Time-correlated quantum amplitude-damping channel

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We analyze the problem of sending classical information through qubit channels where successive uses of the channel are correlated. This work extends the analysis of Macchiavello and Palma to the case of a non-Pauli channel—the amplitude damping channel. Using the channel description outlined by Daffer *et al.*, we derive the correlated amplitude damping channel. We obtain a result similar to that obtained by Macchiavello and Palma, that is, under certain conditions on the degree of channel memory, the use of entangled input signals may enhance the information transmission compared to the use of product input signals.

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Unlike purely classical channels, quantum channels admit more than one well-defined information carrying capacity [1]. For instance, one can transmit classical strings $(x_1, \ldots, x_m)$, with x_i elements from an alphabet set $\mathcal{A} = \{1, \cdots, n\}$, by sending product input states of the form $\pi_{x_1} \otimes \cdots \otimes \pi_{x_m}$, where π_{x_i} are density operators acting on Hilbert space $\mathcal{H}_i$, via a memoryless [10], generally noisy quantum channel Φ, a completely positive, trace preserving map on density operators: $\pi_{x_i} \to \rho_{x_i} = \Phi(\pi_{x_i})$. As in the classical information theory [2], one can define an input probability distribution, $0 \leqslant q_j \leqslant 1$, $\Sigma_j q_j = 1$, on the input states π_j, that is, one can consider an input ensemble $\mathcal{E} = \{q_j, \pi_j\}$, described by $\pi = \Sigma_j q_j \pi_j$. For the above product input states, $\pi_j = \pi_{x_1} \otimes \cdots \otimes \pi_{x_m}$ and $q_j = q_{x_1} \cdots q_{x_m}$, where $0 \leqslant q_{x_i} \leqslant 1$, $\Sigma_{x_i=1}^n q_{x_i} = 1$ is a probability distribution defined on input states π_{x_i}. Measurements are performed on output states $\rho = \Phi(\pi)$ to determine the output probability distribution. The ''distance'' between these distributions is given by the Shannon mutual information for m uses of the quantum channel $I_m(\mathcal{E}, \Phi)$. The classical information capacity of the quantum channel is defined by [3,4] $C = \lim_{m\to\infty} C_m/m$, where $C_m \equiv \sup_{\mathcal{E}} I_m(\mathcal{E}, \Phi)$. By definition, C_m satisfy the following superadditivity property, $C_{m_1} + C_{m_2} \leqslant C_{m_1+m_2}$, and the above limit can be shown to exist. Physically, C represents the largest per symbol amount of classical information that can be transferred reliably using the quantum channel. In Refs. [3,4], Holevo, Schumacher, and Westmoreland have shown that

$$I_m(\mathcal{E}, \Phi) = S(\Phi(\pi)) - \sum_j q_j S(\Phi(\pi_j)), \tag{1}$$

where $S(\sigma) = -\mathrm{tr}(\sigma \log_2 \sigma)$ is the von Neumann entropy of density operator σ expressed in bits. One can recover the classical information case [2] by considering input states $\pi_{x_i} = |\psi_{x_i}\rangle\langle\psi_{x_i}|$, one-dimensional orthonormal projectors. In this case, $C_m = mC_1$ and thus $C = C_1$. In the quantum information theory, one can consider the more general class of entangled input states. This implies the possibility of strict superadditivity of the mutual information and, consequently, $C_1 < C$ [5]. However, there is no such strict superadditivity, at least, for the quantum depolarizing channel [6].

In real physical quantum transmission channels, it is common to have correlated noise acting on consecutive uses. The problem of the classical capacity of quantum channels with time correlated noise was first considered by Macchiavello and Palma [5]. They analyzed the specific case of sending qubits (quantum states belonging to two-dimensional Hilbert spaces, each spanned by orthonormal vectors $\{|0\rangle, |1\rangle\}$) with two consecutive uses of a quantum depolarizing channel with partial memory [5,7]: $\pi \to \rho = \Phi(\pi) = (1-\mu)\Sigma_{i,j=0}^3 D_{ij}^u \pi D_{ij}^{u\dagger} + \mu \Sigma_{k=0}^3 D_{kk}^c \pi D_{kk}^{c\dagger}$, where $0 \leqslant \mu \leqslant 1$. With probability $(1-\mu)$, the noise is uncorrelated and completely specified by the Kraus operators $D_{ij}^u = \sqrt{p_i p_j}\sigma^i \otimes \sigma^j$, while with probability μ the noise is correlated and specified by $D_{kk}^c = \sqrt{p_k}\sigma^k \otimes \sigma^k$. Here, $0 \leqslant p \leqslant 1$, $p_0 = (1-p)$, $p_1 = p_2 = p_3 = \frac{1}{3}p$, and $\sigma^0, \sigma^1, \sigma^2, \sigma^3$ are the identity and the Pauli matrices, respectively. They considered the following equally weighted ensemble of orthonormal input states:

$$\pi = \tfrac{1}{4}(|\pi_1\rangle\langle\pi_1| + |\pi_2\rangle\langle\pi_2| + |\pi_3\rangle\langle\pi_3| + |\pi_4\rangle\langle\pi_4|), \tag{2}$$

where $|\pi_1\rangle = \cos\phi|00\rangle + \sin\phi|11\rangle$, $|\pi_2\rangle = -\sin\phi|00\rangle + \cos\phi|11\rangle$, $|\pi_3\rangle = \cos\phi|01\rangle + \sin\phi|10\rangle$, and $|\pi_4\rangle = -\sin\phi|01\rangle + \cos\phi|10\rangle$, with $0 \leqslant \phi < \pi/2$. They showed that there exists a threshold $\mu = \mu_t$ such that when $\mu > \mu_t$, one may have enhanced information carrying capacity in the case of entangled inputs.

We note that a quantum dephasing channel (Pauli Z channel) with uncorrelated noise can similarly be defined as one specified by the following Kraus operators:

$$Z_{ij}^u = \sqrt{p_i p_j}\sigma^i \otimes \sigma^j, \quad i,j = 0,3 \tag{3}$$

and one with correlated noise by

$$Z_{kk}^c = \sqrt{p_k}\sigma^k \otimes \sigma^k, \quad k = 0,3. \tag{4}$$

The same prescription can be applied to a quantum amplitude damping channel with uncorrelated noise:

$$A_{00}^u = A_0 \otimes A_0, \quad A_{01}^u = A_0 \otimes A_1, \quad A_{10}^u = A_1 \otimes A_0,$$
$$A_{11}^u = A_1 \otimes A_1, \tag{5}$$

where, with $0 \leqslant \chi \leqslant \pi/2$,

$$A_0=\begin{pmatrix}\cos\chi & 0\\ 0 & 1\end{pmatrix},\quad A_1=\begin{pmatrix}0 & 0\\ \sin\chi & 0\end{pmatrix} \tag{6}$$

are the Kraus operators for an amplitude damping channel. Here, $|0\rangle$ and $|1\rangle$ denote the excited and ground states, respectively. However, it is not *a priori* clear how the Kraus operators for a quantum amplitude damping channel with correlated noise could be constructed in a similar manner, if it is at all possible.

Recently, Daffer *et al.* [8] used a special basis of left, $\{L_i\}$, and right, $\{R_i\}$, damping eigenoperators for a Lindblad superoperator, $\Phi(\pi)=\exp(t\mathcal{L})\pi$, where t is time, to calculate explicitly the image of a completely positive, trace-preserving map for a wide class of Markov quantum channels:

$$\pi\rightarrow\rho=\Phi(\pi)=\sum_i \mathrm{tr}(L_i\pi)\exp(\lambda_i t)R_i . \tag{7}$$

For a finite N-dimensional Hilbert space, $\mathcal{L}\pi=-\frac{1}{2}\sum_{i,j=1}^{N^2-1}c_{ij}(F_j^{\dagger}F_i\pi+\pi F_j^{\dagger}F_i-2F_i\pi F_j^{\dagger})$, with the system operators F_i satisfying $\mathrm{tr}(F_i)=0$, $\mathrm{tr}(F_i^{\dagger}F_j)=\delta_{ij}$. The complex c_{ij} form a positive matrix. The right eigenoperators R_i satisfy the eigenvalue equation

$$\mathcal{L}R_i=\lambda_i R_i , \tag{8}$$

and the following duality relation:

$$\mathrm{tr}(L_iR_j)=\delta_{ij} \tag{9}$$

with the left eigenoperators L_i. The amplitude damping and dephasing channels are the examples of quantum Markov channels. The Lindblad equation [8]

$$\mathcal{L}\pi=-\tfrac{1}{2}\alpha(\sigma^{\dagger}\sigma\pi+\pi\sigma^{\dagger}\sigma-2\sigma\pi\sigma^{\dagger}), \tag{10}$$

where α is a parameter analogous to the Einstein coefficient of spontaneous emission, and $\sigma^{\dagger}\equiv\frac{1}{2}(\sigma^1+i\sigma^2)$, $\sigma\equiv\frac{1}{2}(\sigma^1-i\sigma^2)$ are the creation and annihilation operators, respectively, yields the amplitude damping channel, Eq. (6). And, the dephasing channel can be derived from [8]

$$\mathcal{L}\pi=-\tfrac{1}{2}\Gamma(\pi-\sigma^3\pi\sigma^3), \tag{11}$$

where Γ is another parameter.

In this paper, we derive Eq. (4) from the Lindblad equation (12), that is, it gives the quantum dephasing channel with correlated noise. In a similar fashion, we solve Eq. (15) and interpret the resulting completely positive, trace-preserving map as one which describes a quantum amplitude damping channel with correlated noise. Then, we analyze, as in Ref. [5], the action of a quantum amplitude damping channel with partial memory given in Eq. (22). Our results are in agreement with those of Ref. [5]; that is, the transmission of classical information can be enhanced by employing entangled states as carriers of information rather than product states.

We begin by solving the following Lindblad equation:

$$\mathcal{L}^c\pi=-\tfrac{1}{2}\Gamma[\pi-(\sigma^3\otimes\sigma^3)\pi(\sigma^3\otimes\sigma^3)]. \tag{12}$$

Equation (12) is an obvious extension of Eq. (11) with σ^3 replaced by $(\sigma^3\otimes\sigma^3)$. The rationale is that the phase-flip actions of the channel would then be correlated. The method of solution involves first determining the right eigenoperators R_{ij}, which solves Eq. (8):

$$R_{ij}=\tfrac{1}{2}\sigma^i\otimes\sigma^j,\quad i,j=0,1,2,3 \tag{13}$$

with eigenvalues $\lambda_{00}=\lambda_{11}=\lambda_{22}=\lambda_{33}=0$, $\lambda_{01}=\lambda_{10}=\lambda_{02}=\lambda_{20}=-\Gamma$, $\lambda_{03}=\lambda_{30}=0$, $\lambda_{12}=\lambda_{21}=0$, $\lambda_{13}=\lambda_{31}=\lambda_{23}=\lambda_{32}=-\Gamma$. The left eigenoperators are then determined by imposing Eq. (9). Finally, the image of the completely positive, trace-preserving map can be obtained via Eq. (7). In this case, we have $\pi\rightarrow\rho=\sum_{i,j=0}^{3}\mathrm{tr}(L_{ij}\pi)\exp(\lambda_{ij}t)R_{ij}=\sum_{k=0,3}Z_{kk}^{c}\pi Z_{kk}^{c\dagger}$, where Z_{kk}^{c} are given by Eq. (4), with

$$p\equiv\tfrac{1}{2}[1-\exp(-\Gamma t)]. \tag{14}$$

Therefore, Eq. (12) does indeed yield a Pauli Z channel with correlated noise. The Pauli X and Pauli Y channels with correlated noise can similarly be obtained by relpacing $\sigma^3\otimes\sigma^3$, in Eq. (12) with $\sigma^1\otimes\sigma^1$ and $\sigma^2\otimes\sigma^2$, respectively, and solving the resulting equations with Eq. (13), though with different eigenvalues [11]. Equation (13) also solves the following equation: $\mathcal{L}^c\pi=-\frac{1}{2}\Gamma[3\pi-(\sigma^1\otimes\sigma^1)\pi(\sigma^1\otimes\sigma^1)-(\sigma^2\otimes\sigma^2)\pi(\sigma^2\otimes\sigma^2)-(\sigma^3\otimes\sigma^3)\pi(\sigma^3\otimes\sigma^3)]$, giving a depolarizing channel with correlated noise [12].

Next, we solve the Lindblad equation

$$\mathcal{L}^c\pi=-\tfrac{1}{2}\alpha[(\sigma^{\dagger}\otimes\sigma^{\dagger})(\sigma\otimes\sigma)\pi+\pi(\sigma^{\dagger}\otimes\sigma^{\dagger})(\sigma\otimes\sigma)-2(\sigma\otimes\sigma)\pi(\sigma^{\dagger}\otimes\sigma^{\dagger})]. \tag{15}$$

This follows from the same rationale that is behind the construction of Eq. (12). By replacing σ in Eq. (10) with $(\sigma\otimes\sigma)$, we expect the actions of the resulting channel to be correlated. We call it the quantum amplitude damping channel with correlated noise. The right eigenoperators R_{ij}, which solve Eq. (8), are

$$R_{00}=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&\sqrt{2}\end{pmatrix},\quad R_{11}=\begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\quad R_{22}=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix},\quad R_{33}=\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&-1\end{pmatrix}, \tag{16}$$

$$R_{01}^{\pm}=\frac{1}{\sqrt{2}}\begin{pmatrix}0&\pm1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\quad R_{02}^{\pm}=\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&\pm1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{pmatrix},\quad R_{03}^{\pm}=\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0&\pm1\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{pmatrix},\quad R_{12}^{\pm}=\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0&0\\0&0&\pm1&0\\0&1&0&0\\0&0&0&0\end{pmatrix},\tag{17}$$

$$R_{13}^{\pm}=\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0&0\\0&0&0&\pm1\\0&0&0&0\\0&1&0&0\end{pmatrix},\quad R_{23}^{\pm}=\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&\pm1\\0&0&1&0\end{pmatrix},\tag{18}$$

with eigenvalues $\lambda_{00}=\lambda_{11}=\lambda_{22}=0$, $\lambda_{33}=-\alpha$, $\lambda_{01}^{\pm}=\lambda_{02}^{\pm}=\lambda_{03}^{\pm}=-\frac{1}{2}\alpha$, $\lambda_{12}^{\pm}=\lambda_{13}^{\pm}=\lambda_{23}^{\pm}=0$. The left eigenoperators are determined as above, and Eq. (7) becomes $\pi\to\rho=\Sigma_i \mathrm{tr}(L_i\pi)\exp(\lambda_i t)R_i=\Sigma_{j=0}^{1}A_{jj}^{c}\pi A_{jj}^{c\dagger}$, where

$$A_{00}^{c}=\begin{pmatrix}\cos\chi&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix},\quad A_{11}^{c}=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\\sin\chi&0&0&0\end{pmatrix},\tag{19}$$

with

$$\cos\chi\equiv\exp(-\tfrac{1}{2}\alpha t),\quad \sin\chi\equiv\sqrt{1-\exp(-\alpha t)}.\tag{20}$$

We note that, in contrast to Z_{00}^{c} in Eq. (4), A_{00}^{c} cannot be written as a tensor product of two two-by-two matrices. This gives rise to the typical "spooky" action of the channel: $|01\rangle$, $|10\rangle$, and any linear combination of them, and $|11\rangle$ will go through the channel undisturbed, but not $|00\rangle$.

It is interesting to note that Eq. (3) can also be derived by solving the following Lindblad equation: $\mathcal{L}^{u}\pi=-\frac{1}{2}\Gamma[\pi-(\sigma^0\otimes\sigma^3)\pi(\sigma^0\otimes\sigma^3)]-\frac{1}{2}\Gamma[\pi-(\sigma^3\otimes\sigma^0)\pi(\sigma^3\otimes\sigma^0)]$. However, analogous approach for the amplitude damping channel with uncorrelated noise does not work. This is because the amplitude damping channel is by definition nonunital. This is not surprising in view of the fact that although all Lindblad superoperators have a Kraus decomposition, the converse is not true in general.

Before we carry out the same analysis as in Ref. [5], let us characterize a parametrization of single qubit density matrix inputs for the amplitude damping channel. The following proposition is due to a private communication of Fuchs mentioned without proof in Ref. [1]:

Proposition. To calculate the one-qubit capacity of the amplitude-damping channel we need to consider only one-parameter set of pure state inputs:

$$\pi_{\pm}=\begin{pmatrix}1-x&\pm\sqrt{(1-x)x}\\\pm\sqrt{(1-x)x}&x\end{pmatrix}$$

with respective input probabilities of $\frac{1}{2}$ each.

Proof. Consider a pure state input density matrix

$$\pi=\begin{pmatrix}1-a&b^*\\b&a\end{pmatrix},$$

where a is real, $0\leqslant a\leqslant 1$, and b is, in general, complex. This is mapped to

$$\rho=\Phi(\pi)=\begin{pmatrix}(1-p)(1-a)&\sqrt{1-p}\,b^*\\\sqrt{1-p}\,b&(1-p)a+p\end{pmatrix}$$

by sending it through the amplitude-damping channel. Here, $p=\sin^2\chi$. Note that this channel is nonunital and hence we cannot adopt the approach of Ruskai and King, Ref. [9]. Instead, note that the eigenvalues of ρ are given by $\lambda_{\pm}=\frac{1}{2}\pm\frac{1}{2}\sqrt{1-4(1-p)\{(1-a)[(1-p)a+p]-|b|^2\}}$. Consequently, $S(\rho)$ depends only on b via $|b|^2$, and it is sufficient to consider only real b. We note that $S(\rho)$ will have maximum value one by taking $b^2=(1-a)[(1-p)a+p]-1/4(1-p)\leqslant 0$, which satisfy $b^2\leqslant(1-a)[(1-p)a+p]$, required by the positivity of ρ. So, when $b=0$ or $a=(1-2p)/[2(1-p)]$, $S(\rho)$ is maximal and equals one. We also note that $S(\rho)$ will have minimum value zero for fixed a, p by taking $b^2=(1-a)[(1-p)a+p]\geqslant(1-a)a$. Therefore, the mutual information $I_1(\mathcal{E},\Phi)=S(\Phi(\pi))-\Sigma_j q_j S(\Phi(\pi_j))$ is never equal to one unless $p=0$ or $a=1$ in accordance with the physics of the amplitude-damping channel. In fact, $I_1(\mathcal{E},\Phi)\leqslant\max_{x_1}S(\rho(x_1))-\Sigma_{j=1}^{2}q_j\min_{x_2}S(\rho_j(x_2))$ where $\pi=\Sigma_j q_j\pi_i$,

$$\pi(x_1)=\begin{pmatrix}1-x_1&0\\0&x_1\end{pmatrix},$$

$$\pi_j(x_2)=\begin{pmatrix}1-x_2&\pm\sqrt{(1-x_2)x_2}\\\pm\sqrt{(1-x_2)x_2}&x_2\end{pmatrix}.$$

Consequently, it is clear that by taking the inputs prescribed in the statement of the proposition, we can achieve this upper bound.

So, we consider the following set of input states for the one-qubit amplitude-damping channel: $|x\rangle=\cos(\theta/2)|0\rangle+\sin(\theta/2)|1\rangle$, $|y\rangle=\cos(\theta/2)|0\rangle-\sin(\theta/2)|1\rangle$, with $0\leqslant\theta\leqslant\pi$, which correspond to the ansatz described above. We can then obtain a parameterization for input states of the two-qubit amplitude damping channel with partial memory, and instead of Eq. (2):

$$\pi = \frac{1}{4}(|\tilde{\pi}_1\rangle\langle\tilde{\pi}_1| + |\tilde{\pi}_2\rangle\langle\tilde{\pi}_2| + |\tilde{\pi}_3\rangle\langle\tilde{\pi}_3| + |\tilde{\pi}_4\rangle\langle\tilde{\pi}_4|), \tag{21}$$

where $|\tilde{\pi}_1\rangle = \cos\phi|x\rangle|x\rangle + \sin\phi|y\rangle|y\rangle$, $|\tilde{\pi}_2\rangle = -\sin\phi|x\rangle|x\rangle + \cos\phi|y\rangle|y\rangle$, $|\tilde{\pi}_3\rangle = \cos\phi|x\rangle|y\rangle + \sin\phi|y\rangle|x\rangle$, $|\tilde{\pi}_4\rangle = -\sin\phi|x\rangle|y\rangle + \cos\phi|y\rangle|x\rangle$. Note that $|\tilde{\pi}_j\rangle$ are, in general, not normalized. For $\phi=0$, we obtain product states of the original optimal form outlined in the above ansatz. When $\theta = \pi/2$, we obtain Eq. (2). The channel action is given by

$$\pi \to \rho = \Phi(\pi) = (1-\mu)\sum_{i,j=0}^{1} A_{ij}^{u}\pi A_{ij}^{u\dagger} + \mu\sum_{k=0}^{1} A_{kk}^{c}\pi A_{kk}^{c\dagger}. \tag{22}$$

Substituting Eqs. (21) and (22) into Eq. (1) gives

$$I_2(\theta,\phi;\mu,\chi) = S(\Phi(\pi)) - \sum_{j=1}^{4} q_j S(\Phi(\pi_j)), \tag{23}$$

with $\pi_j = 1/\mathrm{tr}(|\tilde{\pi}_j\rangle\langle\tilde{\pi}_j|)|\tilde{\pi}_j\rangle\langle\tilde{\pi}_j|$ and $q_j = \frac{1}{4}\mathrm{tr}(|\tilde{\pi}_j\rangle\langle\tilde{\pi}_j|)$. We carry out a numerical study that exhibits for given values of $\phi=\phi_0$ ($\neq 0$ or $\pi/2$) and χ that there exist threshold values $\mu=\mu_t(\theta)$ for all $0<\theta<\pi$ such that $I_2(\theta,\phi=0;\mu<\mu_t,\chi) < I_2(\theta,\phi=\phi_0;\mu>\mu_t,\chi)$ and thus have shown that we set out to show, since optimizing gives $2C_1 < I_2(\theta,\phi=\phi_0,\mu>\mu_t,\phi) \leqslant C_2$. That is, for each χ there are thresholds μ_t such that for $\mu>\mu_t$, the performance of the entangled states for classical information transmission is better than that of the product states. While, for $\mu<\mu_t$, better performance is achieved by using the product states instead. For instance, numerical calculation of $I_2(\theta,\phi;\mu,\chi)$ for $\phi=\phi_0=\pi/4$ (i.e., entangled states), and $\phi=0$ (i.e., the completely unentangled product states), with $0\leqslant\mu\leqslant 1$ and $0\leqslant\chi\leqslant\pi/2$ allows us to compare the information carrying capacity of both forms of input state. In Fig. 1, we have $\theta=\pi/6$ and $\chi=0.15\pi$, we have $\mu_t\in(0.7,0.9)$. Furthermore, our numerical study shows that for the product states,

$$I_2(\theta,\phi=0;\mu=1,\chi) \geqslant I_2(\theta,\phi=0;\mu=0,\chi). \tag{24}$$

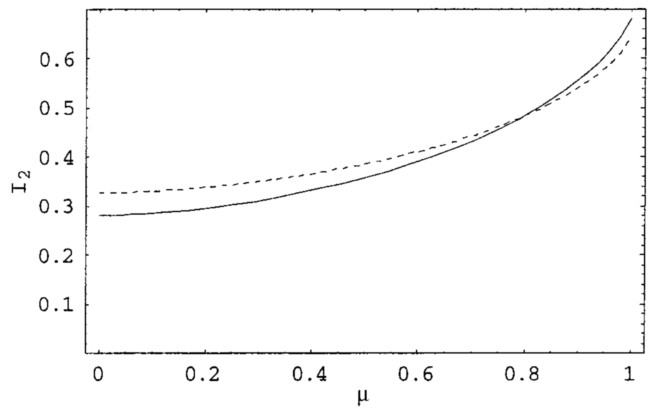

FIG. 1. Mutual information for product states ($\theta=\pi/6,\phi=0$, dotted line) and for entangled states ($\theta=\pi/6,\phi=\phi_0=\pi/4$, solid line) as a function of the degree of memory of the channel, for $\chi=0.15\pi$.

This shows that in the case of quantum amplitude-damping channel with perfect memory, it is possible to obtain enhanced information carrying performance even if we are using product input states. This is the same conclusion reached in Ref. [5] for the case of a depolarizing channel.

In conclusion, we have extended the problem of time-correlated noise (or "channels with memory") as considered in Ref. [5] to the case of the amplitude-damping channel. In the case of sending two qubits by successive uses of an amplitude damping channel with partial memory, we establish numerically that by using entangled states rather than product states as information carriers, we can enhance the transmission of classical information over the quantum channel.

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[10] In the class of memoryless channels, independent (uncorrelated) noise acts on each use of a channel.

[11] For the Pauli X channel with correlated noise, $\lambda_{00}=\lambda_{11}=\lambda_{22}=\lambda_{33}=0$, $\lambda_{01}=\lambda_{10}=0$, $\lambda_{02}=\lambda_{20}=\lambda_{03}=\lambda_{30}=-\Gamma$, $\lambda_{12}=\lambda_{21}=\lambda_{13}=\lambda_{31}=-\Gamma$, $\lambda_{23}=\lambda_{32}=0$. It is easy to guess that the eigenvalues for the correlated Pauli Y channel would be. For the depolarizing channel with correlated noise, $\lambda_{ii}=0$, $\lambda_{jk}=-2\Gamma$ for $i, j\neq k\in\{0,1,2,3\}$.

[12] Formally, we observe that $\mathcal{L}$ in Eq. (11) [$\mathcal{L}^c$ in Eq. (12)] obeys $\mathcal{L}^2\pi=-\Gamma\mathcal{L}\pi$. Consequently, $\mathcal{L}^n\pi=(-1)^{n-1}\Gamma^{n-1}\mathcal{L}\pi$. So, from the formal series expansion of $\exp(t\mathcal{L})\pi$, one can obtain Eq. (14). The same argument applies to the correlated Pauli X and Y channels. It is easy to check that the argument similarly applies to the above equation.