Effect of quantum interference on optical bistability in the three-level V-type atomic system

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The phenomenon of optical bistability is studied for the three-level atomic system in V-configuration confined in a unidirectional optical ring cavity, and the effects of quantum interference and coupling field are investigated. The possibility of obtaining optical multistability in the system by controlling quantum interference and coupling field strength is also discussed.

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The phenomenon of quantum interference is central to many new effects recently discovered in quantum optics. Harris *et al.* [1] considered the V-type three-level atom and demonstrated that the absorption rate can become zero due to the destructive interference and thus it is possible to have light field amplification without population inversion [2]. It was also shown that the quantum interference can lead to line narrowing, black dark line, and removal of spectral emission of driving field frequency in the emission spectrum [3]. It can also produce ultranarrow spectral lines in the fluorescence spectra of a three-level atom [4]. Fluorescence quenching has been experimentally observed in the sodium dimers [5]. Many other related phenomena with quantum interference, e.g., the electromagnetically induced transparency (EIT), refractive index enhancement, modification of spectral features of three-level systems featuring dark resonances, etc., were also studied in recent years [6].

Optical bistability (OB) has been extensively studied both experimentally and theoretically in the recent past [7]. Most of the experimental studies in OB have been devoted to two-level alkali atoms confined in an optical resonator [7,8]. The theoretical models of OB have considered the interaction of a collection of two-level atoms with a single-mode field [7,8]. The perpetual interests in OB and associated phenomena stem from the fact that these phenomena could have wide range of applications such as in optical transistors, memory elements, and all optical switches. Also bistable behaviors were studied theoretically [9,10] and observed experimentally [11] in three-level atomic systems inside optical cavities in recent years.

In the literature, the possible realization of optical multistability (OM) has also been mentioned, which involves interaction of a nonlinear medium with two different optical fields. In particular, Kitano [12] reported optical tristability in a three-level Λ-configuration interacting with two different modes of cavity under the limiting condition of large atomic detuning and no saturation. This work was generalized by Savage *et al.* [13] to include saturation in the dispersion limit, and they predicted that the asymmetric state becomes unstable and gives rise to self-oscillations and a different kind of optical turbulence. Later on Arecchi *et al.* [14] included the effect of ground-state coherence and reported not only the tristability but also higher-order bistability. Recently, a theoretical calculation for OB has been carried out for three-level atoms confined in an optical cavity, and the effect of coherence generated in the system by a control field is studied and possibility of generating multistability is also discussed [9].

Here, in this work we discuss the role of quantum interference in the phenomenon of OB in a three-level V-type atomic system confined in an optical resonator. The atomic system consists of two excited sublevels of same parity and a single ground level of different parity. The effect of quantum interference in spontaneous emission from the upper two levels is included in this investigation of OB. Such a model with quantum interference from spontaneous emission shows great enhancement of the population inversion in one of the optical transitions and can lead to substantial radiation amplification and existence of vacuum-induced quasitrapped states [15]. However, the previous theoretical works studying OB in three-level atomic systems did not include quantum interference in the decay channels in their models.

The model of the three-level atom considered here is depicted in Fig. 1. It is a closed V-type configuration with one single ground-state $|1\rangle$ and two closely lying upper states $|2\rangle$ and $|3\rangle$. The transition between $|1\rangle$ and $|2\rangle$ (with resonant frequency ω_{12}) is mediated by the probe laser field E_P (frequency ω_1), while the transition $|1\rangle$ to $|3\rangle$ (with resonant frequency ω_{13}) is driven by another laser field E_C (frequency ω_2) called coupling field in this work. The atomic dynamics of the system can be described by the Liouville equation for the density operator and the density matrix equations [15] with all decay terms included under rotating-wave approximation are

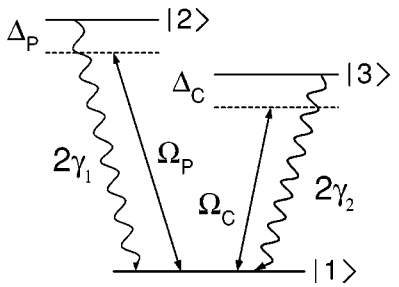

FIG. 1. Schematic diagram of a three-level V-type atom.

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$$\dot{\rho}_{22} = -2\gamma_1\rho_{22} + i\Omega_P e^{-i\delta t}\rho_{12} - i\Omega_P e^{i\delta t}\rho_{21} - \eta\sqrt{\gamma_1\gamma_2}(\rho_{32}+\rho_{23}),$$

$$\dot{\rho}_{33} = -2\gamma_2\rho_{33} + i\Omega_C(\rho_{13}-\rho_{31}) - \eta\sqrt{\gamma_1\gamma_2}(\rho_{32}+\rho_{23}),$$

$$\dot{\rho}_{12} = [-\gamma_1 + i(\Delta_P+\delta)]\rho_{12} + i\Omega_C\rho_{32} + i\Omega_P e^{i\delta t}(\rho_{22}-\rho_{11}) - \eta\sqrt{\gamma_1\gamma_2}\rho_{13},$$

$$\dot{\rho}_{13} = (-\gamma_2 + i\Delta_C)\rho_{13} + i\Omega_P e^{i\delta t}\rho_{23} + i\Omega_C(\rho_{33}-\rho_{11}) - \eta\sqrt{\gamma_1\gamma_2}\rho_{12},$$

$$\dot{\rho}_{32} = -(\gamma_1+\gamma_2)\rho_{32} + i(\Delta_P-\Delta_C+\delta)\rho_{32} + i\Omega_C\rho_{12} - i\Omega_P e^{i\delta t}\rho_{31} - \eta\sqrt{\gamma_1\gamma_2}(\rho_{22}+\rho_{33}),$$

$$\dot{\rho}_{11} = -(\dot{\rho}_{22}+\dot{\rho}_{33}). \quad (1)$$

In these equations the atomic detunings are defined as $\Delta_P=\omega_{21}-\omega_1$, $\Delta_C=\omega_{31}-\omega_2$, and the probe-coupling field frequency detuning is $\delta=\omega_1-\omega_2$. The Rabi frequencies for the probe and coupling fields are $\Omega_P=\vec{d}_{12}\cdot\vec{E}_P/\hbar$ and $\Omega_C=\vec{d}_{13}\cdot\vec{E}_C/\hbar$, respectively. The transition dipole moments $\vec{d}_{12}$ and $\vec{d}_{13}$ can be non-orthogonal and the parameter η measures their alignment, i.e., $\eta=(\vec{d}_{12}\cdot\vec{d}_{13})/(|d_{12}|\cdot|d_{13}|)\equiv\cos(\theta)$. The term $\eta\sqrt{\gamma_1\gamma_2}$ accounts for the spontaneous emission induced quantum interference effect due to coupling between emission-processes in the channels $|2\rangle\rightarrow|1\rangle$ and $|3\rangle\rightarrow|1\rangle$, and $\eta\neq 0$ for $\theta\neq\pi/2$. The ability of controlling η has been experimentally demonstrated recently [5]. The quantum interference terms in Eq. (1) represent the physical situation in which a photon is emitted virtually in channel $|2\rangle\rightarrow|1\rangle$ and its virtual absorption in channel $|1\rangle\rightarrow|3\rangle$ or vice versa. Note that the Rabi frequencies also depend on angle θ. For the sake of convenience in comparison with different values of θ, the Rabi frequencies are kept unchanged by suitably adjusting the field strength for this purpose. In the absence of coupling field the quantum interference terms are significant provided the separation of the upper two levels of V system is about γ_1 (or γ_2). However, with the presence of the coupling field this condition is relaxed [15].

The bistable behavior of the above-described atomic system (N such atoms) will be investigated in the unidirectional cavity (optical ring cavity), as shown in Fig. 2. The intensity reflection and transmission coefficients of mirrors $M1$ and $M2$ are R and T, respectively, such that $R+T=1$. For simplicity, we assume that both the mirrors $M3$ and $M4$ are perfect reflectors. This kind of model is a standard one for studying OB [7]. The three-level atomic system whose dynamics is described by equations in Eq. (1) is a collection of N homogeneously broadened atoms contained in a cell of length L. The total electromagnetic field seen by these atoms is

$$\vec{E} = \vec{E}_P\exp(-i\omega_1 t) + \vec{E}_C\exp(-i\omega_2 t) + \text{c.c.} \quad (2)$$

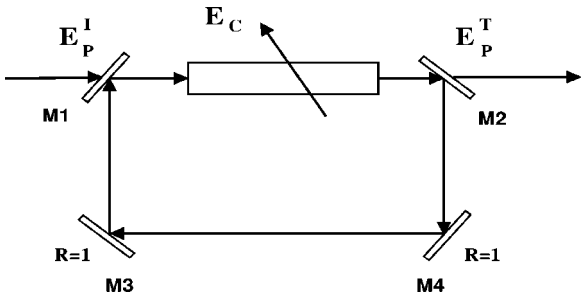

FIG. 2. Schematics of a unidirectional ring cavity with four mirrors and an atomic vapor cell of length L. $M3$ and $M4$ are perfect mirrors. E_P^I and E_P^T are the incident and transmitted fields, respectively, and E_C represents the noncirculating coupling field.

As discussed before, $\vec{E}_P$ is the probe field circulating in the ring cavity and $\vec{E}_C$ is the coupling field that does not circulate in the cavity. So, the dynamics of the probe field in the optical cavity is governed by Maxwell's equation, which, under slowly varying envelope approximation, is given by

$$\frac{\partial E_P}{\partial t} + c\frac{\partial E_P}{\partial z} = 2\pi i\omega_1 d_{12}P(\omega_1), \quad (3)$$

in which $P(\omega_1)$ is the induced polarization in the transition $|1\rangle\rightarrow|2\rangle$ and is given by $P(\omega_1)=Nd_{12}\rho_{12}$. The coherent field E_P^I enters through mirror $M1$, interacts with the atomic sample of length L, circulates in the cavity, and partially comes out from the mirror $M2$ as E_P^T. The probe field at the start of atomic sample is $E_P(0)$ and propagates to the end of the atomic sample to be $E_P(L,t)$ in a single pass. The field boundary conditions in this configuration are

$$E_P^T(t) = \sqrt{T}E_P(L,t),$$

$$E_P(0,t) = \sqrt{T}E_P^I(t) + RE_P(L,t-\Delta t), \quad (4)$$

where Δt is the time for light to travel from $M2$ to $M1$ and the cavity detuning is assumed to be zero. It should be noted that it is the feedback mechanism due to the mirrors for the nonlinear atomic medium, which is responsible for the bistable behavior, e.g., we do not expect any bistability when $R=0$ in Eq. (4) above. In the three-level case, the coupling field can enter the cavity through a polarizing beam splitter and copropagates with the cavity field in the atomic cell as in Ref. [11]. The diameter of the coupling beam is assumed to be much larger than the cavity field.

It is rather difficult to have $P(\omega_1)$ in a simple analytic form in steady state for the three-level atomic system in comparison with a two-level case. Therefore, we solve the density matrix equations numerically [15] and integrate Maxwell's equation (3) over the length of the sample together with the boundary conditions to get the results for OB under various parametric conditions. It should be noticed that in the limiting condition of $\Omega_C\rightarrow 0$, this system reduces to the ordinary two-level atomic system.

We now present details of our numerical studies. In Fig. 3 we plot the bistable behavior of the three-level V-type system subjected to the effect of quantum interference. For this pur-

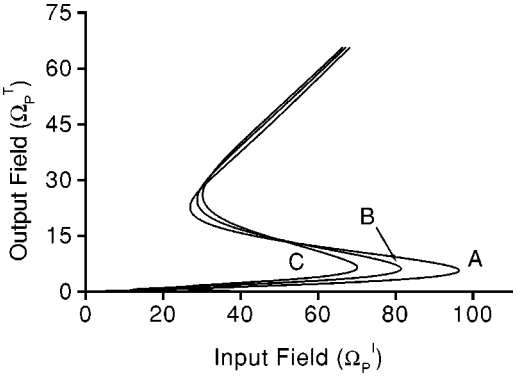

FIG. 3. The input-output field characteristics of the optical cavity field for different values of quantum interference. The parametric conditions are $\gamma_2/\gamma_1=1$, $\Omega_C/\gamma_1=10$, $\Delta_C/\gamma_1=-4.1$, $\Delta_P/\gamma_1=0$, $\delta/\gamma_1=-0.1$, and $C=400$. Curves *A*, *B*, and *C* are for $\eta=0.0$, 0.5, and 0.99, respectively.

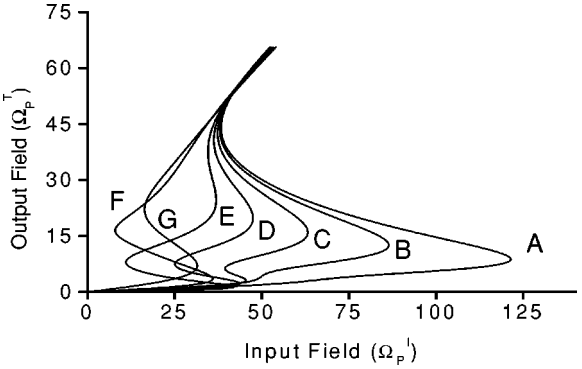

FIG. 5. The input-output field characteristics of the optical cavity field for different values of coupling field strength. The parametric conditions are $\gamma_2/\gamma_1=1$, $\Delta_C/\gamma_1=-1.0$, $\eta/\gamma_1=0$, $\Delta_P/\gamma_1=10.0$, $\delta/\gamma_1=-1.0$, and $C=200$. Curves *A*, *B*, *C*, *D*, *E*, *F*, and *G* are for $\Omega_C/\gamma_1=1$, 3, 5, 7, 9, 11, and 20, respectively.

pose, we have selected some typical parametric conditions, e.g., $\gamma_2/\gamma_1=1$, $\Omega_C/\gamma_1=10$, $\Delta_C/\gamma_1=-4.1$, $\Delta_P/\gamma_1=0$, $\delta/\gamma_1=-0.1$, and $C=400$. The cooperativity parameter C for atoms in a ring cavity is defined as $C=\alpha L/2T$ (in which αL represents single-pass absorption by atomic medium). Curves *A*, *B*, and *C* are for $\eta=0$ (no quantum interference), $\eta=0.5$ and $\eta=0.99$ (large quantum interference), respectively. Clearly, the quantum interference reduces the bistabilty threshold (the point where transition to upper branch takes place), which can be easily explained by reduction in effective saturation intensity since quantum interference suppresses the radiative decay rate from $|2\rangle$ to $|1\rangle$. By keeping all the parameters same as in Fig. 3 except with $\delta/\gamma_1=0$ and redrawing curves *A*, *B*, and *C* in Fig. 4 we get yet another change in the bistable behavior. We do observe reduction of threshold as we go from curve *A* ($\eta=0$) to curve *B* ($\eta=0.5$) due to the quantum interference. But when $\eta=0.99$ (curve *C*), a multistable pattern emerges. For the two-level atoms, the atomic polarization responsible for OB is a ratio of polynomials of first order in Ω_P (in the numerator) and second order in Ω_P (in the denominator). However, the order of these polynomials can go higher (order 5 in numerator and order 6 in denominator) for three-level atoms depending on the relative strengths of various parameters associated with the three-level atoms [9,16]. The observed OM has certainly the roots in this complicated form of polarization $P(\omega_1)$ in terms of the probe field amplitude Ω_P.

One of the advantages of three-level system over a two-level one is the additional controllability offered by the coupling field. By adjusting the coupling field strength, one can alter the absorption and nonlinear optical properties of the atomic medium for the cavity field and, therefore, change the steady-state behaviors. Figure 5 shows another way to achieve OM by varying the coupling field strength without the quantum interference in the two decay channels. So, the situation is like two-level atoms subjected to an additional coupling field on the second channel operating on the common lower level. We have kept the system parameters as $\gamma_2/\gamma_1=1$, $\Delta_C/\gamma_1=-1.0$, $\Delta_P/\gamma_1=10$, $\delta/\gamma_1=-1.0$, and $C=200$. The curves *A*, *B*, *C*, *D*, *E*, *F*, and *G*, are for $\Omega_C/\gamma_1=1$, 3, 5, 7, 9, 11, and 20, respectively. As the coupling field strength increases, threshold for OB decreases because the coupling field will modify the absorption and enhances the nonlinearity of the atomic medium, which makes the cavity field easier to reach saturation. When the coupling field becomes too large, the multistable behavior disappears again due to the reduction in nonlinearity in such case. The origin of OM is same as discussed above and this time it is occurring without the presence of any quantum interference effect in the decay channels.

In summary, we have demonstrated the controllability of atomic OB by using the theoretical model of three-level atoms in V-configuration inside an optical ring cavity. The controlling parameters are the quantum interference in the decay channels whose tunability has been experimentally demonstrated and the coupling field intensity. The possibilities of obtaining OM are also discussed, both by quantum interference and by tuning coupling field strength, which indicate the rich and interesting phenomena in this three-level atomic system inside an optical cavity.

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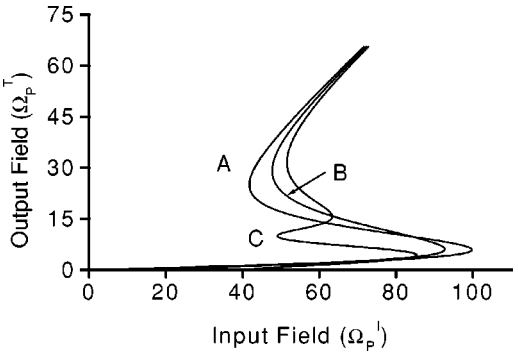

FIG. 4. The input-output field characteristics of the optical cavity field for different values of quantum interference. The parametric conditions are same as in Fig. 3, except $\delta/\gamma_1=0$. Curves *A*, *B*, and *C* are for $\eta=0.0$, 0.5, and 0.99, respectively.

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