

Zero-temperature damping of Bose-Einstein condensate oscillations by vortex-antivortex pair creation

Petr O. Fedichev,^{1,2} Uwe R. Fischer,¹ and Alessio Recati^{1,3}

¹*Institut für Theoretische Physik, Leopold-Franzens-Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria*

²*Russian Research Center Kurchatov Institute, Kurchatov Square, 123182 Moscow, Russia*

³*Dipartimento di Fisica, Università di Trento and BEC-INFN, I-38050 Povo, Italy*

(Received 21 January 2003; published 22 July 2003)

We investigate vortex-antivortex pair creation in a supersonically expanding and contracting quasi-two-dimensional Bose-Einstein condensate at zero temperature. For sufficiently large-amplitude condensate oscillations, pair production provides the leading dissipation mechanism. The condensate oscillations decay in a nonexponential manner, and the dissipation rate depends strongly on the oscillation amplitude. These features allow one to distinguish the decay due to pair creation from other possible damping mechanisms. An experimental observation of the predicted oscillation behavior of the superfluid gas provides a direct confirmation of the hydrodynamical analogy of quantum electrodynamics and quantum vortex dynamics in two spatial dimensions.

DOI: 10.1103/PhysRevA.68.011602

PACS number(s): 03.75.Kk, 03.75.Lm

The process of electron-positron pair creation is well established in quantum electrodynamics since the seminal work of Schwinger [1]. Later on, it became apparent that the hydrodynamics of vortices in two-dimensional (2D) superfluids can be mapped onto 2+1D electrodynamics with vortices playing the role of charged particles, and phonons the role of photons [2]. In this analogy, the superfluid density and the supercurrent act as the magnetic and electric fields on the vortices whose circulation is the charge. The Schwinger vacuum breakdown is a phenomenon occurring whenever the electric field exceeds the magnetic field (in cgs units), which corresponds in the analogy to the instability of a supersonic flow with respect to the spontaneous creation of vortex-antivortex pairs from the superfluid vacuum.

Vortices in Bose-Einstein condensates have been observed and studied experimentally intensely in the past couple of years, e.g., in Refs. [3–7]. Here, we suggest an experiment in a quasi-2D Bose-condensed gas revealing the existence of irreversible condensate dynamics at zero temperature as the result of the Schwinger pair-creation instability. To argue that vortex-antivortex pair creation is the dominant source of dissipation, we use the fact that a quasi-2D Bose-Einstein condensate (BEC) in a time-dependent harmonic trap has a peculiar feature: There is a time dependent transformation (the so-called “scaling transformation”), using which the problem can be exactly solved, because the time dependence can effectively be removed from the Hamiltonian [8–10]. This scaling property also holds for linearized equations describing the evolution of small density and phase perturbations (Bogoliubov quasiparticle excitations), propagating on top of the moving superfluid. Therefore, initial perturbations cannot grow and the instability mechanisms known from classical hydrodynamics play no role [11]. This stability against perturbations implies that at very low temperatures, condensate oscillations are practically undamped (it has been measured that the quality factor $Q \gtrsim 2000$ [12]).

On the other hand, the superfluid velocity in the scaling solution grows linearly towards the condensate border, while the local density decreases. Therefore, the outer region of the

cloud is always supersonic and, according to the Landau criterion for superfluidity, can host instabilities. Vortices are nonlinear excitations above the superfluid ground state, so that they are not protected by the scaling symmetry, which holds for linear excitations. Spontaneous vortex-antivortex pair creation, analogous to the Schwinger process, is an *intrinsic* instability mechanism and constitutes a source of dissipation already at zero temperature, without any need for a symmetry-breaking external perturbation.

In the following, we explicitly analyze the Schwinger instability of a supersonically expanding and contracting BEC in a time-dependent quasi-2D harmonic trap. We show that for sufficiently large condensate oscillations, vortex-antivortex pair production provides the dominant dissipation mechanism. Furthermore, the condensate oscillations decay in a nonexponential manner and the dissipation rate depends strongly on the oscillation amplitude. These features allow one to distinguish experimentally the decay due to pair creation from the previously studied damping mechanisms. We note that the suggested zero temperature damping mechanism is intrinsically different from that discussed in [10], where the dissipation is due to the energy transfer from the radial condensate motion to the longitudinal modes in an elongated cylindrically symmetric condensate. This mechanism can only work if the condensate is sufficiently long, whereas we confine ourselves to the case of a quasi-2D sample, for which any motion along the z axis is suppressed.

The analogy of 2D vortex dynamics with electrodynamics is most easily established by noting that the expression for the 2D Magnus force $\mathbf{F}_M = 2\pi\rho\mathbf{e}_z \times (\dot{\mathbf{X}} - \mathbf{v}_s)$ leads to the identification of $\mathbf{E} = \rho\mathbf{v}_s \times \mathbf{e}_z$ and $\mathbf{B} = -\rho\mathbf{e}_z$ with the “electric” and “magnetic” fields, by comparing with the Lorentz force $\mathbf{F}_L = q(\mathbf{E} + \dot{\mathbf{X}} \times \mathbf{B})$. Here, $\dot{\mathbf{X}}$ and \mathbf{v}_s are vortex and local superflow velocities, respectively, and ρ is the local density. The circulation (2π in our units with $\hbar = m = 1$) is the “charge” q (cf., e.g., Refs. [13–15]). The self-energy of a (widely separated) single vortex pair is $2E_v^0 = 2\pi\rho\Lambda$, with $\Lambda = \ln(R/a_c)$, where R is the size of the pair. We will use in

this expression for the pair energy that the vortex core size in a dilute superfluid is given by $a_c = 1/c_s$, where c_s is the speed of sound. The inertial rest mass of a vortex, stemming from compressibility, $m_v = E_v^0/c_s^2$, is (for large condensates) of “electrodynamical” origin: It stems from the self-interaction of a moving vortex, with the long-range flow and density fields it induces inside the surrounding superfluid medium. Since the “electromagnetic” fields (the density and velocity perturbations) represent “relativistic” particles (phonons), the vortex mass diverges if the velocity of the accelerated vortex approaches the speed of sound, in the same manner in which the mass of a charged ultrarelativistic particle diverges in conventional electrodynamics. We assume in what follows that other possible contributions to the vortex mass (see, e.g., the backflow mass contribution discussed in [16]) remain regular if the local superfluid velocity approaches the speed of sound. These contributions are therefore subdominant for relativistically moving vortices.

We consider a quasi-2D superfluid Bose gas in a time-dependent isotropic harmonic trapping potential $V(\mathbf{x}, t) = \frac{1}{2}\omega^2(t)(x^2 + y^2)$, with $\mathbf{x} = (x, y)$. It is a well-established fact that the hydrodynamic solution for density and velocity of motion in a harmonic potential with arbitrary time dependence may be obtained from a given initial solution by a scaling procedure [8,9]. Defining the scaled coordinate vector $\mathbf{r}_b = \mathbf{x}/b$, the rescaled density and velocity are given by

$$\rho(\mathbf{x}, t) = \frac{1}{b^2} \sigma(\mathbf{r}_b) = \frac{\rho_0}{b^2} \left(1 - \frac{r_b^2}{R_0^2} \right), \quad (1)$$

$$\mathbf{v}_s(\mathbf{x}, t) = \frac{\dot{b}}{b} \mathbf{x}. \quad (2)$$

Here, we assume the superfluid to be described initially within the Thomas-Fermi (TF) approximation (that is, the condensate is large enough to neglect the quantum pressure), ρ_0 is the initial central density, and R_0 the initial TF radius, such that $R = R(t) = b(t)R_0$ is the instantaneous TF radius of the cloud. The energy functional has in the TF approximation the form

$$\mathcal{E}(b, \dot{b}) = \frac{1}{2b^2} \int d^2\mathbf{r}_b \left[\left(\omega^2 + \frac{\dot{b}^2}{b^2} \right) b^4 r_b^2 \sigma + g \sigma^2 \right], \quad (3)$$

with g the interaction strength, which depends in the present quasi-2D case on the tight confinement in the z direction and on the density of the condensate [17]. This leads to an effective Hamiltonian for the dynamical variable b ,

$$\mathcal{E}(b, \dot{b}) = \left(\frac{\alpha}{2} \dot{b}^2 + \frac{\alpha}{2} \omega^2(t) b^2 + \frac{\beta}{2b^2} \right), \quad (4)$$

where $\alpha = \pi\rho_0 R_0^4/6$ and $\beta = \pi\rho_0^2 g R_0^2/3$.

Consider a situation in which the external trap frequency is changed from ω_{in} to $\omega_f \ll \omega_{\text{in}}$, on a time scale much less than the inverse initial trap frequency. As a consequence, the gas undergoes a large-amplitude monopole oscillation with frequency $2\omega_f$ [18]. At sufficiently low temperatures (below

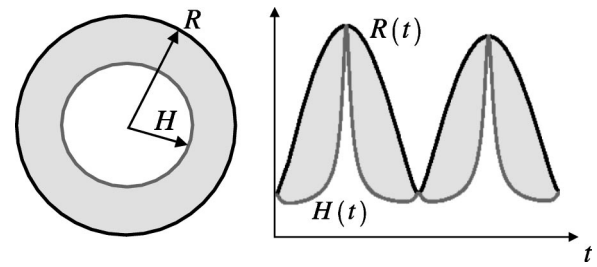


FIG. 1. An oscillating, cylindrically symmetric quasi-2D condensate. The shaded region designates the region of space in which the speed of sound is exceeded by the oscillating condensate, and vortex pair creation takes place; $H = H(t)$ is the horizon location and $R = R(t)$ the Thomas-Fermi radius of the condensate.

the Kosterlitz-Thouless temperature), the initial state of the superfluid contains bound vortex-antivortex pairs, i.e., topological excitations, which can be unbound by the action of the Magnus force in the (time dependent) supersonic flow region. Indeed, for an oscillating condensate, there exists a region, the border of which is called horizon (cf. Fig. 1), where the superfluid velocity magnitude v_s is larger than the local sound velocity $c_s = \sqrt{g\rho}$. The speed of sound is exceeded at the *horizon radius*

$$H(t) = \frac{b(t)R_0}{\sqrt{\gamma^2(t) + 1}}, \quad (5)$$

where $\gamma = \sqrt{2} \dot{b} b / \omega_{\text{in}}$.

Beyond the horizon, the vortices and antivortices get accelerated during condensate evolution and separate at local superflow velocities larger than that of sound. This is analogous to the Schwinger pair-creation process in quantum electrodynamics. It is important to recognize that the flow we consider is inhomogeneous and time dependent by default. Consequently, the argument that there is no pair-creation possible because one can always use the underlying Galilean invariance to “transform away” the background flow, does not apply to our situation.

In a simple model of the 2+1D vacuum pair-creation instability, which exploits directly the analogy to Schwinger pair-creation in quantum electrodynamics, the pair production rate Γ per unit area can be written as [19]

$$\Gamma = \frac{1}{4\pi^2 c_s^2} \mathcal{F}^{3/4} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{3/2}} \exp\left(-\frac{\pi n (E_v^0)^2}{\sqrt{\mathcal{F}}}\right), \quad (6)$$

where we have defined $\mathcal{F} = E^2 c_s^2 - B^2 c_s^4$ and set, within logarithmic accuracy, the vortex pair size in E_v^0 equal to the Thomas-Fermi radius of the condensate. The above relation holds for locally supersonic motion, i.e., if $|\mathbf{E}|/|\mathbf{B}| > c_s$ ($\mathcal{F} > 0$). The value of the prefactor in front of the exponential in the above expression is subject to changes which are due to the microscopic details of vortex motion. We display its value, stemming from taking literally the analogy to quantum electrodynamics also on the level of quantum fluctuations (to one loop order), for numerical concreteness. However, the behavior of Γ for $|\mathbf{E}|/|\mathbf{B}| \gtrsim c_s$ is dominated by the hydrody-

namical exponent, whose value is independent of microscopic physics, and specifically by the $n=1$ term in the above sum.

Assuming that the vortex density is low, the energy dissipation rate $\dot{\epsilon}$ is obtained by multiplying Eq. (6) by the rest energy of the widely separated vortices $2E_v^0$, and integrating over the area of the TF domain. This results in

$$\dot{\epsilon} = \frac{g^{1/2} \Lambda \rho_0^3 R_0^2}{2b^4} \frac{\gamma^8}{(\gamma^2 + 1)^{13/4}} F\left(\frac{\Lambda^2}{g} \frac{1}{\sqrt{\gamma^2 + 1}}\right), \quad (7)$$

where we introduced the function

$$F(\lambda) = \sum_n \frac{(-1)^{n+1}}{n^{3/2}} \int_0^1 d\eta \eta^{3/4} (1-\eta)^{9/4} \times \exp[-\pi n \lambda \sqrt{(1-\eta)/\eta}]. \quad (8)$$

Since γ is proportional to $b\dot{b}$, the Schwinger dissipation rate (7) can give rise to a measurable effect only if the condensate oscillation amplitude is sufficiently large, which implies $\omega_f \ll \omega_{\text{in}}$. In order to provide some analytical results, we consider a simple quasistationary perturbation-theory approach. Consequently, we assume that the dissipation rate is small and therefore the energy of the system in Eq. (3) is a slowly varying function within each oscillation period. Then, the equation of motion for the scaling parameter b can be found from

$$\frac{d}{dt} \mathcal{E}(b, \dot{b}) = -\dot{\epsilon}. \quad (9)$$

In the absence of dissipation ($\dot{\epsilon}=0$), the range of b is between $b_{\text{min}}=1$ and $b_{\text{max}}=\omega_{\text{in}}/\omega_f$. Since $b_{\text{min}}=1 \ll b_{\text{max}}$, we can approximately set $b_{\text{min}} \approx 0$. One can then write $\gamma^2 = 2(\omega_f^2/\omega_{\text{in}}^2)b^2(b_{\text{max}}^2 - b^2)$. In a dilute gas, in the TF limit, the argument of F is large, $\Lambda^2 \gg g\sqrt{\gamma^2 + 1}$, and the dynamical equation (9) for b takes the simpler form

$$\ddot{b} + \omega_f^2 b - \frac{\omega_{\text{in}}^2}{b^3} = -\frac{\mathcal{D}}{\omega_{\text{in}}^5} \dot{b}^7 b^4, \quad (10)$$

where the constant

$$\mathcal{D} = \frac{48}{\pi^8} \frac{g^{7/4} \sqrt{gN}}{(\ln[4\sqrt{gN/\pi}])^{11/2}} \sum_n \frac{(-1)^{n+1}}{n^8}. \quad (11)$$

Using Eq. (10), $b\dot{b}$ can be expressed in terms of b only, $b^2 \dot{b}^2 = \omega_f^2 b^2 (b_{\text{max}}^2 - b^2)$. The oscillation energy lost in a period is then given by

$$I_E = \frac{7\pi}{1024} \mathcal{D} \frac{\omega_f^7}{\omega_{\text{in}}^5} b_{\text{max}}^{12}. \quad (12)$$

The energy decrease rate for b_{max} is obtained from the equation

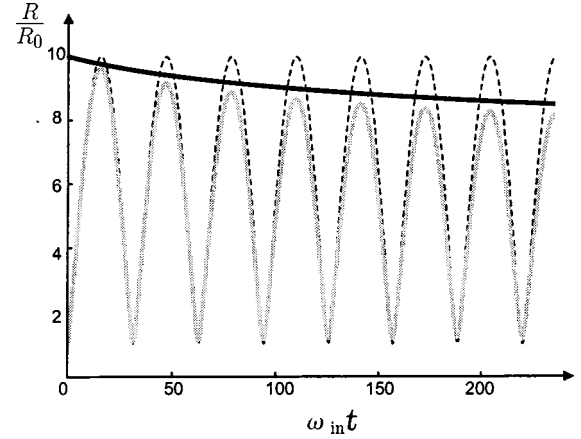


FIG. 2. Damping of condensate oscillations due to vortex-antivortex pair creation, with $N=10^4$, $g=1$, and $\omega_f=0.1\omega_{\text{in}}$, where the radius R is in units of the original Thomas-Fermi size R_0 . The black solid line is the envelope b_{max} from Eq. (14). The gray solid line is the damped breathing mode oscillation obtained from numerically solving Eq. (10). For comparison, the dashed line represents the oscillation of the superfluid gas without pair creation taking place.

$$\frac{d}{dt} \mathcal{E}(b_{\text{max}}) = \frac{\omega_f}{\pi} I_E. \quad (13)$$

Thus, one obtains for the oscillation peak value the following expression:

$$b_{\text{max}}(t) = \frac{b_{\text{max}}(0)}{[1 + \mathcal{D}' b_{\text{max}}^{10}(0) \omega_f t]^{1/10}}, \quad (14)$$

where $\mathcal{D}' = \frac{35}{512} (\omega_f/\omega_{\text{in}})^5 \mathcal{D}$. Our perturbation-theory approach is valid as long as $b_{\text{max}}^{10}(0) \mathcal{D}' \ll 1$.

The damping of condensate oscillations due to vortex-antivortex pair creation is represented in Fig. 2, where we show the numerical solution of the dynamical equation (10) (gray solid line), the approximate solution for the peak amplitude (black solid line), and for comparison the free oscillation without pair creation (dashed line). The parameters used in the numerical integration for the plot are $N=10^4$, $g=1$, and for the final trapping frequency $\omega_f=0.1\omega_{\text{in}}$. These parameters are consistent with the argument of F being large, $\Lambda^2 \gg g\sqrt{\gamma^2 + 1}$, so that Eqs. (10)–(14) hold. The envelope $b_{\text{max}}(t)$ is seen to decay very slowly and in a non-exponential manner, governed by the TF exponent $\frac{1}{10}$ in Eq. (14). For realistic parameters, we conclude from Fig. 2 that an observable damping effect for the condensate oscillations is obtained.

The scaling parameter evolution can be described by Eq. (9) only for sufficiently short times when the total density of vortices produced is still low. At later times, the vortex-antivortex plasma can decrease the superfluid current in the same way as the electron-positron plasma can screen the electric field. This is an interesting collective effect, which however requires a more elaborate treatment.

We described an intrinsic damping mechanism for large-amplitude condensate oscillations in a quasi-2D Bose gas at

zero temperature. The dissipation originates from spontaneous creation of vortex-antivortex pairs and depends on the shape and dynamics of the supersonic flow region. The results we presented therefore depend strongly on the oscillation amplitude. This feature can be used to distinguish the effects of pair production from other possible dissipation mechanisms. The scaling solution not only exists for the discussed monopole modes, but also for quadrupole oscillations, so that, e.g., effects resulting from a rotating superfluid on the pair-creation process may be studied. Observation of the predicted oscillation behavior of the superfluid gas provides a direct confirmation of the hydrodynamical analogy of quantum electrodynamics and quantum vortex dynamics in two spatial dimensions, and would put this analogy to its first real experimental test. Such confirmation would, then, give further motivation to the program of studying analogies between high-energy physics, cosmology, and condensed-matter systems [20].

The outlined mechanism for dissipation is not confined to quasi-2D samples. In strong elongated 3D condensates, the scaling solution also applies, and the vorticity is generated in the form of vortex rings, with the total vorticity integrated over the sample volume still zero. However, as already mentioned above, for a 3D condensate the effect of vortex ring creation can be masked by possibly stronger damping mechanisms, such as the parametric resonance discussed in Ref. [10].

We thank L. P. Pitaevskii and G. E. Volovik for critical remarks and helpful comments on the manuscript, and P. Zoller for discussions. P.O.F. has been supported by the Austrian Science Foundation FWF and the Russian Foundation for Basic Research, U.R.F. by the FWF, and A.R. by the European Union under Contract No. HPRN-CT-2000-00125.

-
- [1] J. Schwinger, Phys. Rev. **82**, 664 (1951).
 [2] V.N. Popov, Zh. Eksp. Teor. Fiz. **64**, 672 (1973) [Sov. Phys. JETP **37**, 341 (1973)].
 [3] K.W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. **84**, 806 (2000).
 [4] C. Raman, J.R. Abo-Shaer, J.M. Vogels, K. Xu, and W. Ketterle, Phys. Rev. Lett. **87**, 210402 (2001).
 [5] J.R. Abo-Shaer, C. Raman, J.M. Vogels, and W. Ketterle, Science **292**, 476 (2001).
 [6] P. Engels, I. Coddington, P.C. Haljan, and E.A. Cornell, Phys. Rev. Lett. **89**, 100403 (2002).
 [7] P. Rosenbusch *et al.*, Phys. Rev. Lett. **88**, 250403 (2002).
 [8] Yu. Kagan, E.L. Surkov, and G.V. Shlyapnikov, Phys. Rev. A **54**, R1753 (1996).
 [9] Y. Castin and R. Dum, Phys. Rev. Lett. **77**, 5315 (1996).
 [10] Yu. Kagan and L.A. Maksimov, Phys. Rev. A **64**, 053610 (2001); e-print, cond-mat/0212377.
 [11] L.D. Landau and E.M. Lifshitz, *Fluid Mechanics* (Pergamon Press, New York 1959).
 [12] F. Chevy, V. Bretin, P. Rosenbusch, K.W. Madison, and J. Dalibard, Phys. Rev. Lett. **88**, 250402 (2002).
 [13] R.J. Donnelly, *Quantized Vortices in Helium II* (Cambridge University Press, Cambridge, 1991).
 [14] D.P. Arovas and J.A. Freire, Phys. Rev. B **55**, 1068 (1997).
 [15] U.R. Fischer, Ann. Phys. (N.Y.) **278**, 62 (1999).
 [16] G. Baym and E. Chandler, J. Low Temp. Phys. **50**, 57 (1983).
 [17] D.S. Petrov, M. Holzmann, and G.V. Shlyapnikov, Phys. Rev. Lett. **84**, 2551 (2000).
 [18] The existence of this mode in two spatial dimensions is related to an underlying SO(2,1) symmetry of the system, see L.P. Pitaevskii and A. Rosch, Phys. Rev. A **55**, R853 (1997).
 [19] R. Iengo and G. Jug, Phys. Rev. B **52**, 7537 (1995).
 [20] G.E. Volovik, Phys. Rep. **351**, 195 (2001); *The Universe in a Helium Droplet* (Oxford University Press, Oxford, 2003).