Superluminal optical pulse propagation in nonlinear coherent media

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We investigate a light-pulse propagation with negative group velocity in a nonlinear coherent medium. We show that the necessary conditions for nonlinear superluminal effects to be observable are realized in a three-level Λ system interacting with a linearly polarized laser field in the presence of a static magnetic field. The initially prepared Zeeman coherence cancels the resonant absorption of the medium almost completely, but preserves its dispersion anomalous and very high. In this medium the light pulse propagates superluminally in the sense that the peak of the transmitted pulse exits the medium before the peak of the incident pulse enters. At the same time the pulse group velocity is intensity dependent, which leads to the self-steepening of the pulse trailing edge due to the fact that the more intense parts of the pulse travel slower. This is in contrast with the shock wave generation in a nonlinear medium with normal dispersion. The predicted effect can be easily observed in the well-known schemes employed in the studies of nonlinear magneto-optical rotation. The upper bound of sample length is found from the criterion that the pulse self-steepening and group-advance time are observable without pulse distortion caused by the group-velocity dispersion.

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I. INTRODUCTION

The propagation of light pulses through the media with a normal dispersion is adequately described by the group velocity, which in this case turns out to be, in fact, the velocity of signal transfer. However, the group velocity loses its role of signal velocity in the region of anomalous dispersion, where it not only exceeds the vacuum speed of light, but at certain frequencies becomes negative; whereas, even when the group velocity is superluminal, no real signal can be transferred faster than the vacuum velocity of light [1]. Nevertheless, contrary to the common assertion [2,3] that superluminal group velocity is not a useful concept, Garrett and McCumber [4] have shown that in the case of superluminal propagation of sufficiently smooth pulses, such as Gaussian wave packets, the motion of the pulse peak is correctly described by the classical expression of group velocity. The effect of superluminality is that the emerging pulse has essentially the same shape and width as that of the incident wave packet, but its peak exits the cell before the incident pulse even enters. This process can be understood in terms of superposition and interference of traveling plane waves that have a distribution of frequencies and add up to form a narrow-band light pulse. In a dispersive medium, the speed of each plane wave is determined by the refractive index at that frequency. Around the resonance in the region of anomalous dispersion, the refractive index varies so steeply and different plane waves interfere in a way that the point of maximal constructive interference is localized at the exit face of the sample. The plane waves appear in the medium much earlier and, hence, the exit pulse is formed long before the peak of the incident pulse enters the medium. When the group velocity is negative, the exit pulse splits into two pulses; one of which moves forward in the vacuum, while the second one propagates backward in the medium and is canceled by the incoming pulse at the entrance of the cell (see Sec. III). Eventually, the superluminal propagation can be represented as a reshaping of the pulse, most of which is absorbed, leaving only a small pulse in the leading edge, which moves superluminally and saves the shape and width of the initial pulse. This picture breaks down, if the envelope of incident wave packet has singularities, for example, step-type modulations, which always move with the light velocity c. All these results demonstrate that for smooth pulses with continuous derivatives the group velocity is physically meaningful even when it is negative. But it cannot be interpreted as a velocity of real signal, which could propagate no faster than light [5]. These predictions have been confirmed experimentally using various schemes, including the anomalous dispersion near an absorption line [6,7], nonlinear gain [8], active plasma medium [9], and the tunneling of one-photon wave packet through a barrier [10]. Another mechanism giving rise to superluminal group velocities is a tachyonic-type dispersion in metastable optical systems. There is a close analogy between superluminal propagation of optical pulses and tachyon dynamics. The particles with negative mass, tachyons, have been introduced in particle physics for construction of renormalizable theory of strong interactions. Although they have not been observed experimentally in a free state, the tachyonic excitations arise very often in unstable physical systems undergoing phase transition [11]. Recently the behavior of laser fields as tachyonic excitations has been shown theoretically in inverted two-level media [12,13] and optical phase conjugators [14]. An experiment in which the superluminal propagation of medium collective modes can be treated as tachyonic excitation has been proposed [12].

An interesting model for a transparent anomalous dispersive medium and a light-pulse propagation with negative group velocity has been proposed by Chiao and co-workers [15,16]. The model is based on the destructive interference between the two close-lying gain transitions. In this scheme

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Wang and co-workers have recently observed a pulse-advance shift of 62 ns in cesium vapor [17].

The results of the papers [4–8,10–18] are based on linear response of the medium where the negative group velocity always exists within the absorption lines [1] and outside gain lines [15–18]. Similar results have been obtained, when a weak probe beam propagates with an effective index of refraction induced by a nonlinear interaction of the medium with a strong pump field. Under these conditions both the probe light ultraslow propagation [19–21] at the electromagnetically induced transparency (EIT) [22] and the negative "group-delay" propagation associated with electromagnetically induced opacity [23,24] have been recently observed experimentally. However, from the point of view of the dependence of medium optical properties on probe field intensity, these systems may be clearly considered as effectively linear optical media with an extraordinary large dispersion.

Intriguing questions arise whether the nonlinear superluminality occurs in a medium, the refractive index of which is modified by the propagating light pulse itself. How this effect is pronounced and what is the form of the superluminal pulse emerging from such a nonlinear medium? It seemed that the answer to these questions could be easily obtained in a two-level atom, which is a basic model for the study of superluminal propagation in linear media [4–8,10,12–18]. Note that this system has been also employed for the investigation of nonlinear slow optical pulse propagation in media with normal dispersion [25,26]. However, a two-state system is unfit for observation of nonlinear superluminal effects. The problem is to observe the pulse superluminal propagation in a medium with an intensity-dependent refractive index, but without accompanying nonlinear processes, which lead to pulse reshaping as well. Since, on the one hand, a two-level atom displays anomalous dispersion within the absorption line and, on the other hand, the nonlinearity in the dispersion is achieved at the intensities that are much larger than the saturation intensity (see the Appendix), then the nonlinear effects such as self-focusing (or defocusing), self-phase modulation, etc., cannot be neglected for two-level atoms. Meanwhile, the systems that are almost transparent and at the same time show large nonlinear dispersion are well known. They have been extensively employed as basic models in theoretical studies of resonant nonlinear magneto-optical effects caused by a laser-induced coherence (see the recent review by Budker *et al.* [27]). The simplest system displaying these properties is a three-level Λ atom with ground-state Zeeman sublevels, which interacts with the right (RCP) and left (LCP) circularly polarized components of the optical light pulse in the presence of weak magnetic field applied along the direction of light-pulse propagation (Fig. 1). Due to the nearly maximal Zeeman coherence, large nonlinear susceptibilities are created at the resonance frequency, analogous to those observed in EIT experiments (for relationship between EIT and nonlinear magneto-optics see Ref. [21]). This leads to very narrow magnetic resonances [28] or enhances significantly the nonlinear Faraday signal [29]. However, the dynamics of resonant light propagation in coherent nonlinear media has not been studied yet. To the best of our knowledge, this analysis is carried out for the first time in the present paper. We analyze the superluminal propagation of a smooth light pulse in a coherent Λ system with the level configuration depicted in (Fig. 1). The initial preparation of Zeeman coherence is necessary to avoid the pulse distortion due to the energy loss needed to initiate the EIT. The Zeeman coherence suppresses the absorption of the medium, while it creates a highly anomalous and intensity-dependent dispersion in the vicinity of the resonance line center. This results in a negative nonlinear group velocity at the values of light intensity, which are many times smaller than the saturation intensity of the corresponding atomic transition. In our scheme the superluminal effects are revealed in the two ways. First, similar to linear media, the peak of the outgoing pulse exits the cell before the peak of the incident pulse with a narrow spectral width enters the medium. Second, the pulse velocity is nonlinear in light intensity: the more intense parts of the pulse travel slower. As a result, the self-steepening of the trailing edge of the pulse takes place in contrast to the shock-wave formation in media with normal dispersion [25,26]. In fact, the steepening effect is the most striking manifestation of anomalous dispersion, if one takes into account that usually the residual absorption causes a misconstruction of the pulse advance. It is worth noting that in a hypothetical case, when the dependence of pulse group velocity on light intensity is ignored, the resulting propagation of the pulse can be again represented as a pulse-reshaping phenomenon without any violation of causality, similar to the case of nonattenuated propagation in the linear media with a gain doublet [15–17]. In our model also there is no attenuation of the pulse after its propagation through the medium (see Fig. 7). However, in contrast to these models, in our case the medium is almost transparent on the resonant transition.

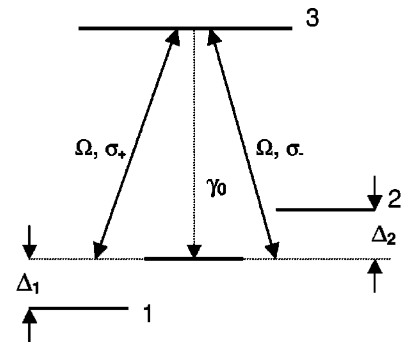

FIG. 1. Open Λ configuration for studying the nonlinear superluminal light propagation. $\sigma_{\pm}$ are components of a linearly polarized laser field with Rabi frequency Ω coupling the level $|3\rangle$ with $|1\rangle$ and $|2\rangle$, respectively. The Zeeman shift of ground-state sublevels $\Delta_{1,2}=\mp\Delta$ is induced by the longitudinal magnetic field B. Radiative decay from $|3\rangle$ to $|1\rangle$ and $|2\rangle$ goes at rate γ and the outside with the rate γ_0.

The paper is organized as follows. In the following section we derive basic equations for the atomic density matrix and transmitted light. The results of numerical calculations for pulse propagation with nonlinear group velocity, which are obtained using the analytic solution for the density matrix, are given in Sec. III. Our conclusions are summarized in Sec. IV. Finally, in the Appendix we calculate the nonlinear

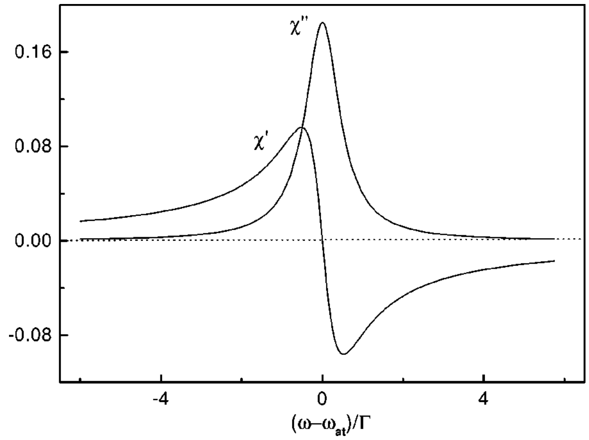

FIG. 2. Real (x') and imaginary (x'') parts of the susceptibility of a two-level atom as a function of the atom-field detaining $\varepsilon = \omega - \omega_{at}$ in units of Γ.

group velocity of a light pulse propagating in the Doppler broadened medium for the cases of two-level atom and three-level Λ system.

II. NONLINEAR NEGATIVE GROUP VELOCITY IN A MEDIUM WITH ZEEMAN COHERENCE

It is well known that in linear media the anomalous dispersion always occurs within an absorption line [1] and leads to negative group velocity for the peak motion of sufficiently smooth pulses [4]. However, as can be seen from Fig. 2 for the case of a two-level atom, in usual materials the light pulses experience very large absorption in the vicinity of sharp atomic resonance that prevents a doubtless observation of high anomalous dispersion [30]. On the other hand, the light-induced Zeeman coherence in a three-level Λ atom changes essentially this absorption-dispersion relationship, making the medium almost transparent, while keeping its dispersion still very large (Fig. 3). Moreover, as is apparent from Fig. 3, there is a frequency range, where the dispersion

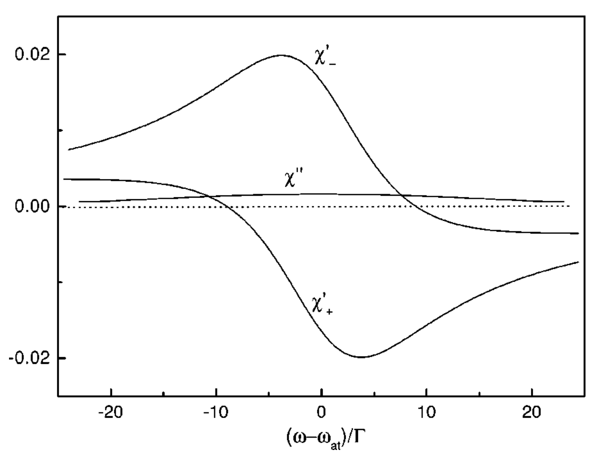

FIG. 3. Imaginary (x'') and real (x_1') parts of susceptibilities at σ_+ and σ_- transitions in a Λ atom as a function of $\varepsilon = \omega - \omega_{at}$ for $\Omega_0 = 0.3\Gamma$, $\Delta = 0.01\Gamma$, $\gamma_c = 10^{-4}\Gamma$. Doppler broadening is neglected.

on both transitions $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |3\rangle$ is the same and anomalous. This means that the RCP and LCP components of linearly polarized laser field travel with the same negative group velocity. As the latter depends on the light intensity, the pulse advance, being the minimum at the pulse peak (see below), is different for different parts of the pulse compared to the same pulse traveling the same distance in a vacuum. As a result, the trailing edge of the pulse should show self-steepening that can be tested in an interferometric experiment similar to that performed by Wang and co-workers [17].

A. Basic equations

The atomic system depicted in Fig. 1 interacts with the fields of RCP and LCP components $E_{1,2}$ of linearly polarized laser light with the frequency ω and wave number k propagating along the z axis. In the presence of a static longitudinal magnetic field B, these components, which are working on the transitions $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |3\rangle$, respectively, are detuned from the atomic resonances by

$$\Delta_{1,2} = \omega - \omega_{31,2} \mp \Delta - \frac{\omega}{c} v = \varepsilon_{1,2} \mp \Delta, \qquad (1)$$

where ω_{ik} is the frequency difference between levels i and k, and $\Delta = g\mu_B B/\hbar$ is the Zeeman shift, which is induced by the magnetic field for the ground-state sublevels $|1\rangle$ and $|2\rangle$ having magnetic quantum numbers ∓ 1. Here g is the ground-state gyromagnetic factor and μ_B is Bohr's magneton. In Eq. (1) we have included the Doppler shift $(\omega/c)v$ for atoms moving with frequency $v_z = v$.

The interaction of fields $E_{1,2}$ with a single atom is described by the Hamiltonian

$$H_{int} = \hbar \Delta_1 \sigma_{11} + \hbar \Delta_2 \sigma_{22} - \hbar(\Omega_1 \sigma_{31} + \Omega_2 \sigma_{32} + \text{H.c.}). \qquad (2)$$

Here $\sigma_{ij} = |i\rangle\langle j|$ are atomic operators. The Rabi frequencies of field polarization components are defined as $\Omega_{1,2} = \mu E_{1,2}/\hbar$, where the dipole moments of the transitions $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |3\rangle$ are taken equal: $\mu_{13} = \mu_{23} = \mu$.

The time evolution of the system's density matrix ρ obeys the master equation

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H_{int}, \rho] + \Lambda \rho, \qquad (3)$$

where the matrix $\Lambda\rho$ accounts for all atomic relaxations. In further, we neglect the collisional broadening of optical transitions compared to the spontaneous decay rate, but we take it into account for the ground-state coherence. This approximation is valid for the gas pressure below 10^{13} cm^{-3} at room temperature that is assumed hereafter. Thus, in our model the longitudinal and transverse optical relaxations of atomic Bloch vectors are determined by the process of spontaneous emission from the upper level $|3\rangle$ to the lower levels $|1\rangle$ and $|2\rangle$ with equal rates $\gamma_1 = \gamma_2 = \gamma$, and outside the three-level Λ system with the rate γ_0. The Zeeman coherence decay rate

γ_c is determined by the collisional and atomic time-of-flight broadening. Thus, the phenomenological relaxation matrix $\Lambda\rho$ has the form

$$\Lambda\rho=\begin{pmatrix} \gamma\rho_{33} & -\gamma_c\rho_{12} & -\Gamma\rho_{13} \\ -\gamma_c\rho_{21} & \gamma\rho_{33} & -\Gamma\rho_{23} \\ -\Gamma\rho_{31} & -\Gamma\rho_{32} & -(\gamma_0+2\gamma)\rho_{33} \end{pmatrix}, \qquad (4)$$

where $2\Gamma=\gamma_0+2\gamma$ is the total spontaneous decay rate of the level $|3\rangle$.

The evolution of the slowly varying field amplitudes $E_{1,2}$ along the z axis is determined by Maxwell's equations

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)E_{1,2}(z,t)=2\pi i\frac{\omega}{c}P_{1,2}(z,t), \qquad (5)$$

where $P_{1,2}(z,t)$ are the field induced polarizations, the real and imaginary parts of which are responsible for the dispersive and absorptive properties of medium at the transitions $|1\rangle\rightarrow|3\rangle$ and $|2\rangle\rightarrow|3\rangle$, respectively. The polarizations are calculated from the Fourier transform

$$P_i(z,t)=\int_{-\infty}^{\infty}d\nu e^{i\nu t}P_i(\nu,z), \quad i=1,2, \qquad (6)$$

where $\nu=\overline{\omega}-\omega$ is the deviation from the field carrier frequency ω, and $P_i(\nu,z)$ are expressed by the corresponding susceptibilities

$$\chi_i(\nu,z)=N\mu^2\rho_{3i}/\hbar\Omega_i, \qquad (7)$$

as

$$P_i(\nu,z)=\chi_i(\nu,z)E_i(\nu,z), \qquad (8)$$

with N being the atomic number density.

We consider the case of smooth incident pulses with a duration much larger than all relaxation times of the medium. Then, the limited bandwidth of field Fourier amplitudes $E_i(\nu,z)$ allows us to approximate $\chi_i(\nu,z)$ by the first few terms of the Taylor series:

$$\chi(\nu,z)=\chi(\omega,z)+\left.\frac{\partial\chi}{\partial\nu}\right|_{\nu=0}\nu+\frac{1}{2}\left.\frac{\partial^2\chi}{\partial\nu^2}\right|_{\nu=0}\nu^2+\cdots. \qquad (9)$$

The substitution of Eq. (9) into Eq. (6) yields

$$P_i(z,t)=\chi_i(\omega,z)E_i(z,t)+i\left.\frac{\partial\chi_i}{\partial\omega}\right|_0\frac{\partial E_i}{\partial t}-\frac{1}{2}\left.\frac{\partial^2\chi_i}{\partial\omega^2}\right|_0\frac{\partial^2 E_i}{\partial t^2}+\cdots. \qquad (10)$$

Then, from Eq. (4) we have

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_i}\frac{\partial}{\partial t}\right)E_i(z,t)=2\pi i\frac{\omega}{c}\left[\chi_i(\omega,z)E_i-\frac{1}{2}\left.\frac{\partial^2\chi_i}{\partial\omega^2}\right|_0\frac{\partial^2E_i}{\partial t^2}\right], \qquad (11)$$

where group velocities v_i ($i=1,2$) for the two polarization components are introduced in the form

$$v_i=c\left[1+2\pi\omega\mathrm{Re}\left.\frac{\partial\chi_i}{\partial\omega}\right|_0\right]^{-1}, \quad i=1,2. \qquad (12)$$

This expression coincides with the classical formula of group velocity $v_{gr}=c\mathrm{Re}[n+\omega dn/d\omega]^{-1}$, if one takes into account that the complex index of refraction of the medium $n=1+2\pi\chi$ and $\mathrm{Re}\chi\ll 1$. The second term in the right-hand side (rhs) of Eq. (11) is responsible for the pulse distortion due to the group-velocity dispersion. In the following section we will show that this term is negligibly small for the length of the atomic sample L, which is restricted by the requirement of smoothness of the pulse.

Since in this paper we restrict our attention to the superluminal pulse propagation, we need only the equation for intensities $I_i(z,t)=c/2\pi|E_i(z,t)|^2$. From Eq. (11) we have

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_i}\frac{\partial}{\partial t}\right)I_i(z,t)=-\alpha_i I_i(z,t), \qquad (13)$$

where the nonlinear absorption coefficients α_i at the frequency ω are defined as

$$\alpha_i=\frac{4\pi\omega}{c}\mathrm{Im}\,\chi_i=\frac{1}{\ell_0}\mathrm{Im}\frac{\rho_{3i}\Gamma}{\Omega_i}. \qquad (14)$$

Here $\ell_0=(\sigma N)^{-1}$ and $\sigma=4\pi\omega\mu^2(\hbar c\Gamma)^{-1}$ is the resonant absorption cross section.

The advance time T_{ad} of the field $E_i(z,t)$ propagating through a medium of length L compared to the pulse passing the same distance in a vacuum is given by

$$T_{ad}=\left(\frac{1}{c}-\frac{1}{v_i}\right)L\approx-2\pi\frac{\omega}{c}\mathrm{Re}\left.\frac{\partial\chi_i}{\partial\omega}\right|_0. \qquad (15)$$

Thus, at the frequencies where the derivative $\partial\chi/\partial\omega$ takes a large negative value, the pulse advance is significant.

B. Solutions

We solve Eq. (13) assuming that the incident laser beam is a Gaussian pulse

$$I(z,t)=I_{in}(t-z/c)=I_0\exp[-(t-z/c)^2/\tau^2], \quad z\leqslant 0, \qquad (16)$$

with a pulse width $\tau\gg\Gamma^{-1}$.

During the propagation of the pulse through the medium the group-velocity dispersion and self-steepening change the pulse width that violates the approximation of truncated Taylor expansion Eq. (10). Our aim is to observe the pulse nonlinear steepening under superluminal propagation without pulse spreading. The source of the latter is the derivative $\partial^2\chi/\partial\omega^2$, which enters in the second term in the rhs of Eq. (11). Since this term is also proportional to the second time derivative of the field $\partial^2E/\partial t^2$, the contribution from the group-velocity dispersion dramatically increases at the distances, where a steep front is formed due to the dependence of group velocity on light intensity. In order to avoid this complication, as well as to use rightly the steady-state solution of Eqs. (2), we restrict the upper bound of the sample length by a value for which the pulse broadening caused by the self-steepening does not exceed the medium absorption line width Γ, i.e., the condition

$$\left|\frac{\partial I(z,t)}{\partial t}\right| \leqslant \Gamma I(z,t), \quad 0 \leqslant z \leqslant L \tag{17}$$

should hold everywhere inside the medium for any point on the pulse envelope. Then, by condition (17), the spreading of the wave packet caused by the group-velocity dispersion can be neglected up to the distances restricted by the value

$$L \ll L_1 = \left| 2\pi \frac{\omega}{c} \Gamma^2 \frac{\partial^2 \chi}{\partial \omega^2}\bigg|_{max} \right|^{-1}. \tag{18}$$

Equation (18) follows from the smallness of the second term in the rhs of Eq. (11). Here we have replaced the derivative $\partial^2\chi/\partial\omega^2$ by its maximal value taking into account that χ is an implicit function of z through its dependence on pulse intensity.

We suppose that the incident pulse does not lose energy for preparation of transparency and its reshaping in the medium is caused solely by the nonlinear dispersion. In Sec. III we show how this situation can be realized. With condition (17) this assumption allows us to use the steady-state solution for atomic density matrix elements. This solution is obtained from Eq. (2) in the form

$$\mathrm{Im}\frac{\rho_{3i}}{\Omega_i} = q\Gamma\Delta^2\delta\rho/D, \quad i = 1,2, \tag{19}$$

$$\mathrm{Re}\frac{\rho_{31}}{\Omega_1} = -\Delta(q\varepsilon\Delta + \Omega^2)\delta\rho/D, \tag{20}$$

$$\mathrm{Re}\frac{\rho_{32}}{\Omega_2} = -\Delta(q\varepsilon\Delta - \Omega^2)\delta\rho/D, \tag{21}$$

$$D = q\Delta^2(\varepsilon^2 + \Gamma^2) + \Omega^4, \quad q = 1 + \frac{\Omega^2\gamma_c}{2\Gamma\Delta^2}, \tag{22}$$

where $\Omega^2 = |\Omega_1|^2 = |\Omega_2|^2$. It is derived under the assumptions of $\Delta, \gamma_c \ll \Gamma$ and $\Gamma^2 \gg \Omega_0^2 \gg \Gamma\gamma_c$, where Ω_0 is the Rabi frequency corresponding to the peak value of pulse intensity. The population difference between the ground-state Zeeman sublevels and the upper state $|3\rangle$ is $\delta\rho$, which is $\delta\rho \simeq \frac{1}{3}$ for equal decay rates $\gamma_0 = \gamma$ and a weak magnetic field. In the linear limit ($\Omega^2 \ll \Delta\Gamma$) these expressions coincide with that for the two-level atom presented graphically in Fig. 2. In the inverse case of $\Omega^2 \gg \Delta\Gamma$, the medium becomes almost transparent near the resonance $\varepsilon = 0$, while its dispersion remains highly anomalous (Fig. 3). This result differs from the well-known fact established for linear media that the optical dispersion of transparent medium is always normal [31]. The latter directly follows from Kramers-Kronig relations, which assume linearity and causality in the response of the medium. In our case this response is nonlinear and the classical Kramers-Kronig relations are no longer valid. The numerical solution of Eq. (3) in steady-state regime is shown in Fig. 3. Our results given by Eqs. (19)–(22) are in excellent agreement with these calculations.

Further, we need to take into account the Doppler broadening by averaging the matrix elements ρ_{ij} over the atomic velocity distribution. Using the results of calculations performed in the Appendix, we finally obtain the group velocity v_{gr} and the absorption coefficient α of two polarization components of the field in the form

$$v_{gr} = \frac{c}{n_g}, \quad n_g = 1 - \frac{c\Gamma\delta\rho}{2\ell_0\Delta_D^2}\frac{1}{(b+1)^2}, \tag{23}$$

$$\alpha = \frac{1}{\ell_0}\frac{\Gamma^2\delta\rho}{\Delta_D^2}\frac{1}{b(b+1)}, \tag{24}$$

where b is given by Eq. (A9). As follows from Eq. (23), the group velocity index n_g reaches its maximum at the peak value of light intensity. As a result, more intense parts of the pulse move slower that leads to the steepening of the trailing edge of the pulse. Also, notice that in the absence of external magnetic field, when $\Delta = 0$, the susceptibilities χ_i vanish proportional to γ_c due to complete population trapping and, hence, the pulse moves with the light vacuum velocity c.

The results of numerical integration of Eq. (13) for pulse propagation using Eqs. (23) and (24) are presented in the following section.

Now, having the solution for susceptibilities Eqs. (A6)–(A8), we can find the upper bound of the cell length from conditions (17) and (18). The maximal value $|\partial^2\chi/\partial\omega^2|_{max} = N\mu^2\delta\rho/4\hbar\Delta_D^2\Gamma$ occurs at $\Omega^2 = \Delta\Gamma$. Substituting it into Eq. (18), we have

$$L_1 = \frac{32\pi\Delta_D^2}{3N\lambda^2\Gamma\gamma\delta\rho}, \tag{25}$$

where μ^2 is expressed by the radiative decay rate $\gamma = 32\pi^3\mu^2/3\hbar\lambda^3$, λ is the wavelength of atomic transition.

Condition (18) can be rewritten in the form

$$\Delta\tau \ll \frac{1}{\Gamma}, \tag{26}$$

where

$$\Delta\tau = 2\pi\omega\Gamma\left|\frac{\partial^2\chi}{\partial\omega^2}\right|\frac{L}{c} \tag{27}$$

is the amount of spreading experienced by a pulse, when traveling through a medium with length L. Thus, condition (26) ensures the truly observable steepening of the backward edge of light pulse without its distortion.

In order to find the upper bound for the cell length from Eq. (17) we consider the situation where the absorption is absent. In this case the reduced equation (13) has a simple wave solution in the form

$$I(z,t)=I_{in}(t-z/v_g), \tag{28}$$

where $I_{in}(z,t)$ is given in Eq. (16). Since at the trailing edge $dI/dt<0$, from Eqs. (27) and (17) we find

$$L\leqslant L_2=\frac{2\ell_0\tau\Delta_D^2}{\Gamma\delta\rho}\frac{(b+1)^3}{b}. \tag{29}$$

Note that Eq. (29) offers the most stringent restriction at the peak intensity where b is close to unity.

The last criterion is that $\Delta\tau$ should be small as compared with the group-advance time T_{ad}, $\Delta\tau\ll T_{ad}$. Using Eqs. (15) and (27) and solutions Eqs. (24)–(26), we obtain

$$\frac{\Gamma\Omega^2\delta\rho}{\Delta\Delta_D^2}\frac{2b+1}{b^2(b+1)}\ll 1. \tag{30}$$

One can easily show that this condition fails only for sufficiently small values of $\Omega\sim\sqrt{\Delta\Gamma}$, so that there is no reason to consider this complication here.

It must be also noted that for the group advance to be observable, it is sufficient that T_{ad} is large with respect to the resolution time of interferometric detection, being at the same time much smaller than the pulse width τ^{-1}, as it was in the recent experiment [17].

III. NUMERICAL RESULTS AND DISCUSSION

Here we give results of numerical calculations based on the equations derived in the preceding section and discuss the relevant physics.

In our simulations for the model of the atomic system we have chosen parameters corresponding to the D_1 absorption line of ^{87}Rb. It is worth noting that an open Λ system (four-level system in Fig. 1) has been employed in Ref. [29] for investigation of nonlinear magneto-optical effects in rubidium. It has been shown that this simplified model, with well-chosen parameters, represents the qualitative physics quite well [29]. In our case the total decay rate of the upper level $|3\rangle$ is chosen $2\Gamma=2\pi\times5\times10^6$ s^{-1}, the wavelength of optical transitions $\lambda=800$ nm, and the inhomogeneous Doppler broadening width $\Delta_D\simeq50\Gamma$. For the atomic number density $N=2\times10^{12}$ cm^{-3} we have $\ell_0\simeq1.25\times10^{-4}$ cm. The external magnetic field is taken $B=30$ mG, which corresponds to $\Delta=0.002\Gamma$ for the ground-state level of rubidium. The peak value of Rabi frequency of the field is chosen to obey the condition $\Omega_0^2=3\Delta\Delta_D$, i.e., $\Omega_0=0.6\Gamma$ ($I_0\approx5$ mW/cm^2), which is below the saturating value for a two-level atom $\Omega_s=2\Gamma$. The Zeeman coherence relaxation rate γ_c is attributed to the collisional (in the presence of buffer gas) and time-of-flight broadening and is taken $\gamma_c\leqslant10^{-4}\Gamma$. Then, for initial pulse duration $\tau=3$ μs and population difference $\Delta\rho\simeq0.25$ the upper bounds of the sample length are estimated from Eqs. (25) and (29) as $L_1\simeq4\times10^4$ cm and $L_2\simeq400$ cm, respectively. This means that in the cell with a length of a few centimeters the superluminal effects, primarily the pulse self-steepening, can be easily observed with negligible pulse distortion.

In Fig. 4 we plot the absorption of the pulse and the group-advance time calculated from Eqs. (24) and (15) as a function of the pulse intensity for the set of parameters given above. It is apparent that the pulse peak is advanced with respect to the same pulse traveling in the vacuum approximately 50 times, which for $L=6$ cm corresponds to $T_{ad}\simeq10$ ns. The advance time increases towards to the pulse wings, and it is about 70 ns for the intensity $I\sim0.1I_0$. Since the absorption is strongly reduced around the pulse peak, where the EIT conditions

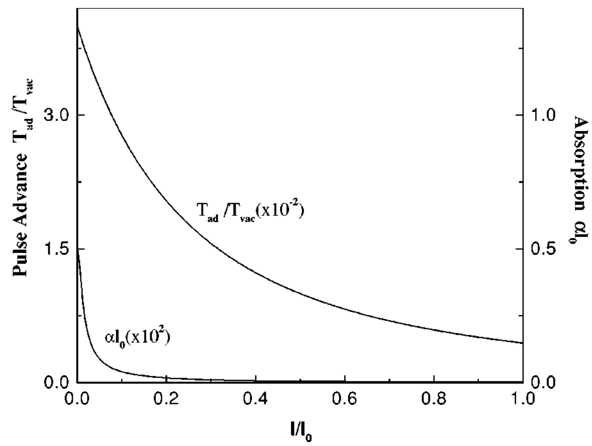

FIG. 4. Absorption α (in units of ℓ_0^{-1}) and group advance time T_{at} (in units of L/c) as a function of light intensity. The parameters are given in the text.

$$\Omega_0^2\gg\Delta\Delta_D,\gamma_c(\Delta_D+\Gamma) \tag{31}$$

are established, the group-advance time should be easily observed with nanosecond resolution time.

As an illustration, in Fig. 5 we give a space-time plot of

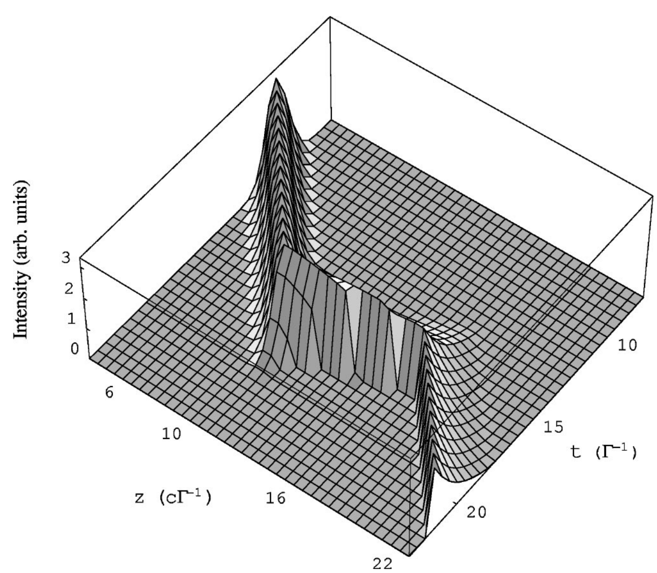

FIG. 5. Space-time image of Gaussian pulse motion with initial pulse duration $\tau=1\Gamma^{-1}$. Times are given in units of Γ^{-1} and distances in $c\Gamma^{-1}$. The medium extends from $z=10$ to 16. The peak of the pulse is at $z=-7$ at $t=0$. The group velocity and absorption coefficient are calculated for the parameters given in the text.

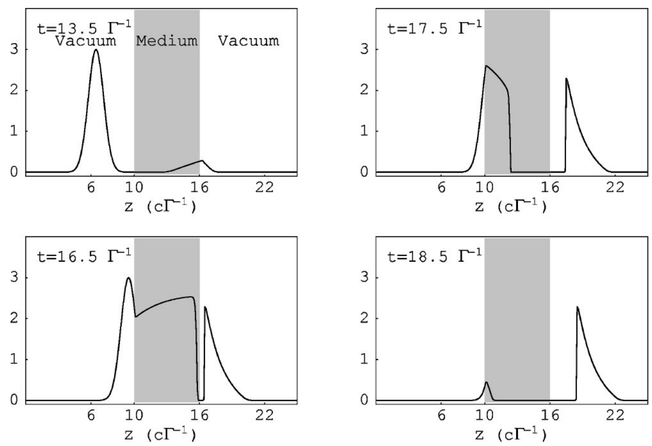

FIG. 6. Diagrams of Gaussian pulse propagation corresponding to sequential instants in Fig. 5.

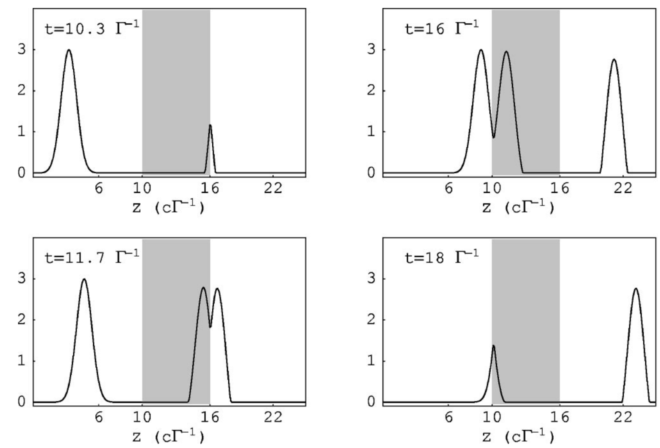

FIG. 7. Same configurations as for Fig. 6, but in the hypothetical case of nonlinear absorption without intensity dependence of group velocity.

the solution to Eq. (13) for the initial Gaussian pulse with duration $\tau \simeq 1\Gamma^{-1}$. The group velocity is negative in the medium. This effect appears as a ''back-in-time'' motion of the pulse. The formation of sharp back front of the pulse to the end of the cell is well visible (Fig. 5).

In order to reconstruct the superluminal propagation in details, we present in Fig. 6 the pulse motion in the form of time-sequence graphs. At first, before the incident pulse enters the medium, a weak pulse is created at the output. Then, this peak increases in time and eventually splits into forward- and backward-moving pulses with very sharp trailing edges. The first of them continues to move in a vacuum with velocity c, and after a long time it represents the final pulse. The second pulse exhibits, however, an unusual behavior. It increases while traveling through the absorbing medium. Nevertheless, there is no inconsistency with the energy conservation law. First, the velocity of energy transport defined as $v_E = \overline{S}/\overline{W}$, where $\overline{S}$ and $\overline{W}$ are the energy flux and field energy densities averaged over a period, is always less than c [32]. Second, the increment of the medium energy in the presence of light field is proportional to $\partial[(\omega n)/\partial\omega]|E|^2$ [31] and is negative for anomalous-dispersive media. Obviously this energy cannot be stored in the medium and it is transformed into the backward-moving wave. However, this takes place until the peak of the incident pulse enters the medium. After that, the real absorption of the light pulse results in a cancellation of backward wave at the entrance of the cell.

In Eq. (13) the nonlinear absorption and intensity-dependent group velocity are responsible for the reshaping of pulse. In order to demonstrate the influence of the two mechanisms separately we solved Eq. (13) in a hypothetical case of nonlinear absorption, assuming that at the same time the group velocity is independent on light intensity. Setting $b=0$ in Eq. (23), we calculated the propagation of the Gaussian pulse with the same set of parameters. The results depicted in Fig. 7 show that in this case the final pulse is built up much earlier than in the case of nonlinear group velocity and it preserves the shape and width of the incident pulse. A weak absorption is observed also.

We have calculated also the propagation of the front half of a Gaussian pulse. If the pulse propagation were not superluminal, then in this case the final pulse would have the original Gaussian shape, since the leading edge of the pulse is the same for the full and half-Gaussian pulses. As is seen from Fig. 8, the half-Gaussian pulse is reproduced. Moreover, the comparison of Figs. 6 and 8 shows that the front half of the Gaussian pulse is absorbed stronger than the back front. This means that we have a superluminal propagation and not a simple pulse reshaping.

In the discussion above, we have considered the steady-state solution for the atomic density matrix assuming that the medium has been prepared in coherent superposition of ground levels before the pulse enters the medium. This can be realized, e.g., when a much stronger linearly polarized control field irradiates the medium at first. It acts at the same transitions and drives the atoms into a coherent superposition of Zeeman sublevels. After the coherent medium is prepared the control field is smoothly turned off, and the signal field, for which the superluminal effects are sought, is switched on within a time interval that is much smaller than the Zeeman coherence decay time ($\sim \gamma_c^{-1}$). A similar method has been used in the light-storage technique [33]. Let us estimate the minimum laser energy that is necessary to establish the medium transparency. As has been shown in Ref. [34], the requirement for the initiation of EIT is that the number of photons in the laser pulse must exceed the number of atoms in a laser path, i.e.,

$$\frac{I\tau}{\hbar\omega} \geqslant NL, \qquad (32)$$

where I is the intensity of laser field, τ its pulse width. For $N \sim 10^{12}$ cm^{-3}, $L \sim 6$ cm, $\tau \sim 3$ μs, and $\hbar\omega \sim 1.5$ eV this condition is fulfilled, if $I > 0.2$ W/cm^2, showing that in our case of $I \sim 5$ mW/cm^2 the EIT cannot be produced by the signal pulse itself and the preliminary preparation of the coherent medium is necessary.

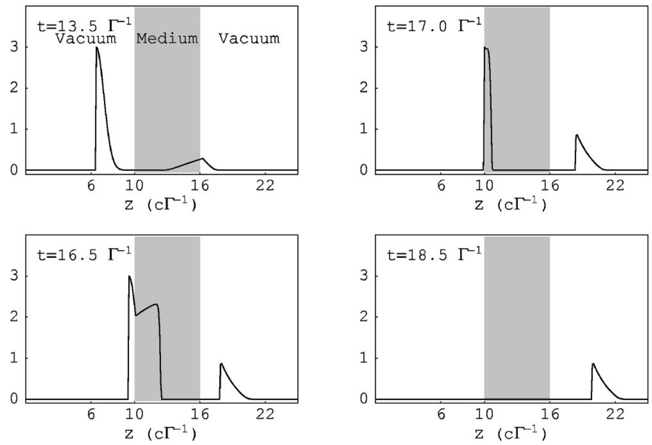

FIG. 8. Same as for Fig. 6, but in the case of front-half pulse propagation only.

IV. SUMMARY

In this paper we have analyzed the superluminal propagation of a light pulse in a nonlinear medium and showed that necessary conditions for these effects to be observable are realized in a three-level Λ system interacting with a linearly polarized laser beam in the presence of a static magnetic field. It is highly important that the nonlinearity of the refractive index of the medium arises in a low power regime, when all other nonlinear processes are negligible. We have shown that the propagation of a light pulse in a transparent-anomalous dispersion medium leads to the formation of an extremely sharp trailing edge and to the lengthening of the leading edge of the pulse. Such a behavior is inverse to the case of nonlinear propagation of the pulse in a medium with normal dispersion where a shock wave is generated. The predicted effect is the most striking manifestation of superluminality and it can be easily observed in the well-known schemes that have been used for studying the nonlinear magneto-optical rotation.

The discussion of many questions remained behind the scope of this paper. In particular, we have not given due attention to the proof of the fact that the predicted effect does not violate the causality. Here we note only that the proof is carried out in the same manner as in previous studies of superluminal propagation [5]. Further, for real atomic systems the complete energy state description of the multilevel structure should be included. This question requires a careful examination and it is in current study, the results of which will be presented in a future publication.

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APPENDIX

Here we calculate the nonlinear group velocity of a light pulse propagating through a Doppler broadened atomic medium. The cases of two-level atom and three-state Λ system are considered. In order to obtain the simple analytical expressions, when averaging the group velocities over the atomic thermal motion, we approximate the usual Gaussian distribution of atomic velocity with the Lorentzian function $W(v)=\Delta_D/\pi[\Delta_D^2+(kv)^2]$, where $2\Delta_D$ is full width half maximum of the Doppler broadening.

In the case of a two-level atom the imaginary and real parts of the susceptibility of medium are given by

$$\mathrm{Im}\chi = N\mu^2\Gamma/\hbar(\Gamma^2+\varepsilon^2+2\Omega^2), \tag{A1}$$

$$\mathrm{Re}\chi = -N\mu^2\varepsilon/\hbar(\Gamma^2+\varepsilon^2+2\Omega^2), \tag{A2}$$

where $\varepsilon=\omega-\omega_{at}-kv$, and Ω is the Rabi frequency of the field, which couples the upper and lower levels of the atom. Straightforward calculation of the velocity-averaged susceptibilities in the first order of $\Delta\omega=\omega-\omega_{at}$ gives

$$\langle \mathrm{Im}\chi\rangle = \frac{N\mu^2\Gamma}{\hbar b_0(b_0+\Delta_D)}, \tag{A3}$$

$$\langle \mathrm{Re}\chi\rangle = -\frac{N\mu^2}{\hbar(b_0+\Delta_D)^2}\Delta\omega, \tag{A4}$$

where $b_0=(\Gamma^2+2\Omega^2)^{1/2}$. For the group velocity of the light pulse from Eq. (A4) we find

$$v_{gr}=c\left(1+2\pi\omega\frac{\partial\langle\mathrm{Re}\chi\rangle}{\partial\omega}\right)^{-1}\approx -\frac{c\hbar(b_0+\Delta_D)^2}{N\omega\mu^2}. \tag{A5}$$

Similarly, for the case of the three-level Λ atom, from Eqs. (19)–(22) we have

$$\langle \mathrm{Im}\chi_i\rangle = \frac{N\mu^2\Gamma}{\hbar\Delta_D^2}\frac{\delta\rho}{b(b+1)}\left[1-\frac{2b+1}{4\Delta_D^2 b^2(b+1)}\Delta\omega^2+\ \dots\right], \tag{A6}$$

$$\langle \mathrm{Re}\chi_1\rangle = -\frac{N\mu^2}{\hbar\Delta_D^2}\frac{\delta\rho}{b+1}\left[\frac{\Omega^2}{qb\Delta}+\frac{\Delta\omega}{b+1} -\frac{\Omega^2}{2q\Delta\Delta_D^2}\frac{2b+1}{b^2(b+1)^2}\Delta\omega^2+\cdots\right], \tag{A7}$$

$$\langle \mathrm{Re}\chi_2\rangle = \langle \mathrm{Re}\chi_1(\Omega^2\to-\Omega^2,\quad q\to q)\rangle, \tag{A8}$$

where

$$b=\frac{\sqrt{\Delta^2\Gamma^2+\Omega^4 q^{-1}}}{\Delta\Delta_D}. \tag{A9}$$

Correspondingly, the group velocity of polarization components of the field is obtained in the form of Eq. (23). The comparison of Eqs. (A5) and (23) shows that, for the case of a two-level atom, the dependence of v_{gr} on light intensity appears at $\Omega \geqslant \Delta_D \gg \Gamma$, whereas in a three-level Λ system it is achieved at much smaller values of $\Omega \geqslant (\Delta\Delta_D)^{1/2}$.

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