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Decoherence from chaotic internal dynamics in two coupled δ-function-kicked rotors

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We show that classical-quantum correspondence of center-of-mass motion in two coupled δ-kicked rotors is enhanced by the entanglement of the center of-mass motion to the internal degree of freedom. The observed correspondence can be attributed to the decoherence generated from chaotic internal dynamics with a few degrees of freedom.

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Classical-quantum correspondence in a classically chaotic system has been one of the most interesting problem in physics for a long time [1,2]. In quantum mechanics, the time evolution of a wave function follows a *linear* Schrödinger equation, and so there is no possibility of the exponential sensitive dependance of solutions on the initial condition, a trademark of classical chaos. Also, chaotic diffusion is suppressed by quantum localization [3]. It has been revealed that some crossover time $t_r = \ln(I/\hbar)/\lambda$ (I is a characteristic action and λ is a Lyapunov exponent) exists so that classical-quantum correspondence breaks down for $t > t_r$ [4].

Recently, the relation between decoherence and the classical-quantum correspondence has been investigated extensively [2,5–15]. Decoherence breaks the purity of initial superposition, which should be conserved in the absence of coupling to environment, and thus only the partial fraction of whole Hilbert space, namely pointer states, are selected by the environment [6]. The dynamics of the system coupled to the environment shows the unique characteristics of the system independent of the coupling strength as long as it is not too large or small [10,11]. In other words, with appropriate coupling to environment, the Lyapunov exponent or entropy production rates, which are important physical quantities characterizing a chaotic system, can be reproduced quantum mechanically.

Usually the environment which interacts as well as decoheres the system is assumed to be an external heat bath with large degrees of freedom. For a composed (macroscopic) system, however, decoherence can be obtained from its own internal dynamics which behaves like a thermal heat bath effectively [13,14]. In this paper, we show that the decoherence of the center-of-mass (c.m.) motion of a composed system can occur through the entanglement of its own subsystems with even a few degrees of freedom due to the chaotic nature of internal dynamics.

Let us consider a classical object governed by a Hamiltonian $H_0 = P^2/2M + V(X)$, where M is the mass of the classical object. Since classical objects are composed of many particles, a complete Hamiltonian will be given by $H = \Sigma_i[p_i^2/2m_i + V_{ext}(x_i)] + \Sigma_{i,j} V_{int}(x_i, x_j)$, where p_i and x_i are momenta and coordinates of the constituents, respectively. Without the external potential V_{ext}, the dynamics of c.m. is separable and becomes trivial; the c.m. of the isolated system moves in a straight line with a constant velocity. We are interested in the case where the dynamics of the whole system (macroscopic object) is chaotic; therefore we assume the existence of the external potential which yields a nonlinear force. With V_{ext}, the dynamics of c.m. is coupled to the internal dynamics [16], and this coupling can induce a decoherence effect on the dynamics of center of mass. In this case, the minor differences between H and H_0 play an important role, and the correct classical-quantum correspondence of the center of mass motion will be obtained not from H_0 but from H.

The decoherence from the internal dynamics of a macroscopic particle has been studied by using a master equation [13,14]. To obtain the master equation, however, it has been assumed that the internal degrees of freedom act like a thermal heat bath. In contrast, we investigate the exact quantum mechanical dynamics of a two-particle system, so that the observed decoherence has purely dynamical origin. We will show the delocalization of wave functions in momentum space and the coincidence of classical and quantum entropy production rates. In our system, the internal dynamics which serves as an environment consists of a few degrees of freedom. One may suspect that the internal degree of freedom is too small to generate the decoherence. However, we show that the chaotic internal dynamics with a single degree of freedom is enough to efficiently produce the decoherence in agreement with other observations [17,18].

As a model for the decoherence generated from the chaotic internal dynamics, we consider the center-of-mass motion of the spatially confined two δ-kicked rotors. The governing Hamiltonian is given by

$$H = \frac{1}{2m}(p_1^2 + p_2^2) + U(r_1, r_2) + k[\cos(r_1) + \cos(r_2)] \sum_i \delta(t - iT), \qquad (1)$$

where r_1 and r_2 are angle variables with range 2π radians and $U(r_1, r_2)$ is the interaction potential which confines two particles within a distance w. If the confinement width w goes to zero, then the system reduces to a usual δ-kicked rotor. Similar systems such as two coupled quantum kicked tops [15], two interacting spins [19], and two interacting par-

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ticles (TIP) in a random potential [20,21] have been studied. The decoherence effect from internal dynamics, however, has not been discussed.

To investigate the center-of-mass motion, we introduce canonical transforms $R=(r_1+r_2)/2$, $r=r_1-r_2$, $P=p_1+p_2$, and $p=(p_1-p_2)/2$. Then, we obtain the following Hamiltonian:

$$H=\frac{P^2}{2M}+\frac{p^2}{2\mu}+U(r)+K\cos(R)\cos\left(\frac{r}{2}\right)\sum_i \delta(t-iT), \quad (2)$$

where $M=2m$, $\mu=m/2$, and $K=2k$. Here, $U(r)$ corresponds to the confining potential with impenetrable walls at $r=\pm w$. δ kicks described by the last term of Eq. (2) yield the interaction between the c.m. motion and the internal degree of freedom, i.e., the motion of the reduced mass μ.

The time evolution of the wave function $\Psi(R,r,t)$ is given by simple maps. Between each kick occurring at $t=iT$ (i is an integer), the c.m. motion and the internal motion evolve independently, so that we obtain

$$\psi(R,r,NT+0^-)=\exp\left(-i\frac{P^2}{2M}\frac{T}{\hbar}\right)\exp\left\{-i\left[\frac{p^2}{2\mu}+U(r)\right]\frac{T}{\hbar}\right\}\Psi(R,r,(N-1)T+0^+). \quad (3)$$

The wave functions just before and after the δ kick at $t=NT$ are related by the following mapping:

$$\psi(R,r,NT+0^+)=\exp\left[-iK\cos(R)\cos\left(\frac{r}{2}\right)\right]\times\Psi(R,r,NT+0^-). \quad (4)$$

Combining the two maps given in Eqs. (3) and (4), we numerically calculate the evolution of the wave function $\Psi(R,r,t)$ for various $\hbar$ and w. For most cases, the number of basis states used for the motion of coordinate R and r are 16 384 and 256, respectively, while 32 768 and 512 basis are used, respectively, for small $\hbar$. The initial condition is chosen to be the ground state $\Psi(R,r,0)=1/\sqrt{w}\cos(\pi r/2w)$.

Classical evolution is obtained from a four-dimensional map for R,P,r, and p derived from Hamiltonian (2). We consider an ensemble of particles which are distributed from $R=0$ to $R=2\pi$ uniformly with $P=0$, and from $r=-w$ to $r=w$ with a probability $\cos^2(\pi r/2w)/w$ with $p=\pm p_0=\pm(\pi\hbar)/(2w)$. From this initial condition, the classical evolution is simulated, and the ensemble average of the normalized variance of center-of-mass momentum, $\Delta^2=2(\langle P^2\rangle-\langle P\rangle^2)/MK^2$, is computed.

As a reference, let us mention the case with a single δ-kicked rotor governed by the Hamiltonian $H=P^2/2M+K\cos(R)\Sigma_i\delta(t-iT)$, which corresponds to the case with $w=0$. For $K=5$, which is used in this work, the system is fully chaotic, and the classical kinetic energy increases diffusively. But the diffusion is suppressed quantum mechanically and the classical-quantum correspondence breaks down, which is the well-known dynamical localization [3].

Now, we consider two δ-kicked rotors. The inset in Fig. 1 shows the differences of classical momentum variances between single and two δ-kicked rotors for various w. As w is decreased, the difference between them vanishes. Meanwhile, shown in Fig. 1 are the differences between the classical and the quantum variances for various w with $\hbar=0.07$. One clearly sees that the quantum localization breaks down when w is increased. The difference of Δ_{cl} and Δ_{qm} nearly disappears for large w, while deviations are observed for small w.

The breakdown of quantum localization, i.e., delocalization, is directly visible in the momentum distribution function in Fig. 2, where the probability distribution of the center-of-mass momentum P are shown at time step $n=500$ for several $\hbar$ and w. The exponential localization is changed into a rather broad Gaussian-like profile as we decrease $\hbar$ or increase w. Let us note that the delocalization of wave functions alone is not enough to prove the occurrence of decoherence. In fact, the delocalization was also observed in the study of TIP in a random potential [20,21]. However, it was not attributed to the decoherence.

Next, we consider the reduced density matrix for the center-of-mass motion in order to show that the observed

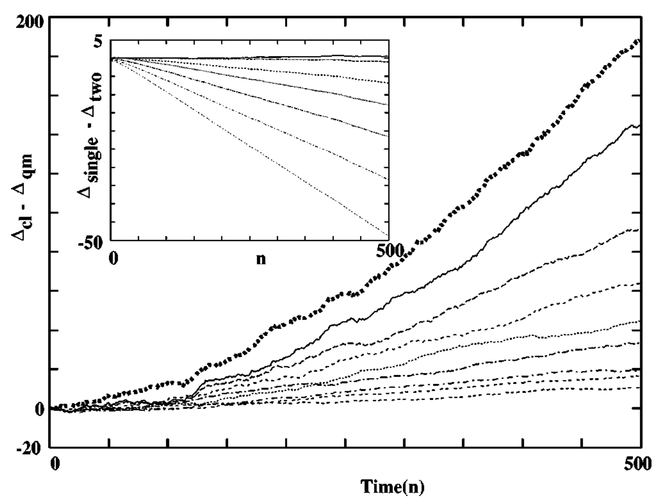

FIG. 1. The differences between classical and quantum momentum variances, $\Delta_{cl}-\Delta_{qm}$, are plotted with $M=1$, $\mu=0.25$, $K=5$, $T=1$, and $\hbar=0.07$. From top to bottom, $w=0.0$ (i.e., single kicked rotor), 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7. (Inset) The difference between variances of single and two coupled δ-kicked rotors, $\Delta_{single}-\Delta_{two}$, for various w are shown. From top to bottom, $w=0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8.

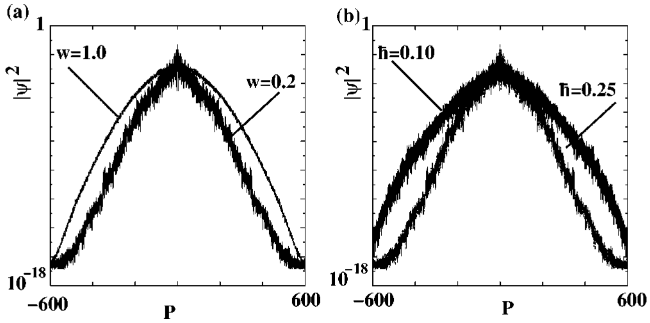

FIG. 2. Momentum distribution functions. (a) $\hbar=0.25$, $w=0.2$ and 1. (b) $w=0.2$, $\hbar=0.1$, and $\hbar=0.25$. Other parameters are the same as in Fig. 1.

classical-quantum correspondence is ascribed to the decoherence caused from the interaction with the internal degree of freedom. The reduced density matrix $\rho_R(R_1,R_2)$ is given by $\mathrm{Tr}_r(\rho)=\Sigma_r\psi(R_1,r)\psi^*(R_2,r)$. As a quantitative measure of decoherence, we calculate the linear entropy $s_l=\mathrm{Tr}(\rho_R-\rho_R^2)$ [6]. Note that for a pure state, $s_l=0$; while for a maximum decoherence, $s_l=1$. Figure 3 shows that for large w the entropy s_l rapidly approaches 1, i.e., maximum decoherence. As we decrease w, the entropy s_l shows rather reduced values and slowly increases in time. In fact, the energy-level spacing of the internal dynamics described by $p^2/2\mu+U(r)$ is proportional to $1/w^2$, which means the smaller w, the larger the level spacing. For a given perturbation determined by K, it is more difficult to excite the internal dynamics in the case of large spacing, which leads to effective decoupling between the c.m. motion and the internal degree of freedom and eventually leads to the decrease of decoherence. This is consistent with the previous result that the breakdown of the localization and the classical-quantum correspondence is easily obtained for large w, i.e., strong decoherence.

Due to the chaotic nature of the internal dynamics, $\cos(r/2)$ in the last term of Hamiltonian (2) can be regarded as an amplitude noise of the kick onto the center-of-mass motion. As w decreases, the noise is reduced and so is the effect of decoherence. Ott *et al.* studied the effect of noise on the single δ-kicked rotor, which showed that, for moderate noise and $\hbar$, the diffusion coefficient D_{qm} is proportional to the square of the noise level, more precisely $D_{qm}\sim\nu^2(K/\hbar)^4$ (ν is a noise strength) [22]. For a very small $\hbar$, it is obtained that $D_{qm}\simeq D_{cl}$ (D_{cl} is a classical diffusion coefficient). If we directly apply this result to our case, we immediately obtain the scaling relation $D_{qm}\sim(w/\hbar)^4K^6$ for moderate $\hbar$ since we can approximately regard $K[1-\cos(r/2)]$ as noise amplitude, and so $\nu\sim Kw^2$. It also implies that the decoherence time $t_d\sim\hbar^2/\nu^2\sim\hbar^2/(K^2w^4)$. Note that the diffusion coefficient D_{qm} is given by Δ_1^2/t_d where $\Delta_1(=K^2/\hbar)$ is localization length of the single δ-kicked rotor in P [22]. The scaling is confirmed by numerically calculating D_{qm} as a function of $(w/\hbar)$ in Fig. 4, where $\hbar=0.25$, and $K=5$. These results also confirm the above proposition that the term $\cos(r/2)$ in Hamiltonian (2) can be

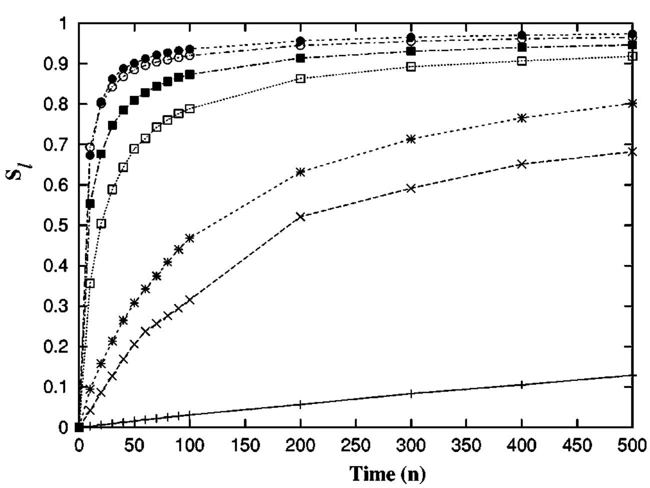

FIG. 3. Linear entropies for various w. From bottom to top, $w=0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7. Other parameters are the same as in Fig. 1.

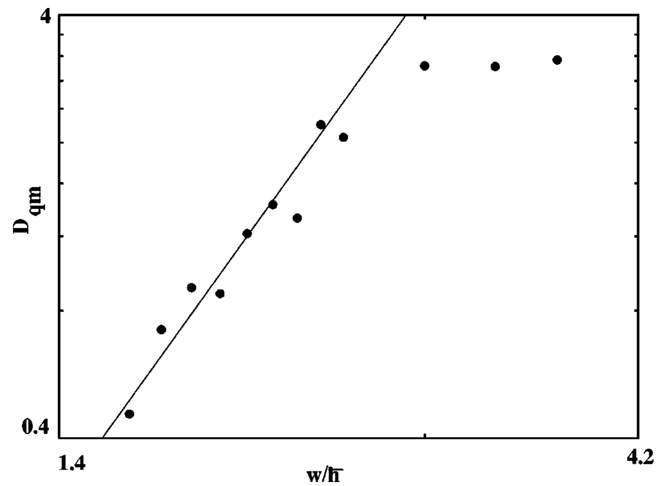

FIG. 4. Quantum diffusion coefficient D_{qm} as a function of $(w/\hbar)$ with $\hbar=0.25$. The solid line corresponds to $D_{qm}\propto(w/\hbar)^4$. Other parameters are the same as in Fig. 1.

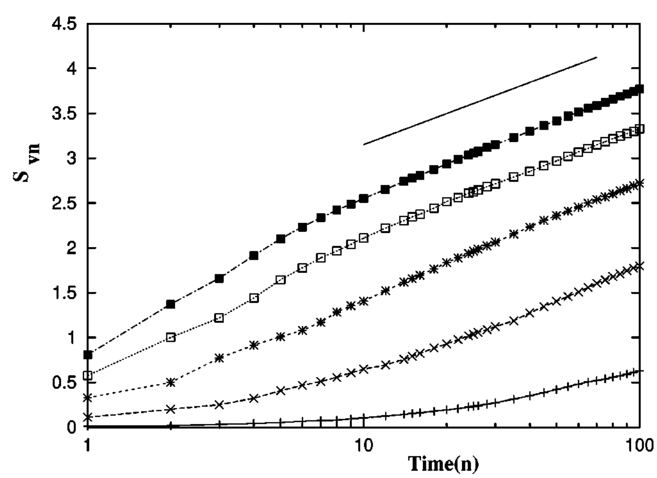

FIG. 5. The von Neumann entropies obtained from the reduced density matrix ρ_R with $\hbar=0.07$. For reference, the $0.5\ln(n)$ dependence of S_{cl} is represented by the solid line. From bottom to top, $w=0.2$, 0.4, 0.6, 0.8, and 1.0.

treated as noise and thus induces decoherence.

Finally, we consider the classical and the quantum entropy production rates of which coincidence has been the criteria for the classical-quantum correspondence of chaotic systems in the studies of decoherence [10]. Quantum mechanically, the von Neumann entropy is given by $S_{vn}=\mathrm{Tr}[\rho_R \ln(\rho_R)]$. Due to the difficulty in computing the eigenvalues of large matrix, we use a smaller number of basis states than that used in the previous simulation; 4096 for the motion of R, and 256 for r. We choose $\hbar=0.07$ and can obtain the quantum evolution up to $n=100$ with this smaller number of basis. The time evolution of the von Neumann entropy consists of two different regimes [23] as shown in Fig. 5. After the initial transient, the von Neumann entropy increases logarithmically in time, which exactly corresponds to its classical counterpart [12]. Let us consider the phase space (R,P) for the center-of-mass motion. Classical distributions are uniform in R and Gaussian in P. The variance of momentum increases diffusively. As a result, the classical entropy is given by $S_{cl}\sim 0.5\ln(n)$. In Fig. 5, the quantum results (dots) show $0.5\ln(n)$ dependence, consistent with the classical prediction.

In summary, we have studied the dynamics of the center-of-mass motion of two coupled δ-kicked rotors, and shown that classical-quantum correspondence is achieved by the decoherence induced by the internal dynamics. Neither external noise nor the outer environment are assumed. The decoherence arising from an environment containing only a single degree of freedom (internal dynamics) should be attributed to the chaotic nature of the internal dynamics. The scaling law of D_{qm} strongly suggests that internal degree of freedom behaves like a thermal noise source due to the chaotic dynamics. We hope that similar results would be observed in experiments using molecules when they are subjected to an external nonlinear force depending on the position of the constituent atoms.

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[16] Let us assume that the mass of each constituent m_i is equal to m. If the force $f_i=-dV_{ext}(x_i)/dx_i$ is linear, the center-of-mass dynamics is decoupled from the internal dynamics since $M\ddot{X}=\Sigma f_i=k\Sigma x_i=NkX$, where X, N, and k are a position of center of mass, the number of constituents, and a constant characterizing the linear force, respectively. If the force f_i is nonlinear, however, Σf_i cannot be described by the single variable X; the center-of-mass motion is coupled to the internal dynamics.

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