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Mechanical stability of a strongly interacting Fermi gas of atoms

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A strongly attractive, two-component Fermi gas of atoms exhibits universal behavior and should be mechanically stable as a consequence of the quantum-mechanical requirement of unitarity. This requirement limits the maximum attractive force to a value smaller than that of the outward Fermi pressure. To experimentally demonstrate this stability, we use all-optical methods to produce a highly degenerate, two-component gas of ^{6}Li atoms in an applied magnetic field near a Feshbach resonance, where strong interactions are observed. We find that gas is stable at densities far exceeding that predicted previously for the onset of mechanical instability. Further, we provide a temperature-corrected measurement of an important, universal, many-body parameter, which determines the stability—the mean-field contribution to the chemical potential in units of the local Fermi energy.

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Strongly interacting Fermi systems are expected to exhibit universal behavior [1]. In atomic gases, such strong forces can be produced in the vicinity of a Feshbach resonance, where a bound molecular state in a closed exit channel is magnetically tuned into coincidence with the total energy of a pair of colliding particles [2]. In this case, the zero-energy scattering length a_S, which characterizes the interactions at low temperature, can be tuned through $\pm\infty$. For very large values of $|a_S|$, the important properties of the system (e.g., the effective mean-field potential, the collision rate, the superfluid transition temperature, etc.) are predicted to lose their dependence on the magnitude and sign of a_S, and instead become proportional to the Fermi energy with different universal proportionality constants. For this reason, tabletop experiments with strongly interacting atomic Fermi gases can provide measurements that are relevant to all strongly interacting Fermi systems [1,3], thus impacting theories in intellectual disciplines outside atomic physics, including materials science and condensed matter physics (superconductivity), nuclear physics (nuclear matter), high-energy physics (effective theories of the strong interactions), and astrophysics (compact stellar objects).

In a gas of degenerate fermions, compression of the gas is resisted by the Pauli exclusion principle, leading to an effective pressure known as the Fermi pressure or the degeneracy pressure. The Fermi pressure plays an important role in nature, providing, for example, the outward force that stabilizes neutron stars against gravitational collapse. Because of the Fermi pressure, a confined cloud of degenerate fermions is always larger than a cloud of bosons with an equivalent temperature and particle number. This effect has been directly observed in two elegant experiments [4,5].

An important question is: At what point, if at all, do strong attractive interactions overcome the Fermi pressure? Previously, it was predicted that an atomic Fermi gas could become mechanically unstable for sufficiently large attractive interactions [6]. If true, this prediction presents a possible roadblock to attempts to use a Feshbach resonance to produce a high-temperature superfluid [7–9], as the gas should be unstable in the required regime. Recently, however, it has become apparent that the gas may indeed be stable. Heiselberg has estimated the maximum ratio of the attractive potential to the local Fermi energy for a strongly interacting Fermi gas. Using a self-consistent many-body approach, he finds that a two-component Fermi gas is mechanically stable, although a gas with more than two components is not [1]. A similar conclusion can also be drawn from an examination of the compressibility of a strongly attractive two-component gas [9].

In this paper, we first show heuristically that the maximum inward force arising from attractive interactions in a two-state gas of atomic fermions is limited by quantum mechanics to a value less than the outward Fermi pressure. Then, we demonstrate this idea experimentally by directly cooling a two-component Fermi gas of ^{6}Li atoms to high degeneracy in an optical trap at magnetic fields near a Feshbach resonance, where strong interactions are observed [10]. The highest densities obtained in the experiments far exceed those predicted previously [6] for the onset of mechanical instability. Imaging the cloud both in the trap and after an abrupt release, we find no evidence of instability. To quantitatively describe the stability, we present a measurement of the relevant universal many-body parameter β—the mean-field contribution to the chemical potential in units of the local Fermi energy. In Ref. [10], we provided a first estimate of β assuming a zero-temperature gas. Here, we present a method to properly account for the nonzero temperature of the gas. This revised measurement of β is in good agreement with a prediction of Ref. [1].

The mechanical stability of the gas can be understood through a heuristic discussion of the forces acting on a gas of fermions in a 50-50 mixture of two spin components. The equation of state for a normal, zero-temperature Fermi gas is [11]

$$\epsilon_F(\mathbf{x})+U_{\mathrm{MF}}(\mathbf{x})+U_{\mathrm{trap}}(\mathbf{x})=\mu, \tag{1}$$

where μ is the chemical potential, $\epsilon_F(\mathbf{x})$ is the local Fermi energy, $U_{\mathrm{MF}}(\mathbf{x})$ is the mean-field contribution to the chemical potential, and $U_{\mathrm{trap}}(\mathbf{x})$ is the trap potential. In the local-density approximation, $\epsilon_F(\mathbf{x})=\hbar^2k_F^2(\mathbf{x})/(2M)$, where k_F is

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the local Fermi wave vector, which is related to the density according to $n(\mathbf{x})=k_F^3(\mathbf{x})/(6\pi^2)$ [12].

For each spin component, there is an effective outward force [13] arising from the Fermi pressure $P_{\mathrm{Fermi}}=2n(\mathbf{x})\,\epsilon_F(\mathbf{x})/5$, even in the absence of interatomic interactions. The outward force per unit volume is $-\boldsymbol{\nabla}P_{\mathrm{Fermi}}$. The corresponding force per particle is $-(\boldsymbol{\nabla}P_{\mathrm{Fermi}})/n$. The local force per particle is then easily shown to be

$$\mathbf{F}_{\mathrm{Fermi}}=-\boldsymbol{\nabla}\epsilon_F(\mathbf{x}). \tag{2}$$

Since the density decreases with distance from the center, $\mathbf{F}_{\mathrm{Fermi}}$ is outward.

Two-body scattering interactions make a contribution U_{MF} to the chemical potential, which is given in the mean-field approximation by

$$U_{\mathrm{MF}}(\mathbf{x})=\frac{4\pi\hbar^2 a_{\mathrm{eff}}}{M}n(\mathbf{x}), \tag{3}$$

where a_{eff} is an effective scattering length, which generally depends on the local thermal average of a momentum-dependent scattering amplitude. The potential is attractive when $a_{\mathrm{eff}}<0$, and repulsive when $a_{\mathrm{eff}}>0$. The local force per particle arising from the mean-field contribution is just

$$\mathbf{F}_{\mathrm{MF}}=-\boldsymbol{\nabla}U_{\mathrm{MF}}(\mathbf{x}). \tag{4}$$

If we assume that a_{eff} is energy independent and equal to a_S for $a_S<0$, it is easy to show that the gas becomes unstable for suitably large values of a_S. In this case, the inward force from the mean-field potential $\mathbf{F}_{\mathrm{MF}}\propto\boldsymbol{\nabla}n(\mathbf{x})$, while the outward force from the Fermi pressure, $\mathbf{F}_{\mathrm{Fermi}}\propto\boldsymbol{\nabla}n^{2/3}(\mathbf{x})$. Then, the inward force exceeds the outward when $|\mathbf{F}_{\mathrm{MF}}|>|\mathbf{F}_{\mathrm{Fermi}}|$, i.e., when $k_F|a_{\mathrm{eff}}|>\pi/2$. The corresponding density for mechanical instability satisfies $n>n_0$, where $n_0=\pi/(48\,|a_S|^3)$. This result matches the previous prediction of Ref. [6], which was derived via a more rigorous calculation.

The assumption that a_{eff} is energy independent, however, is only valid if $k_F\,|a_S|\ll 1$. Outside this regime, this assumption violates the quantum-mechanical requirement of unitarity. At intermediate densities [1], where $a_S\gg k_F^{-1}\propto n^{-1/3}\gg R$, with R the range of the collision potential, two-body scattering is dominant, but the mean-field interaction is proportional to a momentum-dependent two-body T-matrix element, which in turn is proportional to the scattering amplitude $f(k)$. It is well known that f has a magnitude which is limited by unitarity to a maximum of $1/k$ [14]. Hence, one expects that in a zero-temperature Fermi gas, one should use an effective scattering length in Eq. (3) with a maximum magnitude of the order of $|a_{\mathrm{eff}}|=1/k_F$. For $a_{\mathrm{eff}}=-1/k_F$, it is easy to show that the mean-field contribution to the chemical potential is [10]

$$U_{\mathrm{MF}}(\mathbf{x})=\beta\epsilon_F(\mathbf{x}), \tag{5}$$

where the universal parameter β has a maximum value of $-4/(3\pi)=-0.42$ in this heuristic treatment. In this limit, both the inward force $\mathbf{F}_{\mathrm{MF}}$ and the outward force $\mathbf{F}_{\mathrm{Fermi}}$ are proportional to the same power of the atomic density. The ratio of their magnitudes, however, is given by $|\mathbf{F}_{\mathrm{MF}}|/|\mathbf{F}_{\mathrm{Fermi}}|=|\beta|<1$. Thus, one finds that a two-component Fermi gas is mechanically stable as a result of the quantum-mechanical requirement of unitarity. This is in contrast to attractive Bose gases [15,16] and Bose-Fermi mixtures [17], which exhibit dramatic instabilities.

A possible criticism of our heuristic argument is that the two-body relative wave number k lies in the range $0\leqslant k\leqslant k_F$. Since f increases as k decreases, $f(k)$ should be averaged over $W(\mathbf{k})$, the probability distribution for $\mathbf{k}$, where $\int d\mathbf{k}\,W(\mathbf{k})=1$. For a noninteracting gas at zero temperature, one can show

$$W(\mathbf{k})=\frac{6}{\pi k_F^3}\left[1-\frac{3}{2}\frac{k}{k_F}+\frac{1}{2}\left(\frac{k}{k_F}\right)^3\right]\Theta(k_F-k), \tag{6}$$

where $k=|\mathbf{k}|$. Near a Feshbach resonance, where $|a_S|\gg R$, we assume that $f(k)$ takes the form expected for a zero-energy resonance, and that the effective scattering length is determined by the real part of $-f$, i.e., $a_{\mathrm{eff}}=\langle a_S/(1+k^2a_S^2)\rangle$, where $\langle\cdots\rangle$ denotes averaging with Eq. (6). We find that $|a_{\mathrm{eff}}|$ has a maximum value of $1.05/k_F$ when $k_F|a_S|=2$, and decreases slowly for $k_F|a_S|>1$, reaching $0.6/k_F$ at $k_F|a_S|=10$.

Our heuristic estimate, which neglects the effects of the interactions on the free particle wave functions, yields a value of β comparable to that estimated by Heiselberg [1]. In contrast to our heuristic approach, the self-consistent many-body approach of Ref. [1] also predicts that β is independent of the sign and magnitude of k_Fa_S in the intermediate density limit, where $k_F\,|a_S|\gg 1$. Hence, for strongly interacting fermions, β is always negative and the effective mean-field interaction should always be attractive.

To demonstrate that a strongly attractive, two-component Fermi gas of atoms is stable, we employ a 50-50 mixture of the two lowest hyperfine states of ^{6}Li, i.e., $|F=1/2,M=\pm 1/2\rangle$ states in the low field basis. This mixture has a predicted broad Feshbach resonance at 860 G [18,19]. A magnetic field of 910 G is applied to produce a very large and negative zero-energy scattering length.

The gas mixture is prepared and rapidly cooled to degeneracy at 910 G by forced evaporation in our ultrastable CO_2 laser trap, as described for our previous experiments [10]. For our trap, $\omega_\perp=2\pi\times(6625\pm 50\ \mathrm{Hz})$, $\omega_z=2\pi\times(230\pm 20\ \mathrm{Hz})$, and $\bar{\omega}\equiv(\omega_\perp^2\omega_z)^{1/3}=2\pi\times(2160\pm 65\ \mathrm{Hz})$. Following evaporation, the trap is recompressed to full depth over 0.5 s, and allowed to remain for an additional 0.5 s, to ensure thermal equilibrium. The CO_2 laser power is then extinguished and the gas is imaged as described in our previous paper [10]. After release, the gas expands hydrodynamically, rapidly increasing in the transverse dimension while remaining nearly stationary in the axial direction [10].

We determine the number of atoms by numerically integrating the column density (Table I). For a typical number, $N\simeq 8.0\times 10^4$ atoms per state, the corresponding Fermi density for the experiments is calculated to be $n_F\simeq 4.8\times 10^{13}/\mathrm{cm}^3$ per state [12]. The density n_F, can be compared

TABLE I. Value of β: t is the expansion time, N is the number of atoms per state, σ_{xF} is Fermi radius without the mean-field interaction, and $\sigma_x(t)$ is the measured Fermi radius obtained from the finite-temperature fit to the data using Eq. (7).

t (μs)	N	σ_{xF} (μm)	T/T_F	$\sigma_x(t)$ (μm)	β
600	66,000	3.49	0.128	99	-0.237
	86,400	3.65	0.144	102	-0.220
	84,300	3.63	0.140	101	-0.234
	80,200	3.60	0.141	101	-0.208
800	67,200	3.50	0.146	133	-0.176
	87,000	3.65	0.179	131	-0.344
	70,400	3.53	0.151	130	-0.273
	82,000	3.62	0.183	128	-0.382

to the density $n_0 = \pi/(48|a_S|^3)$ predicted for the onset of mechanical instability, assuming a momentum-independent scattering length. Using the best available molecular potentials, which are constrained by measurements of the zero crossing in the s-wave scattering length [19,20], the zero-energy scattering length a_S is estimated to be $\simeq -10^4 a_0$ at 910 G ($a_0 = 0.53\times 10^{-8}$ cm), within a factor of 2 [21]. Then $n_0 = 4.4\times 10^{11}/\text{cm}^3$, showing that the density n_F exceeds n_0 by one to two orders of magnitude.

Although the molecular potentials are constrained by the measured zero crossing in the scattering length [19,20], the precise location of the Feshbach resonance still may be incorrect. Since the scattering length has not been directly measured, it is possible that the attractive potential is not as large as expected. However, a simple argument based on our previous experiments shows that a_S must be large [10]. We observe similar hydrodynamic expansion after release from either full trap depth U_0 or a reduced trap depth of $U_0/100$. In the latter case, we estimate that $k_F^{-1} = 4000 a_0$. Since the observed hydrodynamic expansion appears to be independent of the trap depth, as expected for a unitarity limited interaction, we conclude that in the shallow trap we must have $|k_F a_S| > 1$. Then, we must have $|a_S| > 4000 a_0$ and $n_0 \leqslant 6.9\times 10^{12}/\text{cm}^3$.

Thus, we have clearly demonstrated that the gas is mechanically stable in the intermediate density regime, where $|a_S|$ is large compared to the interparticle spacing. The remainder of this paper provides a revised estimate of the universal parameter β that quantitatively determines the stability of the gas. In our previous estimate [10], we assumed a zero-temperature gas and calculated β from an estimate of the release energy. There, we obtained a value of β, substantially smaller in magnitude than predicted [1]. Here, we determine β from the transverse spatial widths of the expanding gas and properly include both finite-temperature and hydrodynamic scaling effects.

One-dimensional transverse spatial profiles after expansion for 0.4–0.8 ms are very well fit by normalized finite-temperature Thomas-Fermi (T-F) distributions, which determine the width σ_x as well as the ratio of the temperature to the Fermi temperature, T/T_F. We expect that T/T_F is approximately constant during the expansion, since we observe hydrodynamic scaling of the transverse radii consistent with

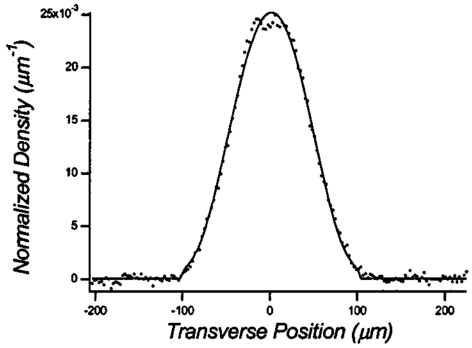

FIG. 1. Transverse spatial profile of the expanding cloud at 600 μs after release.

an effective potential $\propto n^{2/3}$ [10], suggesting an adiabatic process. The T-F shape is not unreasonable despite a potentially large mean field interaction, since the mean-field contribution to the chemical potential, Eq. (5) is proportional to the local Fermi energy. In this case, assuming Eq. (1) for a normal degenerate gas at zero temperature [11], it is easy to show that the mean field should simply scale the Fermi energy of the trapped cloud without changing the shape from that of a T-F distribution [10]. The zero-temperature spatial distribution then corresponds to that of a harmonic oscillator potential, with the frequencies scaled so that $\omega_i' = \omega_i/\sqrt{1+\beta}$, $i = x,y,z$. Hydrodynamic expansion then preserves the shape of the trapped atom spatial distribution [10,11]. Assuming that T/T_F is small, one expects that a finite-temperature spatial distribution for a harmonic potential is a reasonable approximation. A Sommerfeld expansion [22] of the normalized density for $|x| \leqslant \sigma_x$ yields

$$n(x)/N = \frac{16}{5\pi\sigma_x}[f_0(x) + 5\pi^2(T/T_F)^2 f_2(x)], \qquad (7)$$

where $f_0(x) = g^5(x)$, $f_2(x) = g(x)/8 - g^3(x)/6$, and $g(x) = \sqrt{1 - x^2/\sigma_x^2}$.

Equation (7) is used to fit the measured transverse spatial distributions, Fig. 1. We obtain the results given in Table I for four trials each at expansion times t of 0.6 ms and 0.8 ms. Note that Eq. (7) begins to break down near $|x| \simeq \sigma_x$ for $T/T_F > 0.15$. However, an exact treatment using polylogarithm functions yields similar results even for our highest temperatures, where $T/T_F \simeq 0.18$.

The value of the parameter β can be determined from the transverse radii of the trapped cloud, $\sigma_x(0) = \sqrt{2\epsilon_F'/M\omega_\perp'^2}$, where $\epsilon_F' = \hbar\bar{\omega}'(6N)^{1/3}$ is the Fermi energy including the mean-field contribution. Then, $\sigma_x(0) = (1+\beta)^{1/4}\sigma_{xF}$ [23], where $\sigma_{xF} = \sqrt{2\epsilon_F/(M\omega_\perp^2)}$, and $\epsilon_F = \hbar\bar{\omega}(6N)^{1/3}$ is the Fermi energy in the absence of interactions. As we have pointed out previously, the observed anisotropic expansion can arise from unitarity-limited collisional hydrodynamics or superfluid hydrodynamics [10]. In either case, the effective potential is $\propto n^{2/3}$, and we can assume $\sigma_x(t) = \sigma_x(0) b_x(t)$ [10,11]. Hence,

$$\beta = \left(\frac{\sigma_x(t)}{b_x(t)\sigma_{xF}} \right)^4 - 1. \quad (8)$$

For our trap parameters, $b_x(0.6\ \text{ms}) = 29.74$ and $b_x(0.8\ \text{ms}) = 39.88$. The scale factor $b_x(t)$ properly includes the spatial anisotropy of the expansion and the correct hydrodynamic scaling. We obtain from Table I an average value $\beta = -0.26 \pm 0.07$.

This result is consistent with that obtained from the measured transverse release energy. The release energy can be initially calculated from the zero-temperature T-F fits [10] or calculated numerically from the second moment of the density. Since $T/T_F \simeq 0.15$ from the Table I, the measured release energy is larger than the zero-temperature release energy by a factor $\eta \simeq 1 + (2\pi^2/3)(T/T_F)^2 = 1.15$. Reducing the release energy by a factor η and using the method of Ref. [10] yields an average value of β consistent with that given above.

The measured average value of $\beta = -0.26 \pm 0.07$ can be compared with predictions of the energy per particle for the conditions of our experiment, where $k_F a_S = -7.4$. For our equation of state, the sum of the local kinetic and mean-field energy per particle is $(3/5)(1+\beta)\epsilon_F(\mathbf{x})$ [24]. In Ref. [1], Heiselberg provides two estimates for the total energy per particle. His Eq. (11), based on the Galitski equations, yields $\beta_{\text{calc}} = -0.54$ [25]. Using the Wigner-Seitz cell approximation, his Eq. (14) provides an alternative estimate, $\beta_{\text{calc}} = -0.33$. An independent calculation by Steele [3] using effective-field theory yields $\beta = -0.46$. The theoretical predictions are for zero temperature, and the second is in reasonable agreement with our measurements including the temperature correction to the spatial distributions. However, the predictions do not include the additional temperature dependence of β, which arises from a thermal average. Since this important universal many-body parameter can now be experimentally measured, further refinement of both the experimental measurements and the theory is worthwhile and may permit the observation of the superfluid corrections to β. In addition, mixtures of the three lowest hyperfine states of ^{6}Li may permit observation of predicted mechanical instabilities in a three-component Fermi gas [1].

Note added in proof. Recently, other groups have presented preliminary measurements of the mean field energy for two-component mixtures of atomic fermions in the strongly interacting regime. The group of Salomon measures $\beta \simeq -0.3$ near the peak of the broad Feshbach resonance in ^{6}Li [26], while the group of Ketterle observes a cancellation of the mean field shifts from neighboring Feshbach resonances in ^{6}Li [27]. Both results support the possibility of a universal attractive mean field interaction near a Feshbach resonance. Measurements of positive and negative scattering lengths for a Feshbach resonance in ^{40}K by the group of Jim demonstrate the effects of unitarity in limiting the magnitude of the scattering length, but universal behavior is not observed [28].

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[1] H. Heiselberg, Phys. Rev. A **63**, 043606 (2001).
[2] E. Tiesinga, B.J. Verhaar, and H.T.C. Stoof, Phys. Rev. A **47**, 4114 (1993).
[3] J.V. Steele, e-print nucl-th/0010066 (2000).
[4] A.G. Truscott *et al.*, Science **291**, 2570 (2001).
[5] F. Schreck *et al.*, Phys. Rev. Lett. **87**, 080403 (2001).
[6] M. Houbiers *et al.*, Phys. Rev. A **56**, 4864 (1997).
[7] M. Holland *et al.*, Phys. Rev. Lett. **87**, 120406 (2001).
[8] E. Timmermans *et al.*, Phys. Lett. A **285**, 228 (2001).
[9] S.J.J.M.F. Kokkelmans *et al.*, Phys. Rev. A **65**, 053617 (2002).
[10] K.M. O'Hara *et al.*, Science **298**, 2179 (2002).
[11] C. Menotti, P. Pedri, and S. Stringari, Phys. Rev. Lett. **89**, 250402 (2002).
[12] D.A. Butts and D.S. Rokhsar, Phys. Rev. A **55**, 4346 (1997).
[13] Note that Fermi energy in the chemical potential of a noninteracting trapped gas produces an effective outward force, which balances the inward trap force to determine the local density. However, a noninteracting gas expands ballistically after release from the trap.
[14] L.D. Landau and E.M. Lifshitz, *Quantum Mechanics* (Cambridge University Press, Oxford, 1977).
[15] C.A. Sackett *et al.*, Phys. Rev. Lett. **82**, 876 (1999).
[16] J.L. Roberts *et al.*, Phys. Rev. Lett. **86**, 4211 (2001).
[17] G. Modugno *et al.*, Science **297**, 2240 (2002).
[18] M. Houbiers *et al.*, Phys. Rev. A **57**, R1497 (1998).
[19] K.M. O'Hara *et al.*, Phys. Rev. A **66**, 041401(R) (2002).
[20] S. Jochim *et al.*, Phys. Rev. Lett. **89**, 273202 (2002).
[21] Vanessa Venturi (private communication).
[22] N.W. Ashcroft and N.D. Mermin, *Solid State Physics* (Holt, Rinehart, and Winston, Philadelphia, 1976).
[23] The β dependence of the cloud radii was independently derived by S. Stringari (private communication).
[24] Note that the equation of state is obtained from the total energy by functional differentiation with respect to the density, yielding the minimum energy state. S. Stringari (private communication).
[25] Note that $6\pi/35$ in the denominator of Eq. (11) of Ref. [1] should read $6/(35\pi)$, H. Heiselberg (private communication).
[26] T Bourdel *et al.*, e-print cond-mat/0303079.
[27] S. Gupta (unpublished).
[28] C. Regal and D. Jin, e-print cond-mat/0302246