PHYSICAL REVIEW A **67**, 050701(R) (2003)

Three-body loss of trapped ultracold ^{87}Rb atoms due to a Feshbach resonance

V. A. Yurovsky and A. Ben-Reuven
School of Chemistry, Tel Aviv University, 69978 Tel Aviv, Israel
(Received 10 October 2002; published 13 May 2003)

The loss of ultracold trapped atoms in the vicinity of a Feshbach resonance is treated as a two-stage reaction, using the Breit-Wigner theory. The first stage is the formation of a resonant diatomic molecule, and the second one is its deactivation by inelastic collisions with other atoms. This model is applied to the analysis of recent experiments on ^{87}Rb, leading to an estimated value of 7×10^{-11} cm^3/s for the deactivation rate coefficient.

DOI: 10.1103/PhysRevA.67.050701 PACS number(s): 34.50.−s, 32.80.Pj, 03.75.Hh, 03.75.Kk

The phenomenon of Feshbach resonance has recently received an increased attention due to its application to Bose-Einstein condensation (BEC) (see Ref. [1] and references therein). Its most outstanding effect is a drastic change of the elastic scattering length as the collision energy of an atomic pair approaches the energy of a bound level belonging to another electronic or hyperfine state. The resonance can be tuned by applying an external magnetic field, as has been proposed in Ref. [2] in order to control the BEC properties. Applications include a controlled BEC collapse [3] and bright solitons in BEC [4,5], as well as the formation of molecular BEC [1,6–9], an atom-molecule coherent superposition [10,11], and an entangled atomic gas [9].

Another effect of the resonance is the abrupt increase in atom loss due to inelastic collisions of the resonant molecules [1,6,8,12], and to the formation of noncondensed atoms [7,8,13]. The determination of the loss parameters is important for an appreciation of the outcome of applications of Feshbach resonances. We present here an estimate of the rate coefficient for the deactivation of vibrationally excited resonant $^{87}Rb_2$ molecules by collisions with other Rb atoms, based on the results of recent experiments [14].

The theory presented in Refs. [1,6,8,12], based on coupled Gross-Pitaevskii equations for atomic and molecular condensates, cannot be applied to the analysis of these experiments involving a noncondensed thermal gas. The approach used here is based on the Breit-Wigner theory of resonant multichannel collisions (see, e.g., Ref. [15]), as has been proposed for the system under consideration by Ref. [7]. The reaction involving the excited resonant molecule $Rb_2(m)$ includes a reversible input channel of formation from (and dissociation to) a pair of colliding atoms,

$$\mathrm{Rb}+\mathrm{Rb}\rightleftarrows\mathrm{Rb}_2(m), \tag{1}$$

and irreversible output channels of exoergic collisions with a third atom,

$$\mathrm{Rb}_2(m)+\mathrm{Rb}\rightarrow\mathrm{Rb}_2(d)+\mathrm{Rb}, \tag{2}$$

bringing the molecule down to one of the lower-lying rovibrational levels of the same spin state, or to levels belonging to other spin states. (An alternative approach, presented in Refs. [16,17], treats the whole process as a one-stage recombination by a three-body collision.)

Let us consider all atoms, for the time being, as distinguishable particles. According to the standard theory (see Ref. [15]), the natural resonance width Γ_e associated with channel (1) is two times smaller than the corresponding width for the case of indistinguishable atoms presented in Ref. [7] (see also Refs. [1,2]). It exhibits a Wigner threshold dependence of the form

$$\Gamma_e=\frac{|a_a\mu|\Delta}{\hbar^2}p, \tag{3}$$

where a_a is the nonresonant (background) elastic scattering length, μ is the difference of the magnetic momenta of the atomic pair and the $Rb_2(m)$ molecule, Δ is the phenomenological resonance strength (see Refs. [1,2]), and p is the relative momentum of the colliding atoms. These parameters also describe the variation of the elastic scattering length a_{res} as a function of the external magnetic field B in the vicinity of the resonance at $B=B_0$ as (see Refs. [1,2])

$$a_{\mathrm{res}}=a_a\left(1-\frac{\Delta}{B-B_0}\right). \tag{4}$$

The total width Γ_d associated with the deactivation channels (2) can be expressed in terms of a two-body rate coefficient k_d, as

$$\Gamma_d=k_dn \tag{5}$$

is proportional to the atomic density n. The rate coefficient k_d includes the contributions of all the output deactivation channels (d) of Eq. (2).

The Breit-Wigner theory leads to the following expression for the cross section of resonance-enhanced three-body recombination (see Ref. [7]):

$$\sigma=\frac{\pi\hbar^2}{p^2}\frac{\Gamma_e\Gamma_d}{\mu^2(B-B_0)^2/\hbar^2+(\Gamma_e+\Gamma_d)^2/4}. \tag{6}$$

This expression does not take into account the indistinguishability of the three participating atoms, in which case the cross section should be $\sigma_{\mathrm{ind}}=3!\sigma$ (see Ref. [15]).

The resonant molecular state $Rb_2(m)$ can be formed whenever the detuning from the resonance is comparable or less than Γ_e. This state decays producing atoms with a kinetic energy spectrum of width $\hbar\Gamma_e$. Under the conditions of the experiments [14] ($a_a\approx98.96$ a.u., $\mu\approx2.8$ Bohr magnetons, $\Delta\approx0.17$ G for the strongest resonance at 1007.34 G in

 ©2003 The American Physical Society

^{87}Rb, and a collision energy of $p^2/m \approx 2\ \mu$K), the width calculated with Eq. (3) is given by $\hbar\Gamma_e/k_B \approx 7\ \mu$K, where k_B is the Boltzmann constant. Therefore this energy is less than the trap depth of $\approx 20\ \mu$K and a spontaneous dissociation of the resonance molecule (1) cannot lead to a significant loss of trapped atoms (as opposed to the case of BEC—see Refs. [7,8]). Each deactivation event (2) leads to the simultaneous loss of three atoms. Therefore, the loss rate for the atomic density $n(\mathbf{r},t)$ can be written in the form,

$$\dot{n}(\mathbf{r},t) = -3\frac{2p}{m}\sigma_{\mathrm{ind}} n^2(\mathbf{r},t) = -K_3 n^3(\mathbf{r},t), \tag{7}$$

where

$$K_3 = \frac{36\pi\hbar^2 k_d |a_a \mu| \Delta}{m[\mu^2 (B-B_0)^2 + \hbar^2\Gamma_e^2/4]} \tag{8}$$

is the three-body loss rate coefficient. Here the partial inelastic width Γ_d is neglected in the denominator in comparison to Γ_e. Even very close to the resonance, as long as $|B - B_0| > 0.1$ G, the width Γ_e may as well be neglected, leading to an expression similar to Eq. (9) of Ref. [12] for the loss in a BEC. However, the rate coefficient given by Eq. (8) is six times larger than the corresponding rate for a BEC. This difference, due to the effects of quantum statistics, has been predicted for nonresonant three-body recombination in Ref. [18], and observed in experiments [19].

In the case of BEC, the atomic density profile is determined by the repulsive interaction between atoms. This interaction can be neglected whenever its characteristic energy, proportional to the elastic scattering length, is small compared to the kinetic energy of atoms

$$\frac{4\pi}{m}\hbar^2 a_{\mathrm{res}} n \ll k_B T. \tag{9}$$

For the temperature $T = 2\ \mu$K used in the experiments [14], this condition is obeyed whenever $|B - B_0| > 0.01$ G. Therefore we can consider the gas as an ideal one with the equilibrium density profile described by the Boltzmann distribution in the trap potential.

The loss rate given by Eq. (7) is density dependent. In the case of an inhomogeneous trapped gas, the loss processes modify the equilibrium density profile, leading to an atomic drift that tends to compensate for this deformation. The characteristic time for this compensation can be estimated as the trap period. In the experiments [14] the magnetic field that brings the system close to resonance has been applied during a time interval of $t = 50$ ms. This time substantially exceeds the radial trap period (the radial trap frequency is $\omega_r/2\pi = 930$ Hz), but it is less than the axial trap period (the axial trap frequency is $\omega_a/2\pi = 11$ Hz). Therefore we can consider the radial density profile as an equilibrium one, described by a Boltzmann distribution, and write out the atomic density profile as

$$n(\mathbf{r},t) = \frac{\nu(z,t)}{\pi b_r^2} \exp\left(-\frac{x^2+y^2}{b_r^2}\right), \tag{10}$$

where

$$\nu(z,t) = \int dx\, dy\, n(\mathbf{r},t) \tag{11}$$

is a nonequilibrium axial profile and

$$b_r = \frac{1}{\omega_r}\sqrt{\frac{2k_B T}{m}} \tag{12}$$

is the characteristic radius of the atomic cloud.

Neglecting effects of axial atom transport, a kinetic equation for the axial profile can be written in the form

$$\dot{\nu}(z,t) = -K_{1D}\nu^3(z,t), \quad K_{1D} = K_3/(3\pi^2 b_r^4). \tag{13}$$

The solution of Eq. (13) relates the axial profile at time t to the initial one at $t = 0$ as

$$\nu(z,t) = \frac{\nu(z,0)}{\sqrt{1 + 2K_{1D}\nu^2(z,0)t}}. \tag{14}$$

Let us suppose that at $t = 0$ the atoms have a Boltzmann distribution with the temperature T and

$$\nu(z,0) = \nu_0 \exp\left(-\frac{z^2}{b_a^2}\right), \quad \nu_0 = \frac{N_0}{\sqrt{\pi} b_a}, \tag{15}$$

where

$$b_a = \frac{1}{\omega_a}\sqrt{\frac{2k_B T}{m}} \tag{16}$$

is the characteristic half length of the atomic cloud and N_0 is the initial number of atoms. In this case, the number of atoms remaining in the trap can be expressed as

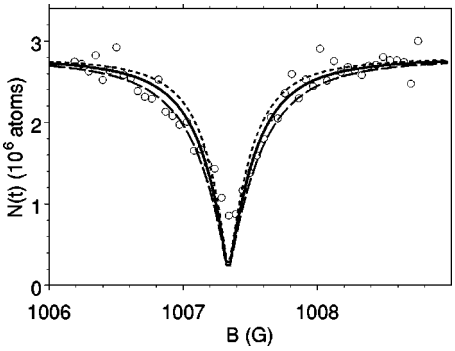

FIG. 1. Number of remaining atoms as a function of the magnetic field in the vicinity of the 1007 G resonance in ^{87}Rb calculated with Eq. (17) for three values of the deactivation rate coefficient, $k_d = 7\times10^{-11}$ cm^3/s (solid line), 10^{-10} cm^3/s (long-dashed line), and 5×10^{-11} cm^3/s (short-dashed line). The circles represent the experimental results of Marte *et al.* [14].

$$N(t)=2\frac{N_0}{\sqrt{\pi}}\int_0^\infty d\zeta\frac{\exp(-\zeta^2)}{\sqrt{1+2K_{1D}\nu_0^2 t\exp(-2\zeta^2)}}, \quad (17)$$

where $\zeta=z/b_a$.

Equation (17) [in combination with Eqs. (3), (8), (13), and (15)] allows us to estimate the value of k_d by a fit to the number of remaining atoms measured in Ref. [14] for $N_0=2.8\times10^6$. The fit produces the optimal value of $k_d=0.7\times10^{-10}$ cm^3/s. This value is comparable to corresponding estimates for Na resonances (1.6×10^{-10} cm^3/s in Ref. [8]; 4×10^{-10} cm^3/s and 10^{-11} cm^3/s in Ref. [20] following the theory of Ref. [1]). The results of calculations for several values of k_d are presented in Fig. 1 in comparison with the experimental results of Ref. [14].

The authors are most grateful to Dr. Stephan Dürr for providing a preprint of Ref. [14] and clarifying details of the experiment.

[1] E. Timmermans, P. Tommasini, M. Hussein, and A. Kerman, Phys. Rep. **315**, 199 (1999).
[2] E. Tiesinga, A.J. Moerdijk, B.J. Verhaar, and H.T.C. Stoof, Phys. Rev. A **46**, R1167 (1992); E. Tiesinga, B.J. Verhaar, and H.T.C. Stoof, *ibid.* **47**, 4114 (1993); A.J. Moerdijk, B.J. Verhaar, and A. Axelsson, *ibid.* **51**, 4852 (1995).
[3] E.A. Donley, R. Claussen, S.L. Cornish, J.L. Roberts, E.A. Cornell, and C.W. Wieman, Nature (London) **412**, 295 (2001).
[4] K. Strecker, G. Partridge, A. Truscott, and R. Hulet, Nature (London) **417**, 150 (2002).
[5] L. Khaykovich, F. Schrenk, T. Bourdel, J. Cubizolles, G. Ferrari, L. Carr, Y. Castin, and C. Salomon, Science **296**, 1290 (2002).
[6] E. Timmermans, P. Tommasini, R. Côté, M. Hussein, and A. Kerman, Phys. Rev. Lett. **83**, 2691 (1999).
[7] F.H. Mies, E. Tiesinga, and P.S. Julienne, Phys. Rev. A **61**, 022721 (2000).
[8] V.A. Yurovsky, A. Ben-Reuven, P.S. Julienne, and C.J. Williams, Phys. Rev. A **62**, 043605 (2000).
[9] V.A. Yurovsky and A. Ben-Reuven, Phys. Rev. A (to be published), e-print cond-mat/0205267.
[10] E.A. Donley, N.R. Claussen, S.T. Thompson, and C.E. Wieman, Nature (London) **417**, 529 (2002)
[11] S.J.J.M.F. Kokkelmans and M.J. Holland, Phys. Rev. Lett. **89**, 180401 (2002).
[12] V.A. Yurovsky, A. Ben-Reuven, P.S. Julienne, and C.J. Williams, Phys. Rev. A **60**, R765 (1999).
[13] M. Holland, J. Park, and R. Walser, Phys. Rev. Lett. **86**, 1915 (2001).
[14] A. Marte, T. Volz, J. Schuster, S. Dürr, G. Rempe, E.G.M. van Kempen, and B.J. Verhaar, Phys. Rev. Lett. **89**, 283202 (2002).
[15] N.F. Mott and H.S.W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, London, 1965).
[16] B.D. Esry, C.H. Greene, and J.P. Burke, Phys. Rev. Lett. **83**, 1751 (1999).
[17] O.I. Kartavtsev and J.H. Macek, Few-Body Syst. **31**, 249 (2002).
[18] Yu. Kagan, B.V. Svistunov, and G.V. Shlyapnikov, JETP Lett. **42**, 209 (1985).
[19] E.A. Burt, R.W. Ghrist, C.J. Myatt, M.J. Holland, E.A. Cornell, and C.E. Wieman, Phys. Rev. Lett. **79**, 337 (1997).
[20] F.A. van Abeelen, and B.J. Verhaar, Phys. Rev. Lett. **83**, 1550 (1999).