Suppression and acceleration effects of measurements on atomic decay in anisotropic photonic crystals

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We study the measurement-induced suppression or acceleration of the radiative decay of an atom embedded in an anisotropic photonic crystal. Due to the presence of a band gap in the electromagnetic density of states, repeated projections onto the excited state of the atom can lead to a suppression or acceleration effect already at rather low repetition rates. It is shown that in contrast to the isotropic band-gap materials, both suppression and acceleration effects are possible, depending on the detuning of the atomic transition from the band edge and the frequency of measurements.

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I. INTRODUCTION

Artificial materials consisting of periodic structures of dielectric media, called photonic crystals, can possess band gaps in the density of electromagnetic modes [1]. As a consequence a modification or even a suppression of radiative decay in these systems has been predicted [2]. Due to the technical difficulties in producing the three-dimensional (3D) photonic band-gap materials with a sufficiently large and complete band gap, an unambiguous experimental proof of this effect has, however, not been achieved until now. On the other hand, it is well known that the radiative decay of an atom can be substantially altered by frequently repeated measurements if the reservoir of radiation modes has a highly structured density of states in the vicinity of the corresponding atomic resonance. This result of the interplay between quantum dynamics and measurement, which is absent in classical measurements, is known as the quantum Zeno (decay suppression) [3] or quantum–anti-Zeno (decay acceleration) effect for sharply varying [4] and for smoothly varying density of states (DOS) [5]. In the case of sharply varying DOS, a smoothing parameter was introduced, which can be chosen to fit different materials such as isotropic and anisotropic photonic crystals. The purpose of the present paper is to analyze as to what extent these suppression and acceleration effects can be observed in a 3D photonic crystal. The most pronounced changes are to be expected in the band-gap materials with an isotropic dispersion relation, as analyzed in Ref. [6], where the density of states shows a singular behavior at the band-edge frequency ω_c. However, we here want to consider the more realistic situation of an anisotropic photonic crystal, where there is no such singularity. We will show that both acceleration and suppression effects may be observed depending on the relative detuning of the atomic transition frequency to the band edge and the frequency of measurements. This is in contrast to the case of an isotropic photonic crystal.

The first experiment on the quantum-Zeno effect was suggested by Cook [7], and was later performed by Itano *et al.* [8] using coherent Rabi oscillations in a three-level atom. The effect has since been studied in a variety of physical systems, ranging from atomic physics [9] to radioactive decay [10] and mesoscopic systems [11]. In Ref. [12], a scheme for slowing down decay into a continuum is proposed by means of a sequence of ultrashort 2π pulses. An opposite phenomenon, the anti-Zeno effect (decay acceleration by frequent measurements), was recently discovered by Kofman and Kurizki [4,5]. Similar predictions have been made in the theoretical and numerical investigations of quantum chaotic systems [13]. Mathematically, a more rigorous treatment of the quantum-Zeno and quantum–anti-Zeno effects has been given for the Friedrichs model [14]. The transition from quantum-Zeno to quantum–anti-Zeno behavior was analyzed in Ref. [15]. In Ref. [16], the first observation of the quantum-Zeno and quantum–anti-Zeno behaviors due to repeated measurements during the nonexponential period of an unstable quantum system was reported.

In this paper, we study the influence of frequent measurements on the evolution of an initially excited two-level atom embedded in an anisotropic photonic crystal with a single (upper) propagation band and a single (lower) stop band. The restriction to a single propagation band seems justified as long as the transition frequency is in the vicinity of the band edge and the measurement frequency is not too high. A more elaborate discussion of the band-gap materials with several propagation bands will be given elsewhere. It is found that the decay process of the atomic excited state may be accelerated or inhibited depending on the detuning from the band edge and the frequency of measurements. Two characteristic values for the atomic transition frequency Ω_2 and for the repetition rate of measurements ν_0 are identified. When the upper level is above Ω_2 and the frequency of the measurements is smaller than ν_0, the decay can be slowed down. Otherwise, the decay is accelerated. These properties are different from the ones for an isotropic photonic crystals [6].

II. MODEL AND STATE EVOLUTION

In an anisotropic photonic crystal, the dispersion relation of the electromagnetic modes is strongly modified by the periodic dielectric structure, and a band gap, is formed on the

surface of the first Brillouin zone in the reciprocal lattice. Near the band edge, the dispersion relation may approximately be expressed by $\omega_{\mathbf{k}}=\omega_c+C|\mathbf{k}-\mathbf{k}_0^n|^2$. C is a model-dependent constant and $\mathbf{k}_0^n$ is a finite collection of symmetry related points in the reciprocal lattice associated with the band edge, e.g., the eight L points in the first Brillouin zone of a diamond photonic crystal. The most important difference between anisotropic and isotropic photonic crystals stems from the dependence of the density of states on the band edge. For an isotropic band-gap material, the density of states is proportional to $(\omega_k-\omega_c)^{-1/2}$ for $(\omega_k>\omega_c)$, which leads to a singularity at the band edge. In the anisotropic case, the density of states, is however, proportional to $(\omega_k-\omega_c)^{1/2}$ for $(\omega_k>\omega_c)$, which is not singular.

Let us consider a two-level atom embedded in an anisotropic photonic crystal. The upper level $|1\rangle$ is coupled by the allowed electromagnetic modes to the lower level $|0\rangle$. The energy of $|0\rangle$ is set to zero. The atomic transition frequency from level $|1\rangle$ to level $|0\rangle$, ω_1, is assumed to be near the band-edge frequency ω_c. In rotating-wave approximation, the Hamiltonian for the system takes the form

$$\hat{H}=\hbar\omega_1|1\rangle\langle 1|+\sum_k \hbar\omega_k b_k^\dagger b_k^\dagger+i\hbar\sum_k g_k(b_k^\dagger|0\rangle\langle 1|-\text{H.c.}). \tag{1}$$

Here, the label k denotes both the wave vector and the polarization of the mode and b_k (b_k^+) is the radiation-field annihilation (creation) operator for the kth mode with frequency ω_k. $g_k=(\omega_1 d_1/\hbar)\sqrt{(\hbar/2\varepsilon_0)\omega_k V_0}\mathbf{e}_k\cdot\mathbf{u}_d$ is the atomic-field coupling constant, where d_1 and $\mathbf{u}_d$ are the magnitude and unit vector of the atomic dipole moment. V_0 denotes the quantization volume, $\mathbf{e}_k$ are the two polarization unit vectors. The atom is assumed to be initially in the excited state $|1\rangle$, and the state vector of the system is given at time t by

$$|\psi(t)\rangle=A(t)e^{-i\omega_1 t}|1,\{0\}\rangle+\sum_k B_k(t)e^{-i\omega_k t}|0,\{1_k\}\rangle, \tag{2}$$

where $|1,\{0\}\rangle$ denotes the atom in the excited state with no photon present and $|0,\{1_k\}\rangle$ describes the atom in the ground state with a single photon in kth mode with frequency ω_k. From Schrödinger equation, we can obtain the following equations for the amplitudes $A(t)$ and $B_k(t)$:

$$\frac{d}{dt}A(t)=-\sum_k g_k e^{-i(\omega_k-\omega_1)t}B_k(t), \tag{3}$$

$$\frac{d}{dt}B_k(t)=g_k A(t)e^{i(\omega_k-\omega_1)t}. \tag{4}$$

Formally integrating Eq. (3) yields

$$B_k(t)=\int_0^t g_k A(t')e^{i(\omega_k-\omega_1)t'}dt'. \tag{5}$$

Inserting Eq. (5) into Eq. (3), we obtain the integro-differential equation

$$\frac{d}{dt}A(t)=-\sum_k g_k e^{-i(\omega_k-\omega_1)t}\int_0^t g_k A(t')e^{i(\omega_k-\omega_1)t'}dt'. \tag{6}$$

Equation (6) is exactly solvable by Laplace transformation. The survival amplitude in the initial state $|1\rangle$, $A(t)$, takes the form [17,18]

$$A(t)=\frac{e^{x_1 t}}{F'(x_1)}+\frac{e^{x_2 t}}{G'(x_2)}-\frac{\sqrt{i}e^{i\omega_{1c}t}}{\pi}\int_0^\infty K(x)e^{-xt}dx, \tag{7}$$

where $\omega_{1c}=\omega_1-\omega_c$. x_1 is the root of

$$F(x)\equiv x-\frac{i\beta^{3/2}}{\sqrt{\omega_c}+\sqrt{-ix-\omega_{1c}}}=0$$

in the region [$\mathrm{Re}(x)>0$ or $\mathrm{Im}(x)>\omega_{1c}$], and x_2 is the root of

$$G(x)\equiv x-\frac{i\beta^{3/2}}{\sqrt{\omega_c}-i\sqrt{ix+\omega_{1c}}}=0$$

in the region [$\mathrm{Re}(x)<0$ and $\mathrm{Im}(x)<\omega_{1c}$]. The functions $F'(x)$, $G'(x)$, and $K(x)$ are defined as

$$F'(x)=1-\frac{x^2}{2\beta^{3/2}\sqrt{-ix-\omega_{1c}}},$$

$$G'(x)=1-\frac{ix^2}{2\beta^{3/2}\sqrt{ix+\omega_{1c}}},$$

$$K(x)=\beta^{3/2}\frac{\sqrt{x}(\omega_c-ix)}{[(\omega_{1c}+ix)(\omega_c-ix)-\sqrt{\omega_c}\beta^{3/2}]^2-i\beta^3 x}.$$

The parameter β is given by

$$\beta^{3/2}=\frac{(\omega_1 d_1)^2\sum_n \sin^2\theta_n}{8\pi\varepsilon_0\hbar C^{3/2}}, \tag{8}$$

where θ_n is the angle between the dipole vector of the atom and the nth $\mathbf{k}_0^n$. Note that the phase angles of $\sqrt{-ix-\omega_{1c}}$ and $\sqrt{ix+\omega_{1c}}$ in the above functions have been defined in the region $(-\pi/2,\pi/2)$.

It can be shown analytically that there are two characteristic frequencies

$$\Omega_1=\omega_c+\frac{\beta^{3/2}}{\omega_c^{1/2}}, \tag{9}$$

$$\Omega_2=2\omega_c-\frac{\beta^{3/2}}{\frac{2}{3}\omega_c^{1/2}+(q_1-q_2)^{1/3}-(q_1+q_2)^{1/3}}, \tag{10}$$

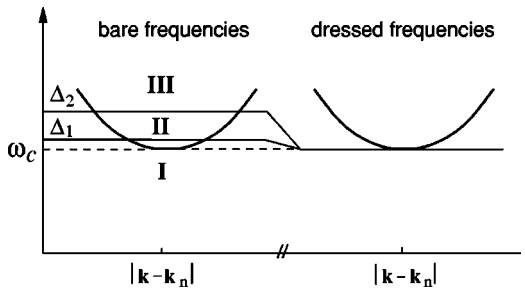

FIG. 1. Frequency regions in a photonic crystal. For bare frequency detunings, $\omega_{1c}=\omega_1-\omega_c$ from the band edge ω_c with $\omega_{1c}<\Delta_1$ (region I), the dressed frequency lies in the forbidden band. For $\Delta_1\leqslant\omega_{1c}\leqslant\Delta_2$ (region II), the dressed frequency is exactly at the band edge, and for $\omega_{1c}>\Delta_2$ (region III) the dressed frequency stays in the allowed band.

with $q_1=[(4\omega_c^3-20\omega_c^{3/2}\beta^{3/2})/27+\beta^3]^{1/2}$ and $q_2=10\omega_c^{3/2}/27-\beta^{3/2}$. If $\omega_1<\Omega_1$ (region I) only a single imaginary root x_1 exists with $\mathrm{Im}[x_1]>\omega_{1c}$. When $\omega_1>\Omega_2$ (region III), there is only one complex root x_2 with a negative real part and an imaginary part being smaller than ω_{1c}. In the intermediate region, i.e., for $\Omega_1<\omega_1<\Omega_2$ (region II), there exists no root of the equations $F(x)=0$ and $G(x)=0$. In this case the corresponding terms in Eq. (7) are set equal to zero. The three frequency regions have a simple physical meaning. In region I, the interaction with the radiation modes shifts the bare atomic transition frequency into the stop band. Here, the atomic excitation will not decay to zero and a bound atom-photon state is generated. In region II, the interaction moves all bare frequencies exactly to the band edge ω_c. In region III, the atomic transition stays in the allowed band even after renormalization due to the interaction with the field. This is illustrated in Fig. 1.

The last term on the right-hand side of Eq. (7) is due to an integration along the cut of the single-valued branches, which is taken along the negative part of the axis in the complex plane. The time-decay behavior of this term is important for the quantum–anti-Zeno effect.

III. SUPPRESSION AND ACCELERATION EFFECTS

We now discuss the influence of the frequent projections onto the excited state on the decay of the atom. Suppose, we detect the atom in the excited state by a von Neumann measurement n times in a time interval T. The projection process itself needs to be short on this time scale and will be assumed instantaneous. The probability of survival in the excited state after time T is then

$$P(T)=|A(T/n)|^{2n}. \tag{11}$$

If the decay of $A(t)$ is exactly exponential, $|A(T/n)|^{2n}=|A(T)|^2$ and the survival probability would not be affected by the measurements. It is, however, known that due to the boundedness of the spectrum from below, the decay is a nonexponential decay. For an atom interacting with free-space electromagnetic modes, this time scale is of the order of the inverse energy of the excited state. If the measurements are made fast enough such that T/n lies within this very short initial time period, the decay will always be suppressed. This phenomenon is called the quantum-Zeno effect. In practice, it is however often impossible to make measurements on such a short time scale. On the other hand, if the transition frequency of an atom in a photon band-gap material is close to the cutoff frquency, a suppression effect may be observed at much smaller repetition rates of the measurements. Furthermore, also the opposite effect is possible if the undisturbed decay slows down for some time. If the decay process is interrupted by a projection before the period of slower decay sets in, an acceleration effect occurs: the effective decay of the system will be accelerated by measurements. These phenomenon can be observed on the decay of the atom embedded in an anisotropic photonic crystal.

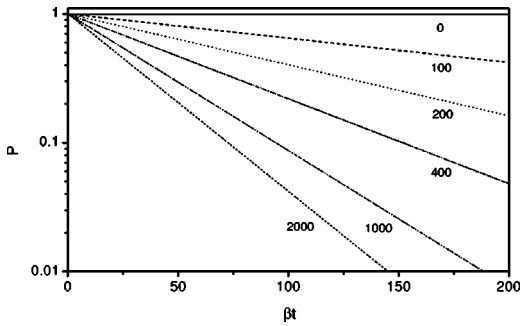

FIG. 2. Decay acceleration effect for an initially excited atom with transition frequency in region I ($\omega_c=100\beta,\omega_{1c}=-\beta$). Shown is the coarse-grained survival probability as a function of time for the projections in intervals T/n with $\beta T=200$. $n=0$ (undisturbed evolution, solid curve), 100 (dashed curve), 200 (dotted curve), 400 (dot-dashed curve), 1000 (dot-dot–dashed curve), and 2000 (short-dashed curve).

When the atomic transition frequency lies in region I (e.g., $\omega_c=100\beta$, $\omega_{1c}=-\beta$), $A(t)$ takes on the following form

$$A(t)=\frac{e^{x_1 t}}{F'(x_1)}-\frac{\sqrt{i}e^{i\omega_{1c}t}}{\pi}\int_0^\infty K(x)e^{-xt}dx. \tag{12}$$

The term $e^{x_1 t}/F'(x_1)$ does not decay with time and leads to a nonvanishing steady-state population in the upper level, which represents a bound atom-photon state. The integral term decays in the manner of a power law, which is fast for short times and is slow for long times. Consequently, frequent projections on the excited state could lead to decay acceleration effect. This is shown in Fig. 2, where we have plotted the (coarse-grained) survival probability as a function of time for different repetition rates of the projection.

When the upper level is within region II, only the integral term is present in the expression for $A(t)$. Here, the survival probability decays to zero and no steady-state population remains in the excited state. Since the time evolution of the integral in Eq. (12) is essentially the same as in the previous case in region II, a decay acceleration effect will be observed when the excited population is measured repeatedly. The time evolution of the survival probability is qualitatively similar to Fig. 2.

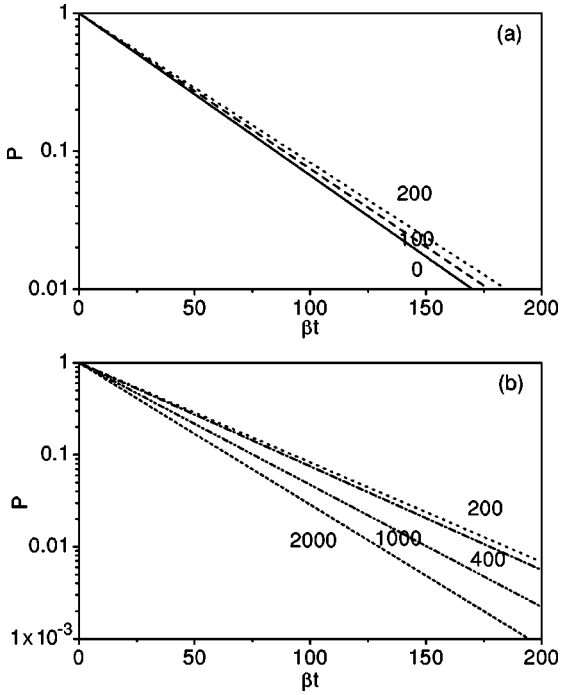

FIG. 3. Suppression and acceleration effects for an atom with transition frequency in region III ($\omega_c=100\beta,\omega_{1c}=2\beta$). Shown is the coarse-grained survival probability as a function of time for projections in intervals T/n, with $\beta T=200$. (a) Suppression effect, $n=0$ (the coherent evolution, solid curve), 100 (dashed curve), 200 (dotted curve); (b) acceleration effect, $n=400$ (dot-dashed curve), 1000 (dot-dot–dashed curve), 2000 (short-dashed curve).

Finally, when the upper level is within region III, $A(t)$ can be written as

$$A(t)=\frac{e^{x_2 t}}{G'(x_2)}-\frac{\sqrt{i}e^{i\omega_{1c}t}}{\pi}\int_0^\infty K(x)e^{-xt}dx. \tag{13}$$

The time evolution of this expression is more complex than in the previous two cases since the two terms in Eq. (13) have different weights at different times. The decay of the first term $e^{x_2 t}/G'(x_2)$ is exponential, and the corresponding decay time is a constant, $(-\mathrm{Re}[x_2])$. Thus, the measurements will not change the contribution of this term to the effective decay rate. The decay of the integral term, on the other hand, slows down with time. For short times, the decay is faster than that of the exponential term, and the total decay time of $|A(t)|$ is smaller than $-1/\mathrm{Re}[x_2]$. For long times $(t\to\infty)$, the decay rate tends to zero and the total decay of $|A(t)|$ is dominated by the exponential term with rate $(-1/\mathrm{Re}[x_2])$. In Fig. 3 we have plotted the time evolution of the survival probability for $\omega_c=100\beta$, $\omega_{1c}=2\beta$ with $T=200/\beta$ and different n. One recognizes that for an increasing repetition rate of measurements, the decay first slows down and then accelerates, i.e., one can observe decay suppression first and then the anti-Zeno behavior as n increases. For reasons discussed above, there will of course again be a Zeno effect in the limit of very high repetition rates, i.e., very large values of n.

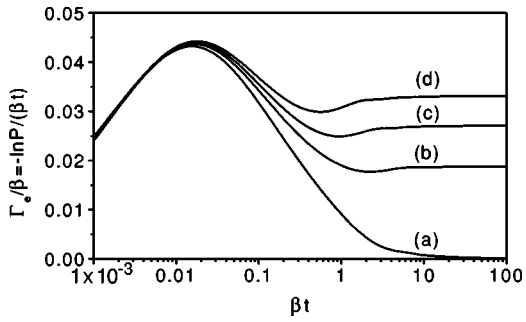

FIG. 4. Effective decay rate $\Gamma_e(t)$ as function of t for $\omega_c=100\beta$ and different relative positions of the upper level from the band gap; (a) $\omega_{1c}=-\beta$, (b) $\omega_{1c}=\beta$, (c) $\omega_{1c}=2\beta$, (d) $\omega_{1c}=3\beta$.

The behavior shown in Fig. 3 can easily be understood if one considers the effective decay rate Γ_e of the atom defined by

$$\Gamma_e(t)=-\frac{\ln P(t)}{t} \tag{14}$$

as a function of time. In Fig. 4, we have plotted Γ_e for $\omega_c=100\beta$ and different relative position of the upper level from the band gap ω_{1c}.

For very short initial time period ($\sim 10^{-2}\beta^{-1}$ in Fig. 4), the effective decay rate increases with time t. It means that there is a quantum-Zeno effect of the excited atomic decay as the time interval between the two measurements is smaller than the time period. However, the measurements on such a short time scale is difficult to make. After the short time period, the effective decay rates for different detuning of the upper level from the band edge have different properties. When the atomic transition frequency is within region I (curve *a*) or region II, the effective decay rate decreases monotonously with time t. If the atomic upper level is within region III (curve *b*, *c*, and *d*), the effective decay rate decreases first to a minimum value and then grows to a constant Γ_0, corresponding to the asymptotic exponential decay. When the atomic upper level moves deeper into the propagation band, the density of state increases and the population decay becomes faster. So the asymptotic decay rate Γ_0 increases as the relative position of the upper level from the band gap ω_{1c} increases. In addition, the time of slowest decay decreases as ω_{1c} increases (as shown in Fig. 4). Thus, the influence of the frequent measurements on the time evolution of the atomic upper-level population depends on both the relative position of the upper level from the band gap and the repetition rate of the measurements. When the frequency of the measurements is larger than a characteristic repetition rate ν_0, there is a quantum–anti-Zeno effect. When the frequency is smaller than ν_0, a suppression effect can be observed. The characteristic repetition rate ν_0 depends on the detuning of the atomic transition frequency from the band edge. This is illustrated in Fig. 5

In regions I and II, i.e., for ω_1 below Ω_2, repeated measurements always lead to an acceleration effect. In region III, i.e., for ω_{1c} above Ω_2 both suppression and acceleration effects are possible depending on the repetition rate ν of the

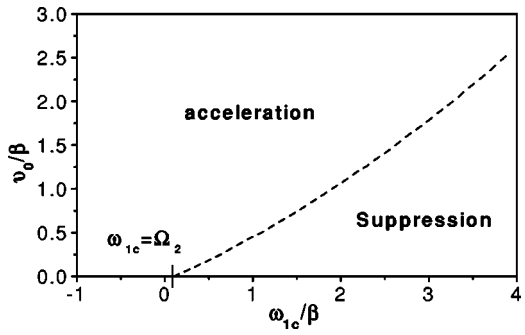

FIG. 5. The characteristic repetition ν_0 as function of the relative position of the upper level from the band gap ω_{1c}. For $\nu < \nu_0$, there is an acceleration effect and for $\nu > \nu_0$ a suppression effect is observed.

measurements. For $\nu < \nu_0$, there is a slowdown of decay, and for $\nu > \nu_0$ the decay is accelerated by measurements. When the upper level lies deep inside the propagation band, the characteristic frequency ν_0 is very large, and the acceleration effect is hard to observe experimentally.

The above results differ from that for an *isotropic* photonic crystal [6]. For a two-level atom in an isotropic photonic crystal, the dispersion relation leads to a singularity in the density of states at the band edge. As a result, the decay of the excited atom is fast with a very short characteristic time scale ($\sim\beta^{-1}$). For the present system, the density of states has no singularity near the cutoff frequency and is much smaller. Thus compared to the case of an atom embedded in an isotropic photonic crystal, the decay discussed here is slower and the corresponding characteristic decay time is substantially larger (e.g., $\sim 10^2\beta^{-1}$ as $\omega_{1c}=0.2\beta$; $\sim 10^3\beta^{-1}$ for $\omega_{1c}=0.1001\beta$). The long decay time in the anisotropic case makes the suppression and acceleration effects more easily observable in an experiment.

IV. CONCLUSION

In this paper, the influence of repeated measurements on the time evolution of an atomic excitation is investigated, when the atom is embedded in an anisotropic photonic crystal. The decay of excitation can be accelerated or suppressed by sufficiently frequent measurements. Which behavior occurs, suppression or acceleration, depends on the relative position of the upper level from the band gap and the frequency of measurements. This is different from the case of an isotropic photonic crystal.

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