing a peace with the special relativity. It should also be kept in mind that in the proposed protocol the teleportation cannot be reliable (i.e., with a probability of success equal to 1). There may happen that $n_0+n_1+n_2+\cdots+n_{2^N-1}$ is even in which case the state at the party 2^N station differs from the original state $|\Psi\rangle_0$ by a relative phase factor. Since no unitary transformations that casts $a|\alpha\rangle-b|-\alpha\rangle$ into $a|\alpha\rangle+b|-\alpha\rangle$ have been available, the teleportation fails. The number of participating parties here is limited to $M=2^N$ just because we have not yet known techniques to produce the state $|\Psi\rangle_{12\cdots M}$, Eq. (3), for any M. If such a technique exists then our formalism remains fully applicable to any M. Yet, looking for other kinds of quantum channel towards an optimal teleportation protocol as well as taking into account effects of noise and decoherence, etc., are worth for further efforts.

Last but not least, we would emphasize that our proposed protocol can potentially be applied to quantum information processing based on continuous variables [19]. Actually, there have appeared tendencies to encode information in quantum states with continuous variables (logical qubits) since such an encoding allows the information to be manipulated much more efficiently than with traditional discrete-variable states (qubits). As mentioned at the beginning, superpositions of coherent states have proved to be, among others, a good candidate for logical qubits to be used as computational bases in quantum computing. Furthermore, the beam-splitter-based measurement (performed here by Alice) for coherent states would be helpful also for quantum cryptography and quantum error correction. As for experimental implementation of our ideas, it is also simple. The 50:50 beam splitters are presently available and the difficulty associated with determination of the parity of photon number in a Fock state by individual parties could greatly be facilitated by additionally employing atom-field dispersive interaction as described in the text.

Discussions with the KIAS's Quantum Information Group and members of the KAIST's Physics Department are gratefully acknowledged. This work was supported by KIAS Research Grant No. 02.0149.001.

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