Pattern recognition on a quantum computer

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(Received 26 August 2002; published 26 June 2003)

By means of a simple example, it is demonstrated that the task of finding and identifying certain patterns in an otherwise (macroscopically) unstructured picture (dataset) can be accomplished efficiently by a quantum computer. Employing the powerful tool of the quantum Fourier transform, the proposed quantum algorithm exhibits an exponential speedup in comparison with its classical counterpart.

DOI: 10.1103/PhysRevA.67.062311 PACS number(s): 03.67.Lx, 42.30.Sy, 89.70.+c

I. INTRODUCTION

Pattern recognition is one of the basic problems in artificial intelligence, see, e.g., Ref. [1]. For example, a glance at Fig. 1 will generally suffice for the human brain to spot the region with the pattern. However, it is a rather nontrivial task to accomplish the same performance with a computer—in particular, if the orientation and structure of the pattern are not known *a priori* and if the pattern is not perfect (in contrast to the one in Fig. 1).

Besides the detection and localization of patterns (for example, identifying seismic waves in the outputs of seismographs), the comparison and matching of the observed pattern to a set of templates (such as face recognition) form another interesting question. These problems can be solved with special classifiers, such as neuronal networks or Fourier analysis, etc., cf. Ref. [1].

The specific properties (i.e., one may consider many possibilities or combinations simultaneously and one is interested in global features only) of the task of pattern recognition give rise to the hope that quantum algorithms may be advantageous in comparison with classical (local) computational methods (with a unique entry).

During the last decade, the topic of quantum-information processing attracted considerable interest, see, e.g., Ref. [2] for a review. It has been shown that quantum algorithms can be enormously faster than the best (known) classical techniques: Shor's factoring algorithm [3], which exhibits an exponential speedup relative to the best (known) classical method; Grover's search routine [4] with a quadratic speedup; and several black-box problems [5–8], some of which also exhibit an exponential speedup; etc.

In the following, a quantum algorithm for the detection, identification, and localization of certain patterns in an otherwise (macroscopically) unstructured dataset is presented. It turns out that this method too is exponentially faster than its classical counterpart. Furthermore, it outperforms the (also extremely fast) method of optical filtering in terms of accuracy and general applicability.

The advantages of using quantum memories and computers for the aforementioned task of template matching (which is somewhat different from pattern detection or localization) have been discussed in Refs. [9–12]. Note, however, that the necessity of loading the complete dataset into a quantum memory may represent a drawback (cf. the next sections). In Ref. [13], an algorithm for data clustering (in pattern recognition problems) is developed, which is based on or inspired by principles of quantum mechanics—though not involving quantum computation.

II. DESCRIPTION OF THE PROBLEM

Let us consider a rectangular $N\times M$ array of unit cells that are either black, i.e., absorptive, or white, i.e., reflective (or, alternatively, transparent), cf. Fig. 1. In the following, we shall assume that these cells are perfectly absorbing or reflecting (transmitting), respectively, although sufficiently small deviations from this idealized behavior resulting in finite absorption, reflection, and transmission coefficients do not alter the following considerations.

The white cells are distributed with a roughly homogeneous density $\varrho<1$ (for example, $\varrho=1/2$) and will later be called points for brevity, i.e., the $N\times M$ array contains $P=\varrho NM$ points. A small fraction χ of these points (say $\chi=1/10$) forms a pattern in a connected (but not necessarily rectangular) region of the size χNM, cf. Fig. 1.

In contrast to Fig. 1, the pattern does not need to be perfect—average symmetries are sufficient, see Sec. V below. For simplicity we restrict our consideration to linear (but again not necessarily rectangular) patterns. That is, the patterns are assumed to be (approximately) invariant under at least two (discrete) symmetry transformations described by global translations into different directions. Geometrically speaking, the angles within the pattern do not change, consider, for example, a set of parallel and equidistant lines or a periodic repetition of small elements as in some wall papers, etc.

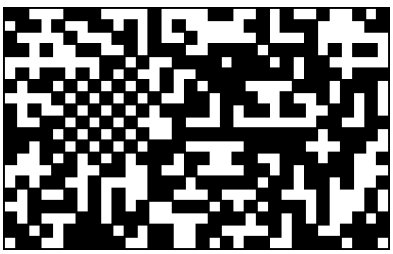

FIG. 1. A 32×20 array half filled with points, i.e., $\varrho=1/2$. In an 8×8 square, they form a pattern (hence, $\chi=1/10$), otherwise the points are randomly distributed.

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Let us assume that we know N, M, and ϱ, since these quantities can easily be measured having at hand the $N \times M$ array of unit cells—but we neither know the size χ of the pattern (or whether there is a pattern at all) nor its structure and orientation (except that it is linear). The task is to find an algorithm for extracting this information.

In principle, we may find out the position of all points (white cells) by shining appropriately focused light beams on the array and measuring the reflection or transmission. However, the array is also assumed to be very sensitive (such as an exposed but not yet developed film, for example) and each absorbed photon causes a certain amount of damage—similar to the Elitzur-Vaidman problem (detection without destruction), cf. Ref. [14]. Therefore, the number of incident photons should be as small as possible.

At the same time, we wish to obtain the characteristic parameters of the pattern with maximum attainable accuracy and minimum effort (i.e., number of subsequent operations). In order to cast the above requirements into a well-defined complexity-theoretic form, we consider the limit of very large arrays $N,M \gg 1$. Furthermore, we assume $N=2^n$ and $M=2^m$ with integers $n,m \in \mathbb{N}$ allowing for a binary representation. (Otherwise, we may enlarge the array accordingly or consider only a part of it.)

III. DATA PROCESSING

It is probably most convenient to view the dataset as a (quantum) black box

$$\mathcal{B}: \begin{bmatrix} |x\rangle \\ |y\rangle \\ |0\rangle \end{bmatrix} \rightarrow \begin{bmatrix} |x\rangle \\ |y\rangle \\ |f(x,y)\rangle \end{bmatrix}, \tag{1}$$

where the input state encodes the coordinates x and y of a potential point (a white cell) in the array as n- and m-qubit strings, respectively, together with a third one-qubit register $|0\rangle$ needed for unitarity. The output function $f(x,y)$ assumes the value 1 if there is a point at these coordinates and 0 if not.

As a possible physical realization one might imagine a configuration such as the following. A focused light beam passes $n+m$ quantum controlled refractors (e.g., nonlinear Kerr media) which change its direction by definite angles φ_j if the control qubit is $|1\rangle$ and do not affect it otherwise. For suitably chosen angles ($\varphi_j^{x,y}=\varphi_0^{x,y}/2^j$, with $j \in \mathbb{N}$), the final direction of the beam encodes the position (x,y) on the $N \times M$ array if the digits of the coordinates $|x\rangle,|y\rangle$ are inserted as the control qubits. Shining the so-directed light beam (which may consist of only one photon) through an aperture mask (as in Fig. 1) on a detector reproduces the action of the black box in Eq. (1).

Note that, in the physical realization described above, it is not necessary to load the complete (classical) information of the array into a quantum memory. Such a loading procedure would slow down the whole process drastically and hence represent a serious drawback. (The same problem limits the region of applicability of Grover's quantum search procedure, for example.) In addition, it is hard to see how this loading could be done without shining a relatively large number of photons on the array and thereby destroying it, cf. Sec. II.

Moreover, the involved number $(n+m)$ of devices (refractors) is very small in this case. Each refractor acts roughly similar to a controlled swap or a switch gate

$$\mathcal{S}: \begin{bmatrix} |\alpha\rangle \\ |\beta\rangle \\ |0\rangle \end{bmatrix} \rightarrow \begin{bmatrix} |\alpha\rangle \\ |\neg\alpha \wedge \beta\rangle \\ |\alpha \wedge \beta\rangle \end{bmatrix}, \tag{2}$$

with $\alpha,\beta=0,1$; but the series connection of these refractors allows for a very efficient data processing.

IV. QUANTUM ALGORITHM

Now we may apply the well-known trick of enquiring for all possible values of the coordinates (x,y) in only one run of the black box (quantum parallelism). To this end, we prepare a state as the superposition of all possibilities by using the Hadamard gate $\mathcal{H}$. For a single qubit in the $|0\rangle,|1\rangle$ basis, the (unitary) Hadamard gate $\mathcal{H}$ acts as $\mathcal{H}|0\rangle=(|0\rangle+|1\rangle)/\sqrt{2}$. By multiple application of $\mathcal{H}$ and running the black box (only once) we arrive at the desired superposition

$$\mathcal{B} \begin{bmatrix} \mathcal{H}^{(n)}|0^{(n)}\rangle \\ \mathcal{H}^{(m)}|0^{(m)}\rangle \\ |0\rangle \end{bmatrix} = \frac{1}{\sqrt{NM}} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \begin{bmatrix} |x\rangle \\ |y\rangle \\ |f(x,y)\rangle \end{bmatrix}. \tag{3}$$

Measuring the third register $|f\rangle$ and obtaining 1 prepares the state $|\Psi\rangle$ as a superposition of the coordinates $|x\rangle$ and $|y\rangle$ of all points. Assuming an ideal black box, the outcome $f=0$ would just imply the complementary set $\varrho \rightarrow 1-\varrho$. However, in the presence of absorbing units, as described in Sec. II, the resulting entanglement with the state of the absorptive cells would destroy the coherence completely in the case $f \neq 1$.

It will be advantageous to reorganize the array by dividing it into M rows of length N and combining them all to one string of length $S=NM$. The coordinate of a given point is now one $n+m=s$-digit binary number $z=x+Ny$ (instead of two numbers x and y). The corresponding quantum state is simply given by $|z\rangle=|x\rangle \otimes |y\rangle$. In terms of this representation, the quantum state $|\Psi\rangle$ prepared by the measurement of $f=1$ reads

$$|\Psi\rangle = \frac{1}{\sqrt{\varrho S}} \sum_{l=1}^{\varrho S} |z_l\rangle, \tag{4}$$

where $0 \geqslant z_l \geqslant S-1$ denotes position of the lth point (as an s-digit binary number).

The next basic part of the quantum algorithm is the application of the quantum Fourier transform (QFT). It acts on a basis element such as $|z\rangle=|110100\ldots\rangle$ as

$$\mathcal{F}: |z\rangle \rightarrow \frac{1}{\sqrt{S}} \sum_{k=0}^{S-1} \exp\left(2\pi i \frac{zk}{S}\right) |k\rangle. \tag{5}$$

Hence, the superposition state $|\Psi\rangle$ in Eq. (4) will be transformed into

$$\mathcal{F}|\Psi\rangle=\sum_{k=0}^{S-1}\sum_{l=1}^{\varrho S}\frac{1}{S\sqrt{\varrho}}\exp\left(2\pi i\frac{z_l k}{S}\right)|k\rangle. \tag{6}$$

Assuming a distribution of points z_l without any (macroscopic) pattern (e.g., purely random), there will be no privileged values of k (except $k=0$) and the measurement of $k>0$ yields just noise. However, the presence of a pattern within the dataset introduces a typical length scale and thus leads to peaks of the factor in front of $|k\rangle$ at certain values of k—which hence can be used as an indicator, see the following section.

In this way, the proposed algorithm efficiently solves the problem of feature selection—i.e., extracting a small amount of relevant quantities (such as wave numbers) from a large dataset: Operating the black box with an element of the computational basis (i.e., classically), a small number $(n+m)$ of refractors enables us to distinguish a large number (NM) of unit cells on the array. Furthermore, feeding in the superposition state in Eq. (3), the measurement of $f=1$ selects one out of a huge number—$O(2^S)$ for $\varrho=1/2$—of different states in Eq. (4), which correspond to the various possible pictures in the array (such as in Fig. 1). However, the distinct quantum states of this huge set are not mutually orthogonal and hence the picture cannot be reproduced from the state in Eq. (4), which indeed extracts a relatively small amount of relevant information from the huge dataset. Of course, this has only been possible by exploiting the, from a classical point of view, nonlocal quantum correlations and quantum parallelism.

V. PATTERN LOCALIZATION

The task of pattern recognition does not only include the mere detection of a pattern, but also its localization and classification. The comparison with a given set of templates will not be discussed here, see, e.g., Refs. [9–12]. The next step is to extract information about the pattern from the peaks in the measurements of k—in close analogy to the reconstruction of the probe structure from the Laue diagram in diffraction experiments.

Consider, for example, a simple pattern consisting of parallel lines such as the one in Fig. 1. In this case, the basic quantities are the distance $D=\text{const}$ of the lines as well as their orientation as described by the (constant) angle $-\pi/2\leqslant\vartheta\leqslant\pi/2$. Here, ϑ denotes the deviation of the line from a vertical one, i.e., after going down R rows the sequence is shifted by $R\tan\vartheta$ columns to the right. So the points z, marking the center of a particular line, are given by

$$z=z_0+[\mathbb{N}(N+\tan\vartheta)]_{\text{integer}}. \tag{7}$$

Note that (in contrast to Fig. 1) the lines do not need to be perfect—it is sufficient if, on an average, the density of points within a linewidth of, say, $D/2$ deviates by a finite amount $\Delta\varrho$ (e.g., $\Delta\varrho=1/4$) from the mean ϱ. [Since the Fourier transform in Eq. (6) involves a sum over many points, all small-scale fluctuations average out and hence the presence of noise inside the pattern as well as outside does not affect the main results, see also Eq. (12) below.]

According to Eq. (6), every row of the pattern generates peaks at

$$k=\left[\mathbb{N}\cos\vartheta\frac{S}{D}\pm O\left(\frac{M}{D\sqrt{\chi}}\right)\right]_{\text{integer}}, \tag{8}$$

with the second term denoting their width. Both, the position and the width of the peaks can be obtained from the associated Laue function $\mathfrak{L}(\xi,\kappa)=\sin^2(\pi\xi\kappa)/\sin^2(\pi\kappa)$ with $k=\kappa\cos\vartheta S/D$ and $\xi=O(N\sqrt{\chi})$ in this case.

However, the sum of all rows interferes constructively only if k is fine tuned according to

$$k=\left[\mathbb{N}\frac{N-\tan\vartheta}{N}M\pm O\left(\frac{1}{\sqrt{\chi}}\right)\right]_{\text{integer}}, \tag{9}$$

which again can be obtained from the associated Laue function with now $\kappa=k(N+\tan\vartheta)/S$ and $\xi=O(M\sqrt{\chi})$. The strongest peaks in the measurements of k occur for values that satisfy both conditions (8) and (9) simultaneously. Accordingly, the wave numbers of these potential peaks read

$$k\approx\left[\mathbb{N}\left(\cos\vartheta\frac{S}{D}-\sin\vartheta\frac{M}{D}\right)\right]_{\text{integer}}, \tag{10}$$

where the corresponding width or uncertainty has been omitted.

However, for large enough D, not every peak in Eq. (8) will contain wave numbers matching Eq. (9), in general. The condition for this to take place is that the integer $\mathbb{N}$ in Eq. (8) multiplied by $\cos\vartheta N/D$ is again close to another integer—the one in Eq. (9)—within an accuracy of $O(1/[D\sqrt{\chi}])$.

Therefore, not all the k values in Eq. (10) do necessarily represent large peaks—the first few of them may be suppressed. On the other hand, the larger D is, the more k values of potential peaks in Eq. (10) are contained in the interval $0<k<S$. Consequently, from the number $D/\cos\vartheta$ of potential peaks in Eq. (10), there must be at least a few $O(1/\sqrt{\chi})$ which match both conditions (8) and (9).

Determining the largest common factor of all the wave numbers of the peaks (within the given accuracy), we obtain a value for the expression in Eq. (10)—possibly multiplied by an integer. Unfortunately, this information alone is not sufficient for extracting D and ϑ. To this end, we may simply transpose the array (e.g., Fig. 1) by interchanging rows and columns $N\leftrightarrow M$ and run the same algorithm again. Since transposing corresponds to $\vartheta\rightarrow\pi/2-\vartheta$ the wave numbers of the peaks are now

$$k'\approx\left[\mathbb{N}\left(\sin\vartheta\frac{S}{D}-\cos\vartheta\frac{N}{D}\right)\right]_{\text{integer}}. \tag{11}$$

Combining the possible values for $D/\cos\vartheta$ from Eq. (10) with the ones for $D/\sin\vartheta$ from Eq. (11), we obtain approximate candidates for D and ϑ. Comparing with the fine-tuning conditions such as Eqs. (8) and (9) and knowing which peaks are suppressed and which do not allow us to

extract the actual values of D and ϑ with high (in fact, maximum attainable) precision $O(1/\sqrt{\chi})$.

Alternatively, if D turns out to be very large, one might decrease the resolution (i.e., average over many cells) and repeat the algorithm until one reaches the minimum resolution for which it is still possible to resolve the pattern. An efficient way to do this is by bisecting the remaining intervals, i.e., after running the algorithm with the maximum resolution $\{M,N\}$, we go over to $M \to M^{1/2}$ as well as $N \to N^{1/2}$ and repeat the algorithm. If we still see peaks, we decrease the resolution even more, i.e., $M \to M^{1/4}$ as well as $N \to N^{1/4}$, and, if not, we increase it again $M \to M^{3/4}$ as well as $N \to N^{3/4}$, etc. In this way, one can determine the minimum resolution necessary for resolving the pattern in $O(\log_2 s) = O(\log_2 \log_2 S)$ steps. In terms of this resolution, the value of D is reasonably small $D = O(1)$ and hence extracting the values of D and ϑ from the peaks in Eqs. (10) and (11) is easy. The above procedure saves (classical) calculation time (determining the largest common factor, etc.), but, on the other hand, requires slightly more runs of the black box $O(\log_2 s) = O(\log_2 \log_2 S)$. Either way, one can always figure out D and ϑ with (arbitrarily) high probability.

The height of the peaks can be estimated by means of Eq. (6). In the resonance case, the sum includes $S\chi\Delta\varrho$ constructively interfering addends, which lead to an amplitude of order $O(\chi\Delta\varrho/\sqrt{\varrho})$. Thus, the probability p of measuring the peaks in Eqs. (10) and (11) is given by

$$p = O\left(\frac{(\chi\Delta\varrho)^2}{\varrho}\right), \tag{12}$$

i.e., independent of N and M—and, therefore, drastically enhanced over the (random) noise.

Consequently, if a number Ω of measurements yields one or more pronounced peaks besides $k=0$, then there exists a pattern larger than $\chi_{\min} = O(1/\sqrt{\Omega})$ and not otherwise (at least with a very high probability). Quite reasonably, the smaller the pattern (i.e., χ) is, the longer one has to search.

After having solved the feature selection problem efficiently by the quantum algorithm, the remaining analysis (peak finding and stopping criteria, etc.) of a small (independent of N and M) amount of measured wave numbers can be accomplished by a classical algorithm.

In this way, one can determine the size of the pattern χ by the frequency of measuring the peaks at k and k' (and their width). Its structure (i.e., the values of D and ϑ) can be inferred from the location of the peaks.

Having found the parameters D, ϑ, and χ of the pattern, it can be localized easily—for example, by dividing the total $N \times M$ array into smaller pieces (according to χ) and running the same quantum algorithm again in the smaller domains.

More complex (but still linear) patterns, such as a regularly recurring pictures[1] (such as in many wall papers), possess more than one characteristic angle ϑ, in general, and therefore generate a richer peak structure—but the main idea remains the same.

VI. COMPLEXITY ANALYSIS

Let us estimate the size of the proposed algorithm, i.e., the number of involved computational steps, and compare it with the classical method in the limit $S \to \infty$ while ϱ, $\Delta\varrho$, and χ remain finite.

In view of Eq. (12), we need only a few—$O(S^0)$ or $O(\log_2 \log_2 S)$, cf. Sec. V—queries of the black box in order to find a pattern of a given size with high probability. Clearly, this is not possible with any classical algorithm—demonstrating the advantage of the global quantum computation over the local (only one point at a time) classical technique. Since the number of queries of the black box corresponds to the total amount of photons shining on the array, the quantum algorithm causes lesser damage (cf. Sec. II) than any classical method, see also Ref. [14].

Given the explicit physical realization of the black box described in Sec. III, it is also possible to compare the total number of fundamental manipulations. For the preparation of the initial state in Eq. (3), one has to apply the Hadamard gate $m+n=s=\log_2 S$ times. The black box itself involves about the same number of operations. The QFT in Eq. (5) requires $O(\log_2^2 S)$ steps for obtaining the exact result and is even faster $O(\log_2 S)$ if we measure [2] the outcome immediately afterwards—as it is the case here.

In contrast, the best (known) classical algorithm, the fast Fourier transform (FFT), implements $O(S \log_2 S)$ operations and is therefore exponentially slower. Note that, since we do not know the typical ''wave numbers'' k associated with the pattern *a priori*, we would have to calculate the FFT for a large number $O(S)$ of possible values of k—whereas the QFT accomplishes all this simultaneously, and automatically gives us the values k with the largest amplitudes in average measurements.

However, it cannot be excluded here that perhaps a classical algorithm exists, which is better than the FFT and may compete with the proposed quantum algorithm (though not in the number of queries of the black box). But since the processing of the coordinates of only one single point already requires $O(\log_2 S)$ operations, one would have to find the pattern by considering a few $O(S^0)$ or $O(\log_2 \log_2 S)$ points in order to outrun the quantum computer—which is apparently not possible.

Nevertheless, in certain situations—e.g., for perfect lines with $D = O(1)$ such as in Fig. 1—it is possible to design an appropriate classical algorithm that determines ϑ by using only $O(\log_2^q S)$ points with $q \geqslant 1$, and thus requires $O(\log_2^{q+1} S)$ computational steps. In this case, the speedup is merely polynomial.

In most cases, however, where D can be very large $D \gg 1$ and the lines are not perfect, it is really difficult to see how one might extract basically the same information as the FFT in (polynomially) logarithmic time with classical methods. [An $O(\sqrt{S})$ algorithm, for example, would also be exponentially slower than the QFT.]

[1] Again, these pictures do not need to be perfect—average features are sufficient.

Assuming that there is indeed no such classical algorithm (or that the set of patterns, for which the Fourier transform is the best classifier, is not empty), the problem under consideration represents another example for the (conjectured) exponential speedup of quantum information processing—based on the power of QFT for problems related to (quasi) periodical structures (which is also the basic factor for Shor's algorithm [3]; though in that case the periodicity is exact—in contrast to the situation considered here).

Of course, such a speedup has only been possible since it was not necessary to load the complete array into a quantum memory (cf. Refs. [9–12])—this would have involved about $O(S)$ operations, and thereby lead to a drastic (exponential) slow-down.

VII. SUMMARY AND OUTLOOK

In summary, quantum algorithms are capable of solving certain problems of pattern recognition (i.e., detection, localization, and classification) aside from template matching [9–12] much faster than their classical counterparts [15]. Although this has been demonstrated explicitly in the present paper for line patterns only, the basic idea applies to more difficult (but still linear) patterns as well. (In some sense, this idea arises from the often successful approach to copy the ingenious solutions which nature reveals to us—such as the possibility of distinguishing a crystal from an amorphous material via x-ray diffraction or the method of optical filtering, see the following section.) The investigation of nonlinear patterns, such as a set of concentric circles, is apparently more involved and would require adapted methods.

Together with the findings in Refs. [9–12], the results of the present paper give rise to the hope that quantum algorithms are also advantageous for more general pattern recognition problems.

VIII. OPTICAL FILTERING

Interestingly, the manipulations involved in the proposed quantum algorithm—apart from the physical realization of the black box itself—can be accomplished (at least, in principle) with present-day optical devices. Hadamard gates as well as the calculation of the quantum Fourier transform with subsequently measuring the outcome can be realized by using beam splitters and classically controlled phase shifters, cf. Ref. [2]. This observation leads to the question of whether one could achieve a similar performance with purely optical techniques.

Indeed, the method of optical filtering reproduces some key features of the proposed quantum algorithm. Shining a plane wave with an appropriate wave number on an aperture mask as in Fig. 1, the far-field diffraction amplitudes are given by the Fourier transform of the object (e.g., Fig. 1) in terms of the perpendicular wave number. Using an ordinary convex lens, one may convert these wave numbers into positions in the focal plane of the lens, see, e.g., Ref. [16].

In this way the described apparatus effectively calculates the desired Fourier transform. However, this method goes along with serious problems and disadvantages: first, it appears to be impossible to achieve the same accuracy as the proposed quantum algorithm, i.e., $O(\log_2 S)$ digits for ϑ and D, without exponentially increasing effort; and, second, the fact that D is not known *a priori* and may vary over several orders of magnitude makes it difficult to select a suitable wave number for the incident light. In addition, the investigation of the array (black or white cell) via incident photons is restricted to a certain frequency window generally.

These obstacles are caused by the main difference between optical filtering and the proposed quantum algorithm. In optical filtering, the relevant quantities (such as wave number, position in the focal plane, ϑ, and D) are directly (i.e., linearly) related to each other—whereas the quantum algorithm employs the digital representation and therewith outperforms the former method.

On the other hand, if we happen to know the order of magnitude of D in advance and would be willing to settle for a limited accuracy of only a few $O(S^0)$ digits, we may detect the pattern with optical filtering by using only a few $O(S^0)$ photons—provided that the corresponding wave number matches with the frequency window of the array. Although much less powerful, i.e., accurate, this method would be significantly faster than the proposed quantum algorithm, which also uses only a few photons shining on the pattern—but it requires $O(\log_2 S)$ photons (or, more generally, qubits) for the subsequent data analysis (QFT, etc.). (This enhancement in speed—although at a certain cost—reflects the fact that a quantum field theoretical object, such as a photon, has more degrees of freedom than just one single qubit.)

ACKNOWLEDGMENTS

The author acknowledges valuable discussions with L. D'Afonseca, D. Curtis, S. Fuchs, Y. Gusev, D. Meyer, N. Pippenger, S. Popescu, G. Schaller, C. Trugenberger, and B. Unruh. This work was supported by the Alexander von Humboldt Foundation and by the Natural Science and Engineering Research Council of Canada.

APPENDIX

If there were no absorptive cells in the array at all, but only (perfectly) reflective (white) and transparent (black) ones, one could (in principle) realize the following generalization of the black box in Eq. (1):

$$\mathcal{B}:\begin{bmatrix}|x\rangle\\ |y\rangle\\ |\alpha\rangle\end{bmatrix}\rightarrow\begin{bmatrix}|x\rangle\\ |y\rangle\\ |\alpha\oplus f(x,y)\rangle\end{bmatrix}, \tag{A1}$$

with $\alpha=0,1$. Here, $\oplus$ denotes summation modulo 2, i.e., $1\oplus 0=0\oplus 1=1$ and $0\oplus 0=1\oplus 1=0$. In this case, one may improve the quantum algorithm by sending the superposition state in Eq. (3) with the third register being $(|0\rangle-|1\rangle)/\sqrt{2}$

instead of $|0\rangle$ to the black box. The third register does not change during this procedure and the resulting state encodes the information about the points in the array in the phases $(+1$ or $-1)$ instead of the amplitudes $(1$ or $0)$

$$|\Psi\rangle = \frac{1}{\sqrt{S}} \sum_{z=0}^{S-1} (-1)^{f(z)} |z\rangle. \tag{A2}$$

Assuming $\varrho = 1/2$, the quantum Fourier transform of this state has certain advantages over the one in Eq. (4)—one gets rid of the (useless) peak at $k=0$ and enhances the probabilities of the other peaks by a factor of 2. Unfortunately, it is hard to see how one might be able to exploit this advantage of the modified black box in the presence of absorbing units.

[1] K. Fukunaga, *Introduction to Statistical Pattern Recognition* (Academic Press, New York, 1972).

[2] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000); see also J. Preskill, Quantum Computation and Information, Lecture Notes (see URL: http://www.theory.caltech.edu/people/preskill/ph229)

[3] P.W. Shor, SIAM J. Comput. **26**, 1484 (1997).

[4] L. Grover, Phys. Rev. Lett. **79**, 325 (1997).

[5] D. Deutsch, Proc. R. Soc. London, Ser. A **400**, 97 (1985).

[6] D. Deutsch and R. Jozsa, Proc. R. Soc. London, Ser. A **439**, 553 (1992).

[7] D.R. Simon, SIAM J. Comput. **26**, 1474 (1997).

[8] E. Bernstein and U. Vazirani, SIAM J. Comput. **26**, 1411 (1997).

[9] C.A. Trugenberger, Phys. Rev. Lett. **87**, 067901 (2001).

[10] C.A. Trugenberger, Phys. Rev. Lett. **89**, 277903 (2002).

[11] M. Sasaki, A. Carlini, and R. Jozsa, Phys. Rev. A **64**, 022317 (2001).

[12] M. Sasaki and A. Carlini, Phys. Rev. A **66**, 022303 (2002).

[13] D. Horn and A. Gottlieb, Phys. Rev. Lett. **88**, 018702 (2002).

[14] A.C. Elitzur and L. Vaidman, Found. Phys. **23**, 987 (1993).

[15] Speculating a little outside the scope of the present paper, one might ask whether the human brain, which is capable of solving such pattern recognition problems very well, actually makes use of similar quantum effects. Although this idea has some appealing aspects (consider optical illusions, for example), it appears to be somewhat unlikely.

[16] B.E.A. Saleh and M.C. Teich, *Fundamentals of Photonics* (Wiley, New York, 1991).