Diffraction phases in atom interferometers

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Diffraction of atoms by lasers is a very important tool for matter wave optics. Although the process is well understood, the phase shifts induced by this diffraction process are not well known. In this paper, we make analytical calculations of these phase shifts in some simple cases and use these results to model the contrast interferometer recently built by Pritchard and co-workers. We thus show that the values of the diffraction phases are large and that they probably contribute to the phase noise observed in this experiment.

I. INTRODUCTION

In atom interferometry, laser diffraction is a very powerful and versatile tool (for overviews, see Refs. [1,2]). The diffraction of matter waves by a standing light wave was proposed by Kapitza and Dirac [3] in the case of electrons and generalized to atoms by Altshuler et al. [4]. Atom diffraction by light has been studied theoretically [5,6] and experimentally [7,8], and this early work was followed by many studies too numerous to be quoted here. The phases of the diffraction amplitudes are rarely discussed in detail, with a few exceptions like the work of Weitz et al. [9] and of Featonby et al. [10], in both cases for Raman adiabatic transfer, and the work of Bordé and co-workers [11,12], which analyzes the general diffraction process in the rotating wave approximation. Unfortunately, this approximation cannot be used for the elastic diffraction studied here.

In an interferometer, the diffraction phases modify the interference signals but this effect is difficult to detect, as it requires accurate phase measurements and it cancels in symmetric interferometers, like the Mach-Zehnder interferometer. The goal of this paper is to present an analytical calculation of diffraction phases in a simple case (elastic diffraction by a laser standing wave) and to show the importance of these diffraction phases in an existing experiment. We consider here diffraction in the Raman-Nath regime and second order Bragg diffraction in the weak field regime, and we apply these results to the contrast interferometer built by Gupta et al. [13]. The calculated diffraction phases are large in this interferometer, and as these phases depend sensitively on the laser power density used for the diffraction process, our calculation may explain the observed phase noise as resulting from fluctuations of this power density.

II. THE PROBLEM

We consider diffraction of slow ground state atoms by a near-resonant laser standing wave of frequency ω_L. For a sufficiently large laser detuning $\delta = \omega_L - \omega_0$, where ω_0 is the resonance transition frequency, the probability of real excitation is negligible and the diffraction process is coherent. In the dressed-atom picture [14], the laser standing wave creates a light shift potential $V(x,t)$:

$$V(x,t) = V_0(t)\cos^2(k_L x)$$
$$= \frac{V_0(t)}{4}[2 + \exp(+2ik_L x) + \exp(-2ik_L x)], \quad (1)$$

where the envelope $V_0(t)$ is proportional to the laser power density divided by the frequency detuning δ, and k_L is the laser wave vector. We are going to forget the x-independent term, which simply shifts the energy zero and therefore has no effect, as long as all atoms experience the same potential. The motion along the y and z directions is free and will not be discussed. The natural energy unit is the atom recoil energy $\hbar\omega_{rec} = \hbar^2 k_L^2/2m$, and we will measure the potential with this unit, by defining $q(t)$ [15,16] as

$$q(t) = V_0(t)/(4\hbar\omega_{rec}). \quad (2)$$

Using a dimensionless time τ defined by $\tau = \omega_{rec}t$, a dimensionless spatial coordinate $X = k_L x$, and a dimensionless wave vector $\kappa = k_x/k_L$, the one-dimensional (1D) Schrödinger equation becomes

$$i\frac{\partial\Psi}{\partial\tau} = -\frac{\partial^2\Psi}{\partial X^2} + q(\tau)[\exp(2iX) + \exp(-2iX)]\Psi. \quad (3)$$

For a constant value of the potential q, the atom eigenstates are Bloch states [17,15,16]. Writing the Hamiltonian matrix corresponding to Eq. (3) in the basis $|\kappa\rangle$ of plane waves of momentum $\hbar\kappa$ and using numerical diagonalization, we get the band structure $\varepsilon(\kappa,p)$, with the pseudomomentum κ belonging to the first Brillouin zone ($-1 < \kappa \leq 1$) and the integer p labeling the bands [16]. Figure 1 presents the energy of the lowest Bloch states as a function of κ for two values of the potential, $q=0$ and $q=1$, with two important features: when q is not equal to zero, band gaps
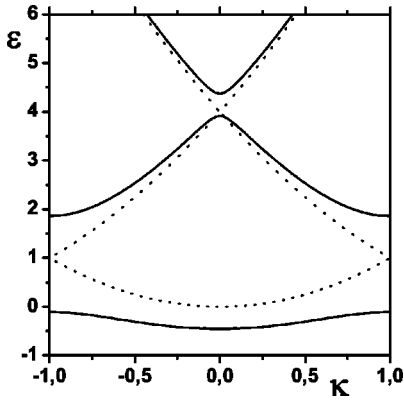
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FIG. 1. Plots of the energies ε of the lowest Bloch states versus the pseudomomentum κ: solid line, $q=1$; dashed line, $q=0$.

appear at each crossing of the $q=0$ folded parabola and energy shifts appear at the same time. These energy shifts are explained by perturbation theory: each free plane wave $|\kappa\rangle$ is coupled to two other states $|\kappa\pm2\rangle$, and the two coupling terms are equal. As the energy denominator is larger for the coupling to the upper state, all the levels are pushed upward (except near the places where gaps open), but the lowest Bloch state is obviously pushed downward.

III. DIFFRACTION PHASES

In order to simplify the calculations, we consider that the atom is initially in a state of zero momentum, $|\psi(\tau=0)\rangle = |0\rangle$. We first consider diffraction in the Raman-Nath regime. This approximation consists in neglecting the dynamics of the atom during the diffraction process produced by a pulse $q(\tau)$ of duration τ_{RN}. This approximation is good if the potential $q(\tau)$ is intense, $q\gg1$, and if the pulse is brief, $\tau_{RN}\ll1$. The validity range of this approximation is given by [16,15]

$$\tau_{RN} < 1/(4\sqrt{q}) \tag{4}$$

and the diffracted wave is a classic result:

$$|\psi(\tau_{RN})\rangle = \sum_p (-i)^{|p|} J_{|p|}(\gamma) |2p\rangle \tag{5}$$

with $\gamma = 2q\tau_{RN}$. We have verified [16] that the Raman-Nath formula accurately predicts the diffraction probability of order 0 and 1, for finite values of the parameter q, as long as condition (4) is satisfied, but we have not tested the phases of these diffraction amplitudes. They could be tested by using the diffraction amplitudes calculated [18] as a power series of $1/q$.

Second order Bragg diffraction is due to the indirect coupling of the $|\pm2\rangle$ free states, through the $|0\rangle$ state. As this coupling is a second order term in q, to make a consistent treatment, we must consider the five lowest-energy states, with $\kappa=0,\pm2,\pm4$. The Hamiltonian matrix has the nonvanishing elements $\langle 2p|H|2p\rangle = 4p^2$ and $\langle 2p|H|2(p\pm1)\rangle = q$.

Up to second order in q, the energy correction of the $|0\rangle$ state is $E_0 = -q^2/2$, and the effective Hamiltonian coupling the states $|-2\rangle$ and $|+2\rangle$ is

$$H_{eff} = \begin{bmatrix} 4+(q^2/6) & (q^2/4) \\ (q^2/4) & 4+(q^2/6) \end{bmatrix}. \tag{6}$$

We have tested the quality of this expansion limited to the q^2 terms, by numerical diagonalization of the Hamiltonian matrix. The neglected terms (in q^4, etc.) are of the order of 1% (10%) of the q^2 terms if $q=0.3$ ($q=1$), thus giving an idea of the validity range of this calculation.

The dynamics is adiabatic if the potential $q(\tau)$ varies slowly, but diffraction remains possible when two free states are degenerate, like the $|\pm2\rangle$ states. The problem is equivalent to a Rabi oscillation exactly at resonance, for which an exact solution is available for any function $q(\tau)$. For a pulse extending from τ_1 to τ_2, the Rabi phase φ_r at the end of the pulse is given by

$$\varphi_r = \int_{\tau_1}^{\tau_2} (q^2/2) d\tau, \tag{7}$$

and if $|\psi(\tau_1)\rangle = |\pm2\rangle$ the final state is

$$|\psi(\tau_2)\rangle = e^{\{-i[4(\tau_2-\tau_1)+(\varphi_r/3)]\}} \left[\cos\left(\frac{\varphi_r}{2}\right) |\pm2\rangle \right.$$
$$\left. - i \sin\left(\frac{\varphi_r}{2}\right) |\mp2\rangle \right] \tag{8}$$

where the phase shift of the $|\pm2\rangle$ states due to their mean energy shift has been expressed as a fraction of the Rabi phase. When $|\psi(\tau_1)\rangle = |0\rangle$, the final state is the $|0\rangle$ state with an extra phase shift, also due to its energy shift:

$$|\psi(\tau_2)\rangle = e^{i\varphi_r} |0\rangle. \tag{9}$$

From now on, we consider a $\varphi_r = \pi$ pulse. If the wave function at time τ_1 is given by

$$|\psi(\tau_1)\rangle = \sum_{p=-2,0,+2} a_p(\tau_1) |p\rangle, \tag{10}$$

the wave function at time τ_2 is given by

$$|\psi(\tau_2)\rangle = e^{i\pi} a_0(\tau_1) |0\rangle + e^{[-4i(\tau_2-\tau_1)-(5i\pi/6)]} [a_{-2}(\tau_1) |+2\rangle + a_{+2}(\tau_1) |-2\rangle]. \tag{11}$$

The phase factor $\exp[-4i(\tau_2-\tau_1)]$ is due to the free propagation of the $|\pm2\rangle$ states and is not linked to the diffraction process. The interesting results are the diffraction phases equal to $(+\pi)$ for the $|0\rangle$ state and $(-5\pi/6)$ for the $|\pm2\rangle$ states. The opposite signs of the diffraction phases are a consequence of the opposite signs of the energy shifts of these levels. In the resulting phase difference, the level shift contribution, equal to $4\pi/3$, is proportional to the Rabi phase φ_r, taken equal to π. In an experiment, the phase difference
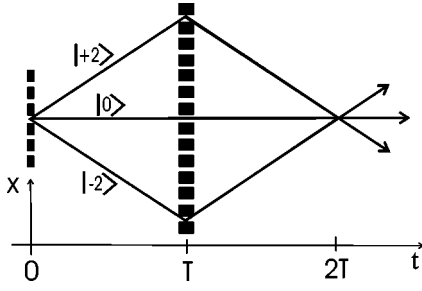
FIG. 2. In the (x,t) plane, representation of the atomic paths followed by the wave packets in the interferometer of Gupta *et al.* [13]: Raman-Nath diffraction at time $t=0$, second order Bragg diffraction at time $t=T$, detection near time $t=2T$.

may differ from this calculated value, as a result of an imperfect π pulse or of other effects neglected here (e.g., $\kappa \neq 0$).

IV. SIMPLE MODEL OF THE CONTRAST INTERFEROMETER OF GUPTA *et al.*

We now calculate the output signal of the contrast interferometer developed by Gupta *et al.* [13]. This interferometer uses second order Bragg diffraction and Raman-Nath diffraction, and the atomic paths are represented in Fig. 2. The initial state is a Bose-Einstein condensate, approximated here by a $|\kappa=0\rangle$ state. A first intense and brief pulse from $\tau=0$ to τ_{RN} is used to diffract this initial state in three coherent states $|0\rangle, |\pm 2\rangle$. Within the Raman-Nath approximation, the wave function for τ_{RN} is given by

$$|\psi(\tau_{RN})\rangle = J_0|0\rangle - iJ_1[|+2\rangle + |-2\rangle], \qquad (12)$$

the argument γ of the Bessel functions being omitted for compactness. The best contrast [13] would be obtained with diffraction probabilities equal to 50% for the $|0\rangle$ state and 25% for each of the $|\pm 2\rangle$ states. It is impossible to perfectly satisfy these two conditions simultaneously, as the first one implies $\gamma=1.13$ whereas the second one implies $\gamma=1.21$. We can nevertheless suppose that $\gamma \approx 1.17$. Although $J_2(1.17) \approx 0.15$, we will neglect here the second order diffraction amplitudes, as was done in Ref. [13]. We assume that τ_{RN} is negligible so that free propagation starts at $\tau=0$ and lasts until the Bragg diffraction pulse, which extends from τ_1 to τ_2. Using Eq. (11), we get the wave function after this pulse:

$$|\psi(\tau_2)\rangle = J_0 e^{i\pi}|0\rangle + J_1 e^{-4i\tau_2} e^{-4i\pi/3}[|+2\rangle + |-2\rangle]. \tag{13}$$

Free propagation goes on until a time τ where the matter grating formed by the interference of these three states is read by the reflection of a laser beam. The atomic density as a function of X and τ is deduced from the wave function:

$$|\langle X|\psi(\tau)\rangle|^2 = J_0^2 + 2J_1^2[1+\cos(4X)]$$

$$+ 4J_0 J_1 \cos(2X)\cos\left(4\tau+\frac{7\pi}{3}\right). \tag{14}$$

The experimental signal $S(\tau)$ is the intensity of the light reflected by this grating. This homodyne detection signal is proportional to the square of the $\cos(2X)$ modulation of the atomic density, with the following time dependence:

$$S(\tau) \propto \cos^2\left(4\tau+\frac{7\pi}{3}\right), \tag{15}$$

while the equation used by Gupta *et al.* is

$$S(\tau) \propto \sin^2(4\tau). \tag{16}$$

The difference between Eqs. (15) and (16) is important only if one wants to make an absolute prediction of the phase, but it has no consequence in the analysis carried out by Gupta *et al.* [13], because their fitted value of ω_{rec} comes from the derivative of the phase with the time interval T [19]. However, our result remains interesting as it may explain a large part of the observed phase noise, 200 mrad from shot to shot. In the $7\pi/3$ phase of Eq. (15), $4\pi/3$ is proportional to the Rabi phase, which is itself proportional to q^2, i.e., to the square of the laser power density during the Bragg pulse. Therefore, a 1% variation of the laser power density changes the diffraction phase by 84 mrad.

Our calculation relies on several approximations, some of them being not very accurate in the experimental conditions of Gupta *et al.* [13].

(i) The $\kappa=0$ approximation is an oversimplification but the calculation with $\kappa \neq 0$ is more complex.

(ii) The first diffraction pulse used in the experiment is 1 μs long, corresponding to $\tau_{RN}=0.157$. Assuming $\gamma \approx 1.17$, we get $q \approx 3.7$ and the validity condition (4) requires $\tau \leq 0.13$. Therefore, the corrections to the Raman-Nath phases are not fully negligible. We have also neglected the second order diffraction beams, which contribute to the signal.

(iii) As for the perturbation expansion used to describe Bragg diffraction, the π pulse used is a Gaussian with a width of 7.6 μs [13]. Assuming that $q=q_{max}\exp[-(t-T)^2/(2\sigma_t^2)]$, with $\sigma_t=3.8$ μs, i.e., $\sigma_\tau \approx 0.6$, we get the value $q_{max} \approx 2.4$, well outside the validity range of our second order perturbation expansion. Higher-order terms in q^n with $n=4,6,\ldots$ contribute to the phases and the sensitivity of the diffraction phase to the laser power density may even be larger than predicted above.

Obviously, to describe this experiment very accurately, a full numerical modelization is needed and is feasible, as the problem reduces to a 1D Schrödinger equation, if atom-atom interactions are neglected. But, as noted by Gupta *et al.*, the mean field effect of the condensate can also modify atomic propagation and this effect has not been considered here.

V. CONCLUSION

In this paper, we have made a simple and tutorial calculation of the phase shifts of atomic waves due to the elastic diffraction process by a laser standing wave. We have calculated the associated phase shift for the contrast interferometer of Pritchard and co-workers [13], thus showing that it should

be possible to make an experimental test of the dependence of the diffraction phase shifts on potential strength and interaction time. The present calculations are simple because of our assumptions: the Raman-Nath limit or perturbative regime, and vanishing initial momentum $\kappa = 0$. An accurate modelling of a real experiment requires numerical integration of the Schrödinger equation to describe the diffraction dynamics without any approximation.

We have considered only first and second order diffraction. Higher diffraction orders up to order 8 have been observed [20–22] with moderate laser power densities. The leading term of the coupling matrix element responsible for diffraction order n behaves like q^n [20], whereas the leading terms of the energy shifts, responsible for the diffraction phase shifts, are always in q^2. Therefore, for diffraction orders $n > 2$, the control of the phase shifts will require a full knowledge of the pulse shape. For the second order of diffraction, the diffraction phase shifts and the Rabi phase are simply related, as long as second order perturbation theory is a good approximation.

We have made a systematic use of atomic Bloch states to describe atom diffraction by a laser, following our previous paper [16]. The introduction of Bloch states to describe atoms in a laser standing waves is due to Letokhov and Minogin [23,24] in 1978 and also to Castin and Dalibard [25] in 1991. Their use is rapidly expanding, in particular to treat Bose-Einstein condensates in an optical lattice, as reviewed by Rolston and Phillips [26]. When coupled with reduced units as here, the atomic Bloch states represent a very efficient tool to get a simple understanding of the diffraction process.

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