Generation of entangled *N*-photon states in a two-mode Jaynes-Cummings model

C. Wildfeuer* and D. H. Schiller
Fachbereich Physik, Universität Siegen, D-57068 Siegen, Germany
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We describe a mathematical solution for the generation of entangled *N*-photon states in two field modes. A simple and compact solution is presented for a two-mode Jaynes-Cummings model by combining the two field modes in a way that only one of the two resulting quasimodes enters in the interaction term. The formalism developed is then applied to calculate various generation probabilities analytically. We show that entanglement, starting from an initial field and an atom in one defined state may be obtained in a single step. We also show that entanglement may be built up in the case of an empty cavity and excited atoms whose final states are detected, as well as in the case when the final states of the initially excited atoms are not detected.

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I. INTRODUCTION

Entangled states are one of the building blocks in quantum information processing and nonlocality tests [1]. They can be used, in the case of the electromagnetic field, to improve the sensitivity of interferometric measurements [2–5], and may help to overcome the classical Rayleigh diffraction limit in quantum optical lithography [6]. A feasible way to generate such states is given by the atom-field interaction in the framework of one- or two-mode Jaynes-Cummings (JC) models [7–12].

Here we consider the generation of entangled two-mode field states by different schemes inspired partly by Refs. [7,8,13]. We let two-level atoms interact, one at a time, with two degenerate modes of a lossless cavity. Solving the corresponding JC model algebraically by an SU(2) transformation, we discuss the generation of entangled *N*-photon states of the general form

$$|\Psi_N\rangle=\sum_{k=0}^{N} c_k^{(N)}|N-k,k\rangle, \tag{1}$$

which comprises the maximally entangled Bell states

$$|\Psi_N^{\pm}\rangle=\frac{1}{\sqrt{2}}(|N,0\rangle\pm|0,N\rangle). \tag{2}$$

The field states are defined in terms of the usual two-mode Fock states $|n_1,n_2\rangle:=|n_1\rangle_1|n_2\rangle_2$, with n_1 (n_2) photons in mode 1 (2). The two modes have the same energy and are in resonance with the two-level atom. We solve the model algebraically by combining the two field modes into two quasimodes, of which only one enters in the interaction term, yielding an effective one-mode JC model [14]. Using its known solution and the transformation between mode and quasimode Fock states, the generation probabilities of the entangled states are found for three different schemes.

*Electronic address: wildfeuer@physik.uni-siegen.de

II. ALGEBRAIC SOLUTION OF THE TWO-MODE JAYNES-CUMMINGS MODEL

The JC Hamiltonian for resonant interaction of a two-level atom ($|e\rangle,|g\rangle$) with two field modes (a_1,a_2) in the dipole and rotating wave approximation is given by $H=H_0+H_{\rm int}$, where

$$H_0=\hbar\omega\left[\frac{\sigma_z+\mathbf{1}}{2}+(a_1^\dagger a_1+a_2^\dagger a_2)\mathbf{1}\right], \tag{3}$$

$$H_{\rm int}=\hbar[\sigma^+(g_1a_1+g_2a_2)+\sigma^-(g_1^*a_1^\dagger+g_2^*a_2^\dagger)]. \tag{4}$$

Here $\sigma_z:=|e\rangle\langle e|-|g\rangle\langle g|$, $\sigma^+:=|e\rangle\langle g|$, $\sigma^-:=|g\rangle\langle e|$, and $\mathbf{1}=|e\rangle\langle e|+|g\rangle\langle g|$ are operators for the two-level atom, g_i is the coupling constant of the ith mode with the atom, and $\hbar\omega$ is the photon energy. We introduce the *quasimode* operators

$$A_1=\gamma_1a_1+\gamma_2a_2,\quad A_2=-\gamma_2^*a_1+\gamma_1^*a_2, \tag{5}$$

where $\gamma_i:=g_i/g$ and $g:=\sqrt{|g_1|^2+|g_2|^2}$. Equation (5) defines an SU(2) transformation of the mode operators a_1,a_2, leaving the commutation relations and the number-sum operator $a_1^\dagger a_1+a_2^\dagger a_2=A_1^\dagger A_1+A_2^\dagger A_2$ invariant. The transformed Hamiltonian then reads

$$H_0=\hbar\omega\left[\frac{\sigma_z+\mathbf{1}}{2}+(A_1^\dagger A_1+A_2^\dagger A_2)\mathbf{1}\right], \tag{6}$$

$$H_{\rm int}=\hbar g(\sigma^+A_1+\sigma^-A_1^\dagger), \tag{7}$$

representing a JC Hamiltonian for the quasimode A_1 decoupled from noninteracting quasimode A_2. Since $H_{\rm int}$ depends only on quasimode 1 and $[H_0,H_{\rm int}]=0$, the time evolution operator $U(t)=\exp(-iH_{\rm int}t/\hbar)$ in the interaction picture is the same as for a one-mode JC model. Expanding U in the atom basis $\{|e\rangle,|g\rangle\}$

$$U=U_{ee}|e\rangle\langle e|+U_{ge}|g\rangle\langle e|+U_{eg}|e\rangle\langle g|+U_{gg}|g\rangle\langle g|, \tag{8}$$

the matrix elements $U_{ab}(t)$ are given by [15]

$$U_{ee}=\cos(\tau\sqrt{A_1^\dagger A_1+1}),\quad U_{ge}=A_1^\dagger\frac{\sin(\tau\sqrt{A_1^\dagger A_1+1})}{i\sqrt{A_1^\dagger A_1+1}},$$

$$U_{eg}=\frac{\sin(\tau\sqrt{A_1^\dagger A_1+1})}{i\sqrt{A_1^\dagger A_1+1}}A_1,\quad U_{gg}=\cos(\tau\sqrt{A_1^\dagger A_1}),\qquad(9)$$

where $\tau:=gt$ is the dimensionless "interaction time." The model can be solved in the usual way in terms of quasimode Fock states defined as the common eigenstates of $A_1^\dagger A_1$ and $A_2^\dagger A_2$. The complete solution is then found by giving the relation between the quasimode and the mode Fock states.

The quasimode operators A_i, $A_i^\dagger$, $i=1,2$, obey the same algebra as the mode operators a_i, $a_i^\dagger$, so that two-quasimode Fock states (denoted by a double-ket) can be defined by

$$|n_1,n_2\rangle\rangle:=\frac{A_1^{\dagger n_1}A_2^{\dagger n_2}}{\sqrt{n_1!n_2!}}|0,0\rangle\rangle.\qquad(10)$$

To find the transformation between the two-mode Fock states $|n_1,n_2\rangle$ and the two-quasi-mode Fock states $|n_1,n_2\rangle\rangle$, we use Schwinger's oscillator model [16] and introduce angular-momentum states $|j,m\rangle$ and $|j,m\rangle\rangle$, where $j=(n_1+n_2)/2$ and $m=(n_1-n_2)/2$. In cases where it is not obvious, we shall write subscript S on the state vectors to indicate the Schwinger angular-momentum basis, e.g., $|2,0\rangle=|1,1\rangle_S$. Inserting Eq. (5) into Eq. (10) and identifying the two vacua $|0,0\rangle\rangle$ and $|0,0\rangle$, we obtain

$$|j,m\rangle\rangle:=\frac{(\gamma_1^* a_1^\dagger+\gamma_2^* a_2^\dagger)^{j+m}(-\gamma_2 a_1^\dagger+\gamma_1 a_2^\dagger)^{j-m}}{\sqrt{(j+m)!(j-m)!}}|0,0\rangle.$$

Expanding the products, rearranging the terms [17] and using the definition of the Fock basis $|n_1,n_2\rangle$ in terms of $a_1^\dagger$ and $a_2^\dagger$, we obtain the important relation between the quasimode and the mode Fock bases,

$$|j,m\rangle\rangle=\sum_{m'=-j}^{j}D^{(j)}_{m'm}(\varphi,\vartheta,\chi)|j,m'\rangle,\qquad(11)$$

$$|j,m\rangle=\sum_{m'=-j}^{j}D^{(j)^\dagger}_{m'm}(\varphi,\vartheta,\chi)|j,m'\rangle\rangle.\qquad(12)$$

Here $D^{(j)}_{m'm}(\varphi,\vartheta,\chi)=\exp[-i(m'\varphi+m\chi)]d^{(j)}_{m'm}(\vartheta)$ are the Wigner D-matrix elements of the SU(2) group [16,17], with arguments determined by $\varphi=\varphi_1-\varphi_2$, $\chi=\varphi_1+\varphi_2$, $\cos(\vartheta/2):=|\gamma_1|$, $\sin(\vartheta/2):=|\gamma_2|$, and $\gamma_i=|\gamma_i|\exp(i\varphi_i)$. It follows that the mode and quasimode Fock states belonging to the same total number of photons, $n_1+n_2=2j$, are related by an irreducible rotation matrix of weight j and with Euler angles determined solely by the interaction constants.

The action of U_{ab} on the field states is easily calculated in the quasimode Fock basis

$$U_{ee}(\tau)|j,m\rangle\rangle=\cos(\tau\sqrt{j+m+1})|j,m\rangle\rangle,$$

$$U_{ge}(\tau)|j,m\rangle\rangle=-i\sin(\tau\sqrt{j+m+1})|j+\tfrac{1}{2},m+\tfrac{1}{2}\rangle\rangle,$$

$$U_{eg}(\tau)|j,m\rangle\rangle=-i\sin(\tau\sqrt{j+m})|j-\tfrac{1}{2},m-\tfrac{1}{2}\rangle\rangle,$$

$$U_{gg}(\tau)|j,m\rangle\rangle=\cos(\tau\sqrt{j+m})|j,m\rangle\rangle,\qquad(13)$$

showing that U_{ee} and U_{gg} do not change the number of quasiphotons, whereas U_{ge} (U_{eg}) act as creation (annihilation) operators of quasimode 1. Using Eqs. (11) and (12), we find for the action on the usual Fock states,

$$U_{ee}(\tau)|j,m\rangle=\sum_{m'=-j}^{j}C^{j}_{m'm}(\tau)|j,m'\rangle,$$

$$U_{ge}(\tau)|j,m\rangle=\sum_{m'=-j-1/2}^{j+1/2}S^{j}_{m'm}(\tau)|j+\tfrac{1}{2},m'\rangle,$$

$$U_{eg}(\tau)|j,m\rangle=\sum_{m'=-j+1/2}^{j-1/2}\overline{S}^{j}_{m'm}(\tau)|j-\tfrac{1}{2},m'\rangle,$$

$$U_{gg}(\tau)|j,m\rangle=\sum_{m'=-j}^{j}\overline{C}^{j}_{m'm}(\tau)|j,m'\rangle,\qquad(14)$$

where we have introduced the following coefficients:

$$C^{j}_{m'm}(\tau)=\sum_{\nu=-j}^{j}\cos(\tau\sqrt{j+\nu+1})D^{(j)}_{m'\nu}D^{(j)^\dagger}_{\nu m},$$

$$S^{j}_{m'm}(\tau)=-i\sum_{\nu=-j}^{j}\sin(\tau\sqrt{j+\nu+1})D^{(j+1/2)}_{m',\nu+1/2}D^{(j)^\dagger}_{\nu m},$$

$$\overline{S}^{j}_{m'm}(\tau)=-i\sum_{\nu=-j}^{j}\sin(\tau\sqrt{j+\nu})D^{(j-1/2)}_{m',\nu-1/2}D^{(j)^\dagger}_{\nu m},$$

$$\overline{C}^{j}_{m'm}(\tau)=\sum_{\nu=-j}^{j}\cos(\tau\sqrt{j+\nu})D^{(j)}_{m'\nu}D^{(j)^\dagger}_{\nu m}.\qquad(15)$$

Given the above equations, we now have all the ingredients to calculate the time evolution of the density operator according to $\rho(t)=U(t)\rho(0)U^\dagger(t)$.

III. GENERATION OF ENTANGLEMENT IN ONE STEP

We start with the calculation of the probability to find at time τ the field state $|\Psi_N\rangle$ in Eq. (1), assuming an initial field state $|\xi\rangle$ and an atom entering the cavity in either the excited or ground state. The analytical calculation is straightforward. The initial field state is expanded according to

$$|\xi\rangle=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}b_{n_1n_2}|n_1,n_2\rangle=\sum_{j=0}^{\infty}{}'\sum_{m=-j}^{j}\tilde{b}_{jm}|j,m\rangle\rangle,\qquad(16)$$

where the primed summation symbol indicates a sum over integer and half-integer values of j. The expansion coefficients with respect to the Fock and Schwinger bases are related by $b_{j+m,j-m}=\tilde{b}_{j,m}$. State $|\Psi_N\rangle$ is given in the Schwinger basis by

$$|\Psi_N\rangle=\sum_{m=-N/2}^{N/2}{}' \tilde{c}_{(N/2)m}|N/2,m\rangle_{\mathrm{S}}, \tag{17}$$

with density operator $\rho_{\Psi_N}=|\Psi_N\rangle\langle\Psi_N|$. From the time-evolved initial states

$$U|e;\xi\rangle=U_{ee}|e;\xi\rangle+U_{ge}|g;\xi\rangle,$$

$$U|g;\xi\rangle=U_{eg}|e;\xi\rangle+U_{gg}|g;\xi\rangle,$$

we obtain the reduced density operator of the field by tracing out the atomic degrees of freedom:

$$\rho_{\mathrm{F}}^{(a)}(t)=\mathrm{tr}_{\mathrm{A}}(U(t)|a;\xi\rangle\langle a;\xi|U^{\dagger}(t)), \quad a=e \text{ or } g.$$

The probability of finding $|\Psi_N\rangle$ at time t follows from $\langle\rho_{\Psi_N}^{(a)}\rangle=\mathrm{tr}(\rho_{\mathrm{F}}^{(a)}(t)\rho_{\Psi_N})$ and is given by

$$\langle\rho_{\Psi_N}^{(e)}\rangle=\left|\sum_{m=-N/2}^{N/2}\sum_{m'=-N/2}^{N/2}{}' \tilde{b}_{(N/2)m}\tilde{c}^{*}_{(N/2)m'}C^{N/2}_{m'm}(\tau)\right|^2 + \left|\sum_{m=-(N-1)/2}^{(N-1)/2}\sum_{m'=-N/2}^{N/2}{}' \tilde{b}_{(N-1/2)m} \times\tilde{c}^{*}_{(N/2)m'}S^{(N-1)/2}_{m'm}(\tau)\right|^2 \tag{18}$$

for the initial atom-field state $|e;\xi\rangle$, and by

$$\langle\rho_{\Psi_N}^{(g)}\rangle=\left|\sum_{m=-N/2}^{N/2}\sum_{m'=-N/2}^{N/2}{}' \tilde{b}_{(N/2)m}\tilde{c}^{*}_{(N/2)m'}\bar{C}^{N/2}_{m'm}(\tau)\right|^2 + \left|\sum_{m=-(N+1)/2}^{(N+1)/2}\sum_{m'=-N/2}^{N/2}{}' \tilde{b}_{(N+1)/2m} \times\tilde{c}^{*}_{(N/2)m'}\bar{S}^{(N+1)/2}_{m'm}(\tau)\right|^2 \tag{19}$$

for the initial state $|g;\xi\rangle$. It follows that in order to obtain nonvanishing probabilities at time τ, the initial field state must contain at least one of the Fock states from the set

$$\{|N,0\rangle,|N-1,1\rangle,\ldots,|0,N\rangle\} \cup\{|N-1,0\rangle,|N-2,1\rangle,\ldots,|0,N-1\rangle\}, \tag{20}$$

if the atom is initially in the excited state, or from the set

$$\{|N,0\rangle,|N-1,1\rangle,\ldots,|0,N\rangle\} \cup\{|N+1,0\rangle,|N,1\rangle,\ldots,|0,N+1\rangle\}, \tag{21}$$

if it is in the ground state.

For $N=1$, the set of contributing atom-field states according to Eq. (20) is

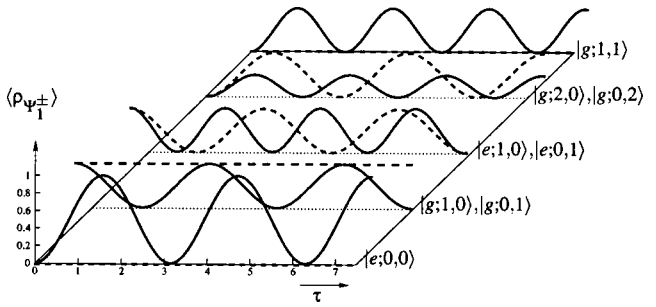

FIG. 1. Parametric plot of the generation probabilities $\langle\rho_{\Psi_1^{+}}\rangle$ (solid) and $\langle\rho_{\Psi_1^{-}}\rangle$ (dashed) as a function of time $\tau=gt$, for different initial atom-field states shown at the right.

$$\{|e;1,0\rangle,|e;0,1\rangle\}\cup\{|e;0,0\rangle\} \tag{22}$$

and according to Eq. (21)

$$\{|g;1,0\rangle,|g;0,1\rangle\}\cup\{|g;2,0\rangle,|g;1,1\rangle,|g;0,2\rangle\}. \tag{23}$$

We illustrate the probabilities for the generation of $|\Psi_1^{\pm}\rangle=(|1,0\rangle\pm|0,1\rangle)/\sqrt{2}$ in Fig. 1, where we have taken $g_1=g_2$, $\varphi_1=\varphi_2=0$ (real coupling constants).

The states shown on the right are just the initial atom-field states from Eqs. (22) and (23). The interesting case is $|e;0,0\rangle$, where the state $|\Psi_1^{+}\rangle$ is produced periodically with probability 1 at the times $\tau_n=(n+1/2)\pi$, for $n=0,1,\cdots$.

Next we consider the creation of various Bell states, Eq. (2), resulting from the initial atom-field state $|e;N,0\rangle=|e;N/2,N/2\rangle_{\mathrm{S}}$. We obtain from Eq. (18) the entangled N-photon field states $|\Psi_N^{\pm}\rangle$ with probabilities

$$\langle\rho_{\Psi_N^{\pm}}^{(e)}\rangle=\frac{1}{2}|C^{N/2}_{(N/2)(N/2)}(\tau)\pm C^{N/2}_{-(N/2)(N/2)}(\tau)|^2, \tag{24}$$

as well as the entangled $(N+1)$-photon states $|\Psi_{N+1}^{\pm}\rangle$ with probabilities

$$\langle\rho_{\Psi_{N+1}^{\pm}}^{(e)}\rangle=\frac{1}{2}|S^{N/2}_{[(N+1)/2](N/2)}(\tau)\pm S^{(N/2)}_{-[(N+1)/2](N/2)}(\tau)|^2. \tag{25}$$

In the case of Eqs. (25) the Bell states $|\Psi_{N+1}^{\pm}\rangle$ have no overlap with the initial field state $|N,0\rangle$. The probabilities at time τ, however, may come close to 1 for some particular values of the coupling constants and interaction time. In this case we may say that $|\Psi_{N+1}^{\pm}\rangle$ have been generated in a *single step* or single shot. This property of the two-mode JC model can be understood if we think of the atom (re)emitting photons into and (re)absorbing photons from the two modes many times during the interaction time τ.

IV. CONDITIONAL GENERATION

Next we present a *conditional* scheme for the generation of N-photon entangled states starting with an empty cavity [7,8]. The scheme implies sending consecutively atoms in the excited state through a two-mode cavity and detecting

them in the ground state. We start with an initial atom-field state $|e;0,0\rangle = |e;0,0\rangle\rangle_S$ and let the first atom interact for a time τ_1. By using Eq. (13), we obtain the state

$$U(\tau_1)|e;0,0\rangle\rangle_S = \cos(\tau_1)|e;0,0\rangle\rangle_S - i\sin(\tau_1)|g;\tfrac{1}{2},\tfrac{1}{2}\rangle\rangle_S .$$

Detecting the atom in the ground state leaves the field in the state $|\chi_1\rangle = K_1(-i)\sin(\tau_1)|\frac{1}{2},\frac{1}{2}\rangle\rangle_S$, where $K_1 = |\sin(\tau_1)|^{-1}\exp(i\alpha_1)$ is a normalization constant. By choosing phase α_1 appropriately, the factor entering the normalized state may be set equal to 1, yielding the state $|\frac{1}{2},\frac{1}{2}\rangle\rangle_S$. Proceeding this way, the field state obtained after N conditional steps is simply given by

$$|\chi_N\rangle = |N/2, N/2\rangle\rangle_S = \sum_{k=0}^{N} D^{N/2}_{N/2-k,N/2}(\varphi,\vartheta,\chi)|N-k,k\rangle, \tag{26}$$

where we have used Eq. (11) and Fock-state notation on the right-hand side. This is precisely a state of the form given in Eq. (1) with coefficients determined by the Wigner rotation matrix elements. Since these elements depend solely on the coupling constants, the generated entangled state is sensitive to their magnitudes and phases. The state in Eq. (26) corresponds to the quasimode Fock state $|N,0\rangle\rangle$, implying that each conditional step generates one photon in quasimode 1. The generation probabilities of states $|\Psi_N\rangle$ and $|\Psi_N^{\pm}\rangle$ after N conditional steps are given by

$$|\langle\Psi_N|\chi_N\rangle|^2 = \left|\sum_{m=-N/2}^{N/2} \tilde{c}^*_{(N/2)m} D^{N/2}_{mN/2}\right|^2, \tag{27}$$

$$|\langle\Psi_N^{\pm}|\chi_N\rangle|^2 = \frac{1}{2}|D^{N/2}_{(N/2)(N/2)} \pm D^{N/2}_{-(N/2)(N/2)}|^2. \tag{28}$$

We shall show that the probability of detecting the atoms N times consecutively in the ground state is a rapidly decaying function of N. But, as discussed below, it is not essential to rely on this assumption. Actually, it is sufficient to detect them in a sequence of n ($\geqslant N$) steps N times in the ground state.

V. NONCONDITIONAL GENERATION

In the following we consider a *nonconditional* scheme. We start with an empty cavity and send a sequence of excited atoms through it without detecting their final states. The reduced density operator of the field after the passage of the first atom (interaction time τ_1) is given by

$$\rho_F^{(1)}(\tau_1) = \cos^2(\tau_1)|0,0\rangle\rangle_S\langle\langle 0,0| + \sin^2(\tau_1)|\tfrac{1}{2},\tfrac{1}{2}\rangle\rangle_S\langle\langle\tfrac{1}{2},\tfrac{1}{2}|$$

and serves as the ''initial'' field configuration for the second excited atom. Proceeding this way, the reduced density operator of the field after n steps turns out to be of the form

$$\rho_F^{(n)}(\{\tau_n\}) = \sum_{j=0}^{n/2}{}' p_j^{(n)}(\{\tau_n\})|j,j\rangle\rangle_S\langle\langle j,j|, \tag{29}$$

where the coefficients $p_j^{(n)}$ are given recursively by

$$p_0^{(n)} = \cos^2(\tau_n)p_0^{(n-1)},$$

$$p_j^{(n)} = \cos^2(\tau_n\sqrt{2j+1})p_j^{(n-1)} + \sin^2(\tau_n\sqrt{2j})p_{j-1/2}^{(n-1)},$$

$$p_{n/2}^{(n)} = \sin^2(\tau_n\sqrt{n})p_{(n-1)/2}^{(n-1)}, \tag{30}$$

for $1/2 \leqslant j \leqslant (n-1)/2$ and $p_0^{(0)} = 1$. The argument $\{\tau_n\}$ stands for all interaction times $(\tau_1, \ldots, \tau_n)$ of the n steps. Equation (29), which is obviously true for $n=1$ and $n=2$ [see Eq. (31)], can be proven by induction.

The coefficients $p_j^{(n)}$ in Eq. (29) are the probabilities to find the field after n nonconditional steps in the quasimode state $|j,j\rangle\rangle_S$. In particular, $p_0^{(n)} = \cos^2(\tau_1)\cos^2(\tau_2\sqrt{2})\cdots\cos^2(\tau_n\sqrt{n})$ and $p_{n/2}^{(n)} = \sin^2(\tau_1)\sin^2(\tau_2\sqrt{2})\cdots\sin^2(\tau_n\sqrt{n})$ correspond to the cases, where in n steps the initially excited atoms emerge n times in the excited and ground state, respectively. The intermediate $p_j^{(n)}$'s correspond to the cases, where the n atoms emerge $2j$ times in the ground state and $n-2j$ times in the excited state, irrespective of the order of appearance. The coefficient $p_j^{(n)}$ consists of a sum of $\binom{n}{2j}$ terms, each of which corresponds to a particular sequence of $|g\rangle$'s and $|e\rangle$'s contributing, respectively, a sine-squared and cosine-squared factor. There are altogether 2^n terms in Eq. (29). All this is easily seen by giving $\rho_F^{(2)}$ as an example:

$$\begin{aligned}\rho_F^{(2)} = {} & \cos^2\tau_1\cos^2\tau_2|0,0\rangle\rangle_S\langle\langle 0,0| + [\cos^2\tau_1\sin^2\tau_2 \\ & + \sin^2\tau_1\cos^2(\tau_2\sqrt{2})]|\tfrac{1}{2},\tfrac{1}{2}\rangle\rangle_S\langle\langle\tfrac{1}{2},\tfrac{1}{2}| \\ & + \sin^2\tau_1\sin^2(\tau_2\sqrt{2})|1,1\rangle\rangle_S\langle\langle 1,1|.\end{aligned} \tag{31}$$

Here the four terms correspond to the final-state sequences (e,e), (e,g), (g,e), and (g,g).

States $|\Psi_N\rangle$ and $|\Psi_N^{\pm}\rangle$ are generated in a nonconditional n-step process with probabilities

$$\langle\rho_{\Psi_N}\rangle = p_{N/2}^{(n)}\left|\sum_{m=-N/2}^{N/2} \tilde{c}^*_{(N/2)m} D^{N/2}_{mN/2}\right|^2, \tag{32}$$

$$\langle\rho_{\Psi_N^{\pm}}\rangle = \frac{1}{2}p_{N/2}^{(n)}|D^{N/2}_{(N/2)(N/2)} \pm D^{N/2}_{-(N/2)(N/2)}|^2, \tag{33}$$

which are the *conditional* probabilities found before, multiplied by the probability $p_{N/2}^{(n)}$. Here the interaction times must

be chosen such that $p_{N/2}^{(n)} \neq 0$, which amounts to control the n parameters $(\tau_1, \ldots, \tau_n)$. State $|\Psi_N\rangle$ can be generated in a minimum number of $n=N$ steps with probability $p_{N/2}^{(N)}$ which, however is a rapidly decaying function of N.

In the nonconditional scheme, all field states $|j,j\rangle\rangle_S = |2j,0\rangle\rangle$ for $j=0,1/2,\ldots,n/2$ are produced, Eq. (29). On the contrary, in the *conditional* scheme only the entangled N-photon state $|N,0\rangle\rangle$, Eq. (26), is generated, if in n steps N atoms are detected in the ground state. To produce $|\Psi_N\rangle$ it is, therefore, not crucial that the atoms have been detected N times consecutively in their ground state. Any sequence of the ground and excited states containing N times the ground state will do it. Finally, we note that there is a particular choice of the interaction time of the ℓth atom, given by $\tau_\ell = \pi/(2\sqrt{\ell})$ for which both the conditional and nonconditional schemes give (with probability 1) the same entangled state $|N,0\rangle\rangle$ in Eq. (26).

VI. CONCLUSION

To conclude, we have solved the two-mode JC model algebraically by reducing it to an effective one-quasi-mode JC model. The mode and quasimode picture are unitarily related by an SU(2) transformation. The solution found is used to discuss three different schemes for the generation of entangled states of the two field modes. To generate entangled N-photon states in a single step, the initial state must contain at least $N-1$ photons and an excited atom. Starting from the vacuum we need at least $n \geqslant N$ steps to produce pure (mixed) field states in the conditional (nonconditional) scheme presented.

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