Construction of a quantum repeater with linear optics

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We study the mechanism and complexity of an efficient quantum repeater, employing double-photon guns, for long-distance optical quantum communication. The guns create polarization-entangled photon pairs on demand. One such source might be a semiconducter quantum dot, which has the distinct advantage over parametric down-conversion that the probability of creating a photon pair is close to 1, while the probability of creating multiple pairs vanishes. The swapping and purifying components are implemented by polarizing beam splitters and probabilistic optical controlled-NOT gates. We also show that the bottleneck in the efficiency of this repeater is due to detector losses.

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I. INTRODUCTION

Quantum repeaters are essential for quantum communication over distances longer than the decoherence length of the communication channels [1,2]. Repeaters employ a combination of entanglement swapping [3] and entanglement purification or distillation [4,5]; that is, multiple pairs of degraded entangled states are condensed into (fewer) maximally entangled states, after which swapping is used to extend the (now maximal) entanglement over greater distances. Both entanglement distillation and swapping have been demonstrated experimentally [6,7]. In this paper, we present a protocol for optical quantum repeaters based on linear optics, a double-photon gun, and a quantum memory. We give an estimate of the number of components in the repeater stations.

II. THE DOUBLE-PHOTON GUN

Currently, the source for polarization-entangled photon pairs consists of parametric down-converters, where a strong pump laser is sent through a nonlinear crystal. The interaction between the laser and the crystal results in entangled photon pairs. However, the output of these devices are not clean, maximally entangled, two-photon states, but rather a coherent superposition of multiple pairs. In this section, we argue why these coherent sources are unsuitable for large-scale quantum communication, and advocate the use of an alternative entanglement source: the double-photon gun.

Suppose the effective interaction Hamiltonian of a type-II parametric down-converter is given by

$$\hat{H} = i\kappa\hat{L}_{+} - i\kappa^{*}\hat{L}_{-}, \tag{1}$$

where $\hat{L}_{+} = \hat{a}_H^{\dagger}\hat{b}_V^{\dagger} - \hat{a}_V^{\dagger}\hat{b}_H^{\dagger} = \hat{L}_{-}^{\dagger}$ [8]. Here, $\hat{a}^{\dagger}$ and $\hat{b}^{\dagger}$ are the usual creation operators of the two optical modes, and H and V are orthogonal polarization directions. The operator, $\hat{L}_{+}$ ($\hat{L}_{-}$) is the creation (annihilation) operator for entangled photon pairs (in this case polarization singlets). The outgoing state of a spontaneous parametric down-converter is then given by

$$|\Psi_{\rm out}\rangle = \exp(i\hat{H}t/\hbar)|0\rangle = \sum_{n=0}^{\infty} \mathcal{N}_n(\epsilon L_{+})^n|0\rangle. \tag{2}$$

This expression is obtained by normal ordering $\exp(i\hat{H}t/\hbar)$, where $\epsilon \equiv \hat{\boldsymbol{\kappa}}\tanh(\kappa t/\hbar)$ and $\mathcal{N}_n^{-1}$ is the multiple-pair normalization $\sqrt{n!(n+1)!}$, which is analgous to the normalization factor $\sqrt{n!}$ of the ordinary creation operator $\hat{a}^{\dagger n}$ acting on the vacuum. The complex number ϵ is the probability amplitude of creating the maximally entangled state $|H,V\rangle - |V,H\rangle$. Therefore, down-converters only produce single pairs when $|\epsilon| \ll 1$, and by far, the major contribution to the state is the vacuum $|0\rangle$.

For large-scale applications, such as a quantum repeater, there is a more serious drawback to down-conversion. You typically need many entangled photon pairs, which would require, say, N down-converters to fire in unison. This happens with probability $|\epsilon|^{2N}$. This is already extremely small (current experiments are aimed at achieving $N=3$ events), but with approximately the same probability, the first down-converter produces N photon pairs, while the others produce nothing. Worse still, *any* distribution of N photon pairs scales proportional to $|\epsilon|^{2N}$. As shown in Refs. [8] and [9], this will seriously affect the performance for most applications.

We would therefore like to have a source with the following properties: (1) whenever we push the button of our entanglement source, we produce, on demand, a polarization-entangled photon pair, and (2) the fidelity of the output of our entanglement source must be very close to 1. We call a source with these properties a double-photon gun.

One entanglement source that very nearly meets our requirements has been proposed by Yamamoto and co-workers (see Fig. 1) [10]. A quantum dot separating p-type and n-type GaAs is sandwiched between two Bragg mirrors. The entire structure is therefore an optical microcavity, and electron-hole recombination will result in the creation of an entangled photon pair. Critically, due to Pauli's exclusion principle, only one electron and one hole are recombined at a time,

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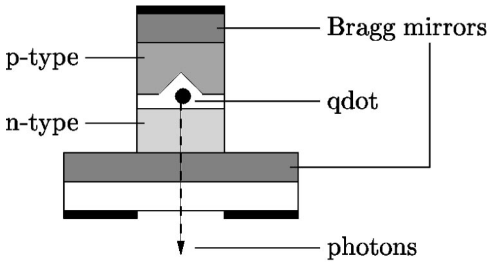

FIG. 1. The entanglement source by Benson *et al.* [10]. A quantum dot separating *p*-type and *n*-type GaAs is sandwiched between two Bragg mirrors. Electron-hole recombination will result in the creation of an entangled photon pair. Due to the Pauli exclusion principle, multiple pair production is suppressed. The efficiency of this proposed source is predicted to reach values up to 90%.

resulting in at most one photon pair. Furthermore, this process is triggered by applying a potential difference over the microcavity, which allows for greater control over the creation of a pair. In particular, the probability of creating a pair can theoretically be as high as $p_s = 0.9$ (although low collection efficiencies reduce this number by two orders of magnitude). Consequently, this source satisfies the required double-photon gun properties outlined above. These double-photon guns operate in the temperature range between 20 K and 50 K.

The two entangled photons from this source have different frequencies, which allows us to spatially separate them by means of a dichroic mirror. Interference phenomena at beam splitters, however, rely on the indistinguishability of the incoming photons, and the nondegenerate frequencies might render the photons distinguishable. Special care needs to be taken to arrange the setup in such a way that only photons with equal frequencies enter any particular optical element. However, this will not present any fundamental difficulties, and we will return to this point in the following section.

These double-photon guns do not yet exist. However, recently, Moreau *et al.* demonstrated quantum correlations between two photons that were generated in a single quantum dot [11]. Furthermore, Santori *et al.* and Pelton *et al.* have demonstrated efficient single-photon sources and interference effects in their output states using quantum dots in microcavities [12]. We therefore expect that the double-photon guns will also be constructed (and improved upon) in the near future. However, before they can be assembled in a large array, the double-photon guns must be manufactured such that they are almost identical in order for interference to take place. Also, so far, the two photons from the double-photon guns are correllated, but not entangled [13]. Obviously, this has to be overcome before these sources can be used in a repeater. In the following section, we study how we can construct an optical quantum repeater based on these sources.

III. THE QUANTUM REPEATER

A quantum repeater works as follows [1]: suppose Alice and Bob need to share a maximally entangled state, but they are far apart. Alice can prepare the entangled state and send one-half to Bob. However, the further Alice and Bob are apart, the further the quantum system has to travel, and the fidelity F of the total state will decrease due to decoherence effects. We may assume that the fidelity behaves exponentially, that is, $F \propto \exp(-L\gamma)$, where L is the distance between Alice and Bob, and γ is the characteristic rate of deterioration for the traveling quantum system. Alice and Bob can use purification protocols to extract maximal entanglement, but such protocols break down below a minimum fidelity F_0. The maximum distance of unaided quantum communication therefore has an upper bound.

A quantum repeater overcomes this difficulty by purifying an ensemble of entangled states after a certain distance L/N, when $F > F_0$. This process is repeated N times in series (N "legs"), where the ith repeater station holds one part of leg i and one part of leg $i+1$. After successful purification of all the N legs of the distance from Alice to Bob, the repeater stations apply entanglement swapping between legs i and $i+1$, and the result is a shared maximally entangled state between Alice and Bob.

In this case, the fidelity decreases with a factor $\alpha = \exp(-L\gamma/N)$, which is an exponential improvement. When the decrease in the fidelity is due to the attenuation of an optical beam in a fiber, the probability that a photon emitted by Alice reached Bob, without repeaters, is α^N. Therefore, Alice must send α^{-N} photons to Bob, in order to share one maximally entangled photon pair on average. Using quantum repeaters, every leg needs only $1/\alpha$ photons, and the total number of photons in all the legs is $1/(N\alpha)$. Thus, the quantum repeater transforms an exponential overhead into polynomial overhead. The two essential ingredients for the repeater are *entanglement purification* and *entanglement swapping*. In the next two sections we will discuss the optical implementations of these ingredients.

A. Entanglement purification

We consider quantum communication protocols that use any of the four two-qubit Bell states: $|\Phi^\pm\rangle = (|H,H\rangle \pm |V,V\rangle)/\sqrt{2}$ and $|\Psi^\pm\rangle = (|H,V\rangle \pm |V,H\rangle)/\sqrt{2}$, with H and V the polarization directions. These states can be locally transformed into each other by means of simple qubit operations, and we therefore may assume that the above entanglement source (entangler) can make any of the four Bell states. A difficulty is that these states change due to transport. They may pick up a relative phase, undergo a polarization rotation, they might become mixed, or they may be lost altogether. We therefore need to distill a single maximally entangled state from an ensemble of nonmaximally entangled states.

In order to distill maximally entangled quantum states, we use entanglement purification. Suppose Alice and Bob share two pairs of nonmaximally entangled states. Bennett *et al.* showed that, with some finite probability, it is possible to extract a single maximally entangled state [4]. To do this, both Alice and Bob apply a controlled-NOT (CNOT), where the halves of the first entangled pair serve as the control qubit, and the halves of the second as the target. The target qubits are then measured in the computational basis (deter-

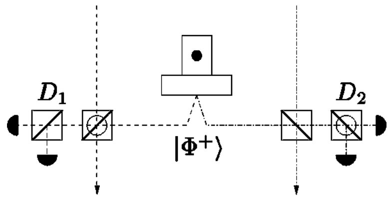

FIG. 2. The probabilistic CNOT gate of Pittman *et al*. [17]. Conditioned on a specific detector outcome in D_1 and D_2, the setup performs a controlled not. The boxed beam splitter is a linear polarization beam splitter and the circled box is a circular polarization beam splitter.

mined by the participants prior to the communication, e.g., $|H\rangle$ and $|V\rangle$), and conditioned on a parallel coincidence (like $|H\rangle_A|H\rangle_B$ and $|V\rangle_A|V\rangle_B$), Alice and Bob now share a maximally entangled state in the remaining two qubits. The probability of purification depends on the fidelity of the incoming entangled state, and therefore on the channel noise factor γ.

Additionally, in some cases the modes of the control qubit might be empty, because the entanglement source failed to create a photon. In order to rule out these events, we can employ the single-photon quantum nondemolition (QND) measurement scheme proposed by Kok *et al*. [14]. This is a probabilistic scheme that can be set up to signal the presence of a a single-photon wave packet in an optical mode without changing its (unknown) polarization. In the teleportation-based configuration, the success rate of this device is $p_{\mathrm{QND}} = \frac{1}{2}$. This device employs two photodetectors, as well as an entanglement source to create the quantum channel for teleportation.

Essential for the success of the repeater protocol is the ability to perform the controlled-NOT operation. In Fig. 2 we show the schematic setup for the probabilistic CNOT designed by Pittman *et al*. [17]. The main ingredients are a $|\Phi^+\rangle$ source and four polarization beam splitters, two of which separate *circular* polarization. The control qubit enters a linear polarizing beam splitter, and the target enters a circular polarizing beam splitter. The secondary input ports of these two beam splitters are fed by the two components of a $|\Phi^+\rangle$ Bell state.

A successful CNOT operation is now conditioned on detecting a linearly polarized photon after the circularly polarizing beam splitter (D_1 in Fig. 2), and a circularly polarized photon after the linearly polarizing beam splitter (D_2 in Fig. 2). These detections can be implemented with suitable polarizing beam splitters and ordinary photodetectors [17]. The probability of this CNOT operation is given by $p_{\mathrm{CNOT}} = \frac{1}{4}$. Recently, entanglement distillation was demonstrated by Yamamoto *et al*. [18].

B. Entanglement swapping

The entanglement-swapping component (swapper) of the quantum repeater is essentially nothing more than a Bell detector. It is well known that it is impossible to make a deterministic, complete, Bell measurement with linear optics [15], but one can distinguish two out of four two-qubit Bell states with a simple beam splitter configuration [16]. Recently, Franson and co-workers have shown that a CNOT—and hence a Bell measurement—is possible *probabilistically* with only projective measurements and entangled input states [17]. The probability of success for this CNOT is not large enough to make the Bell measurement more efficient.

A partial Bell measurement can be performed using a beam splitter. This is extensively described by Braunstein and Mann [16]. When two photons enter the beam splitter, one at each input, a detector coincidence in the two output modes collapses the input state onto $|\Psi^-\rangle$, that is, the singlet state. When both photons end up in one spatial output mode with opposite polarizations H and V, then the state is collapsed onto $|\Psi^+\rangle$. Other detector outcomes do not project the state onto a Bell state. The probability of success is therefore 1/2.

In order to build a complete quantum repeater, we have to integrate the components described above into a circuit [1,2]. The separate components are (see Fig. 3) the *entangler* E, the *purifier* P, and the *swapper* S. The assembled quantum repeater, shown in Fig. 4, is a circuit involving E, P, and S, together with classical communication between the different stations. This classical channel is necessary to exchange information about the measurement outcomes of the purifiers and about the location of the purified (and swapped) entanglement.

IV. REPEATER ASSEMBLY

In this section, we show how to assemble the quantum repeater and assess its probability of success. In Fig. 4 it is shown how a two-leg communication system with one quantum repeater is used to share maximal entanglement between Alice and Bob. First, the entanglement stations E distribute

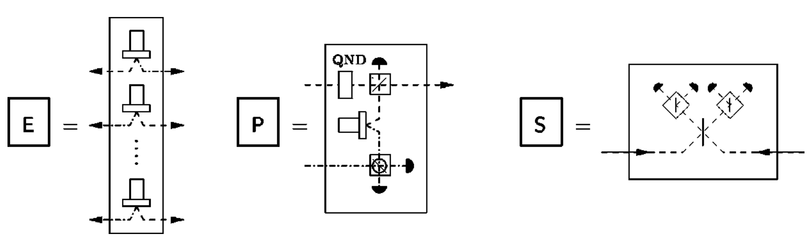

FIG. 3. The components of the quantum repeater. The three boxes E, P, and S denote the entanglement station (entangler), the purifier, and the swapping element (swapper), respectively. The entanglers are drawn as little top hats. The dashed and the dash-dotted lines represent the fact that the two output modes have different frequencies. The purifier element contains a QND device, an optical CNOT gate, and a detector on one output mode. The swapper implements a partial Bell measurement.

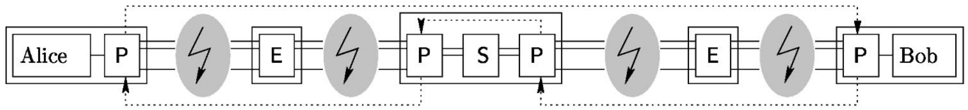

FIG. 4. The assembled quantum repeater. The lines between the entanglers, purifiers, and the swapper represent the quantum channels, and the dotted lines denote classical communication channels. The distance is included in the shaded region. This is also where the "decoherence devil" resides. Note that the entanglement sources are separate stations. Alternatively, the entanglement station can be placed near Alice and Bob.

imperfect entanglement between two adjacent repeater stations. In the purifier P, two modes are fed into the CNOT. Since the double-photon gun creates two photons with different frequencies ω_1 and ω_2, the guns must be oriented in such a way that two photons of equal frequencies enter the polarization beam spliters in the CNOT gates. However, this does not present any fundamental difficulties to the protocol.

Second, to purify two entangled photon pairs, the target output of the CNOT is detected in the computational basis $\{|H\rangle,|V\rangle\}$, and the control qubit is stored in a quantum memory. Alice sends her detector outcomes to the adjacent repeater station, which in turn selects the qubits that are to be swapped.

We now have to take into account the probability of success for the individual components, as well as the losses in the system. For the purification part (P to P in Fig. 4) we have five double-photon guns (two for the photon sources in E, one in the QND device, and one for every CNOT) and eight detectors with quantum efficiency η (three per CNOT and two in the QND device). Furthermore, let the noise parameters due to the attenuation be given by γ for the dephasing (reducing the fidelity, this includes the probability of successful purification) and ζ for the photon loss over the channel. The probability for purifying a single pair of entangled photons is then given by

$$p_{\text{pur}}=(1-\gamma)\zeta p_{\text{s}}^5\eta^8 p_{\text{CNOT}}^2 p_{\text{qnd}}, \tag{3}$$

where p_{s} is the probability of success of the double-photon gun. It is immediately obvious that a reduced detector efficiency η will strongly contribute to the deterioration of the success rate, due to the η^8 behavior. In addition, the double-photon guns also need a high probability of success.

Next, we start with two purified pairs and perform entanglement swapping on their two halves. As argued above, this swapping protocol is not deterministic, and is subject to losses as well. As can be seen in Fig. 3, the swapping element requires a twofold detector coincidence. Furthermore, a complete Bell detection occurs only 50% of the time. The probability of success for entanglement swapping is therefore given by

TABLE I. The number of components in the quantum repeater for different values of the detector efficiency η.

η	N_{pur}	N_{swap}	N_{total}
0.3	1.7×10^6	25	7.3×10^7
0.8	650	4	4×10^3
1	110	2	435

$$p_{\text{swap}}=\frac{\eta^2}{2}. \tag{4}$$

To estimate the size of the repeater stations, let us now insert some values of the several components. We will use different values for the detector efficiency, since this is the most important parameter. Choose, for example, $p_{\text{s}}=0.9$, $\gamma=\frac{1}{2}$, $\zeta=\frac{1}{2}$, $p_{\text{CNOT}}=\frac{1}{4}$, and $p_{\text{QND}}=\frac{1}{8}$. For three different values of η, this gives rise to Table I (with $N_{\text{pur}}=p_{\text{pur}}^{-1}$ and $N_{\text{swap}}=p_{\text{swap}}^{-1}$). Since a repeater needs two purifiers and one swapper, the total number of components (N_{total}) is given by $N_{\text{total}}=2N_{\text{pur}}N_{\text{swap}}$. The results of Table I should be compared with the number of transistors on a Pentium chip, which is of the order 10^7. This calculation does not take into account the added complexity of a quantum memory, which is required for purification.

It is immediately clear that an improvement in the detector efficiency yields a substantial gain in the efficiency of the protocol, due to the factor η^8 in Eq. (3). Even though detector efficiencies of 0.8 are quoted, experimental values are as bad as 0.3. Therefore, in order to operate the repeater more efficiently, better detectors are needed.

Note also that intelligent switching, conditioned on detector outcomes and classical communication between the components, is needed both to purify and to correlate purified entanglement in the swapping procedure. This results in an additional overhead in the number of components.

V. CONCLUSION

We studied an optical implementation for a quantum repeater employing double-photon guns, probabilistic CNOT operations, and quantum nondemolition measurements. The protocol uses available and almost available technology. Possible drawbacks are the conditional switching and the low quantum efficiencies of state-of-the-art photodetectors.

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