Interference between the halves of a double-well trap containing a Bose-Einstein condensate

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Interference between the halves of a double-well trap containing a Bose-Einstein condensate is studied. It is found that when the atoms in the two wells are initially in the coherent state, the intensity exhibits collapses and revivals, but it does not for the initial Fock states. Whether the initial states are in the coherent states or in a Fock state, the fidelity time has nothing to do with collision. We point out that interference and its fidelity can be adjusted experimentally by properly preparing the number and initial states of the system.

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I. INTRODUCTION

Since the recent experimental realization of Bose-Einstein condensation (BEC) in small atomic samples [1–4], there has been much theoretical interest focused on the physical properties and nature of Bose-Einstein condensed systems such as coherent tunneling, and the collapses and revivals in both the macroscopic wave function and the interference patterns [5–12]. It is hoped that the study of these experimental systems will give new insight into the physics of BEC. Since the current understanding of BEC is largely influenced by the concept of a macroscopic wave function, the study of this feature is of foremost importance. The investigation of interference phenomena should be perfectly suited for this purpose. Another motivation for the study of interference properties is the envisioned development of a new source of atoms, based on BEC, with high flux and coherence. It is expected to stimulate atomic interference experiments.

Recently, Javanainen *et al.* [13] have theoretically studied atom number fluctuations between the halves of a double-well trap containing a Bose-Einstein condensate, in which the two-mode approximation is used, which assumes that only two one-particle states are involved. They have developed an analytical harmonic-oscillator-like model, and verified numerically for both stationary fluctuations in the ground state of the system and for the fluctuations resulting from splitting of a single trap by dynamically erecting a barrier in the middle.

This paper is organized as follows. Section II gives the solution of model. Section III studies collapses and revivals of interference intensity. Section IV investigates fidelity of interference. A conclusion is given in the last section.

II. MODEL

In a symmetric double-well potential, the ground state of a single particle is represented by an even wave function ψ_g that belongs equally to both wells of the potential. Provided that barrier between the halves of the potential is tall enough so that the tunneling rate between the potential wells is small, nearby lies an excited odd state ψ_e that likewise belongs to both halves of the double well. The starting point is that we take only two one-particle states ψ_g and ψ_e to be available to the N bosons. The reason for this choice of including only the two lowest lying modes for the double-well potential is that the other modes are energetically inaccessible.

We adopt the usual two-particle contact interaction $U(\mathbf{r}_1,\mathbf{r}_2)=(4\pi\hbar^2 a/m)\delta(\mathbf{r}_1-\mathbf{r}_2)$, where a is the s-wave scattering length and m is the atomic mass. Given the restricted state space of precisely two one-particle states, the many-particle Hamiltonian is [13]

$$H=\tfrac{1}{2}(\epsilon_1+\epsilon_2)(a_1^+a_1+a_2^+a_2)+\tfrac{1}{2}(\epsilon_1-\epsilon_2)(a_1^+a_1-a_2^+a_2)+K_{11}a_1^{+2}a_1^2+K_{22}a_2^{+2}a_2^2+K_{12}(a_1^{+2}a_2^2+a_2^{+2}a_1^2+4a_1^+a_1a_2^+a_2), \tag{1}$$

where we set $\hbar=1$, and correspondingly use the terms energy and (angular) frequency, interchangeably. In Eq. (1), a_1 and a_2 are the boson operators for the excited and ground wave functions. The constants ϵ and K are the one- and two-particle matrix elements

$$\epsilon_1=\int d^3\mathbf{r}\psi_e(\mathbf{r})\left[-\frac{1}{2m}\nabla^2+V(\mathbf{r})\right]\psi_e(\mathbf{r}), \tag{2}$$

$$\epsilon_2=\int d^3\mathbf{r}\psi_g(\mathbf{r})\left[-\frac{1}{2m}\nabla^2+V(\mathbf{r})\right]\psi_g(\mathbf{r}), \tag{3}$$

$$K_{11}=\frac{2\pi a}{m}\int d^3\mathbf{r}|\psi_e(\mathbf{r})|^4, \tag{4}$$

$$K_{22}=\frac{2\pi a}{m}\int d^3\mathbf{r}|\psi_g(\mathbf{r})|^4, \tag{5}$$

$$K_{12}=K_{21}=\frac{2\pi a}{m}\int d^3\mathbf{r}|\psi_e(\mathbf{r})|^2|\psi_g(\mathbf{r})|^2, \tag{6}$$

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where $V(\mathbf{r})$ is the symmetric double-well binding potential. Without restricting the generality, we assume that the wave functions $\psi_{e,g}$ are real. To simplify the discussion, we set $K_{12}=K_{11}=K_{22}=K$ and $\epsilon_1=\epsilon_2=\epsilon$.

In order to solve Eq. (1), we introduce the following transformations:

$$a_1=\frac{1}{\sqrt{2}}(A_1e^{ikt}-iA_2e^{-iKt}), \quad (7)$$

$$a_2=\frac{1}{\sqrt{2}}(A_1e^{ikt}+iA_2e^{-iKt}), \quad (8)$$

where $[A_i,A_j^+]=\delta_{ij}$. We have, from Eq. (1),

$$H=\epsilon(A_1^+A_1+A_2^+A_2)+K[(A_1^+A_1+A_2^+A_2)^2-3(A_1^+A_1+A_2^+A_2)-3A_1^+A_1A_2^+A_2+(A_1^+A_1)^2+(A_2^+A_2)^2]. \quad (9)$$

If we define two bases as follows:

$$|n,m)=\frac{1}{\sqrt{n!m!}}A_1^{+n}A_2^{+m}0,0), \quad (10)$$

$$|n,m\rangle=\frac{1}{\sqrt{n!m!}}a_1^{+n}a_2^{+m}|0,0\rangle, \quad (11)$$

we have

$$H|n,m)=E_{n,m}|n,m), \quad (12)$$

with

$$E_{n,m}=\epsilon(n+m)+K(2m^2+2n^2-mn-3n-3m). \quad (13)$$

We now define two-mode coherent states as follows:

$$|\alpha_1,\alpha_2\rangle=D_{a_1}(\alpha_1)D_{a_2}(\alpha_2)|0,0\rangle, \quad (14)$$

$$|u_1,u_2\rangle=D_{A_1}(u_1)D_{A_2}(u_2)|0,0\rangle, \quad (15)$$

where the displacement operators are defined by

$$D_{a_i}(\alpha_i)=\exp[\alpha_i^*a_i-\alpha_ia_i^+] \quad (i=1,2), \quad (16)$$

$$D_{A_i}(u_i)=\exp[u_i^*A_i-u_iA_i^+] \quad (i=1,2). \quad (17)$$

It is easy to see that

$$|\alpha_1,\alpha_2\rangle=\left|\frac{1}{\sqrt{2}}(\alpha_1+\alpha_2)e^{-iKt},\frac{i}{\sqrt{2}}(\alpha_1-\alpha_2)e^{iKt}\right). \quad (18)$$

Considering the arguments of Bose broken symmetry, we assume that two condensates are initially in the coherent state. So that the wave function of the system at time t can be written as

$$|\psi(t)\rangle=e^{-N/2}\sum_{n,m=0}^{\infty}\frac{1}{\sqrt{n!m!}}(u_1e^{-iKt})^n(iu_2e^{iKt})^m\times\exp[-iE_{n,m}t]|n,m), \quad (19)$$

where

$$u_1=\frac{1}{\sqrt{2}}(\alpha_1+\alpha_2), \quad u_2=\frac{1}{\sqrt{2}}(\alpha_1-\alpha_2), \quad (20)$$

$$N=|\alpha_1|^2+|\alpha_2|^2=|u_1|^2+|u_2|^2. \quad (21)$$

III. COLLAPSES AND REVIVALS OF INTERFERENCE INTENSITY

For convenience, we rewrite Hamiltonian (9) as follows:

$$H=(\epsilon-3K)N_1+(\epsilon-3K)N_2+2KN_1^2+2KN_2^2-KN_1N_2, \quad (22)$$

where $N_i=A_i^+A_i$ $(i=1,2)$, $2KN_1^2$ and $2KN_2^2$ stand for two-body hard-sphere collisions, and $-KN_1N_2$ describes the collision between the atoms of the two wells.

The dissipation is included by considering the master equation [3,10]

$$\frac{\partial\rho}{\partial t}=-i[H,\rho]+\sum_{j=1,2}\gamma_j(2A_j\rho A_j^+-A_j^+A_j\rho-\rho A_j^+A_j), \quad (23)$$

where γ_j $(j=1,2)$ denotes the dissipation or loss rate due to some relaxation processes such as the coupling of the atoms in the two wells with the environment. To solve Eq. (23), we can introduce a transformation $\tilde{c}=\exp(iHt)c\exp(-iHt)$, then Eq. (23) becomes

$$\frac{\partial\tilde{\rho}}{\partial t}=\sum_{j=1,2}\gamma_j(2\tilde{A}_j\tilde{\rho}\tilde{A}_j^+-\tilde{A}_j^+\tilde{A}_j\tilde{\rho}-\tilde{\rho}\tilde{A}_j^+\tilde{A}_j). \quad (24)$$

The master equation [Eq. (24)] can be solved exactly for any chosen initial state. In particular, when the atoms in the two wells are initially in the coherent state (cs) $|\alpha_1,\alpha_2\rangle$ or in a Fock state (Fs) $|n,m\rangle$, the corresponding density matrices are given by, respectively,

$$\tilde{\rho}^{(cs)}(t)=|\alpha_1e^{-\gamma_1t},\alpha_2e^{-\gamma_2t}\rangle\langle\alpha_1e^{-\gamma_1t},\alpha_2e^{-\gamma_2t}| \quad (25)$$

and

$$\tilde{\rho}^{(Fs)}(t)=\sum_{l=0}^{n}\sum_{k=0}^{m}(e^{-2\gamma_1t})^{n-l}(1-e^{-2\gamma_1t})^l(e^{-2\gamma_2t})^{m-k}(1-e^{-2\gamma_2t})^kC_n^lC_m^k|n-l,m-k\rangle\langle n-l,m-k|, \quad (26)$$

where

$$A_1|\alpha_1\rangle=\alpha_1|\alpha_1\rangle, \quad A_2|\alpha_2\rangle=\alpha_2|\alpha_2\rangle,$$

$$C_m^n=m!/n!(m-n)!. \quad (27)$$

The Schrödinger picture field operator for the sum of the two modes is $\psi=(A_1+A_2)/\sqrt{2}$, where the spatial dependence has been suppressed [6,7]. The corresponding operator for the intensity of the atomic pattern is $\psi^{+}\psi$ and its time-varying expression can be obtained by the trace operator $I(t)=\mathrm{Tr}[\rho(t)\psi^{+}\psi]$. When the atoms in the two wells are initially in the cs $|\alpha_1,\alpha_2\rangle$, one has

$$I^{(cs)}(t)=\frac{1}{2}\sum_{j=1,2}|\alpha_j|^2\exp(-2\gamma_j t)+|\alpha_1\alpha_2|\times\exp[-\Gamma(t)]\cos\phi(t), \tag{28}$$

where

$$\Gamma(t)=(\gamma_1+\gamma_2)t+2\sum_{j=1,2}|\alpha_j(t)|^2\sin^2\frac{5}{2}Kt, \tag{29}$$

$$\phi(t)=\beta+\sum_{j=1,2}(-1)^j|\alpha_j(t)|^2\sin 5Kt, \tag{30}$$

$$|\alpha_j(t)|=|\alpha_j|\exp(-\gamma_j t), \quad \alpha_1^*\alpha_2=|\alpha_1\alpha_2|\exp(-i\beta). \tag{31}$$

Here, we have set $\alpha_1=|\alpha_1|\exp(i\phi_{\alpha_1})$, $\alpha_2=|\alpha_2|\exp(i\phi_{\alpha_2})$, and $\beta=\phi_{\alpha_1}-\phi_{\alpha_2}$. Equation (28) can be expanded as the form

$$I^{(cs)}(t)=\frac{1}{2}\sum_{j=1,2}|\alpha_j|^2\exp(-2\gamma_j t)+|\alpha_1\alpha_2|\times\exp[-(\gamma_1+\gamma_2)t]\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\times I_m(|\alpha_1(t)|^2)I_n(|\alpha_2(t)|^2)J_p(|\alpha_1(t)|^2)J_l(|\alpha_2(t)|^2)\times\cos(\beta+[5m-5p+5n+5l]Kt), \tag{32}$$

where $J_p(x)$ and $I_m(x)$ stand for the Bessel and modified Bessel functions, respectively.

It is clear that when the dissipations are neglected ($\gamma_j=0$ and $j=1,2$) and we take the terms

$$m-p+n+l=0, \tag{33}$$

we obtain a nonzero time-averaged value of the intensity of the atomic pattern

$$I^{(cs)}=\frac{1}{2}(|\alpha_1|^2+|\alpha_2|^2)+|\alpha_1\alpha_2|\cos\beta\times\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}I_m(|\alpha_1|^2)I_n(|\alpha_2|^2)\times J_{m+n+l}(|\alpha_1|^2)J_l(|\alpha_2|^2). \tag{34}$$

Equations (32) and (34) show that the intensity exhibits the revivals and collapses. This phenomena also can be easily seen from Fig. 1.

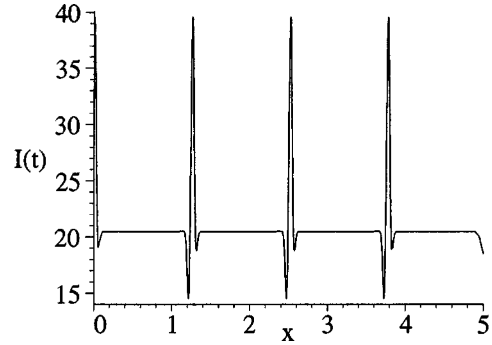

FIG. 1. Diagram of the time evolution of $I^{(cs)}(t)$. The time is in units of $x=Kt$. The result is shown for the case of $\gamma_1=\gamma_2=0$, when the total number of atoms in the two wells is $N=41$ with $|\alpha_1|=5$ and $|\alpha_2|=4$. Here, $\beta=\pi/6$.

On the other hand, when the atoms in the two wells are initially in a Fs $|n,m\rangle$, one has

$$I^{(Fs)}(t)=\frac{1}{2}\sum_{l=0}^{n}\sum_{k=0}^{m}(n+m-k-l)(e^{-2\gamma_1 t})^{n-l}\times(1-e^{-2\gamma_1 t})^l(e^{-2\gamma_2 t})^{m-k}(1-e^{-2\gamma_2 t})^k C_n^l C_m^k, \tag{35}$$

which shows that the intensity does not exhibit collapses and revivals (see Fig. 2).

IV. FIDELITY OF INTERFERENCE BETWEEN THE ATOMS IN THE TWO WELLS

The fidelity and its loss rate of interference between the atoms in the two wells may be characterized by [14]

$$\tilde{F}=\langle\psi_0|\tilde{\rho}(t)|\psi_0\rangle, \tag{36}$$

$$\tilde{L}=-\left\langle\psi_0\left|\frac{\partial\tilde{\rho}}{\partial t}\right|\psi_0\right\rangle\Bigg|_{t=0}, \tag{37}$$

where $|\psi_0\rangle$ is the initial state of the system, and $\tilde{\rho}(t)$ and $\partial\tilde{\rho}/\partial t$ satisfy Eq. (24).

We now turn to study the fidelity of interference between the atoms in the two wells. When the atoms in the two wells

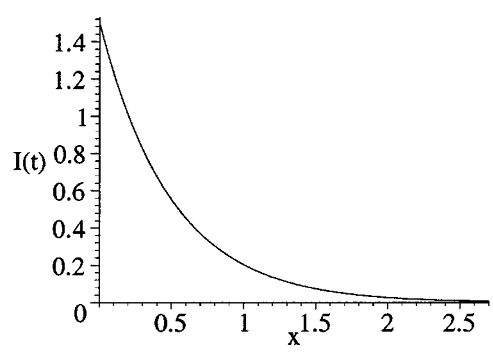

FIG. 2. Diagram of the time evolution of $I^{(Fs)}(t)$. The time is in units of $x=\gamma_1 t=\gamma_2 t$; we have set $\gamma_1=\gamma_2$.

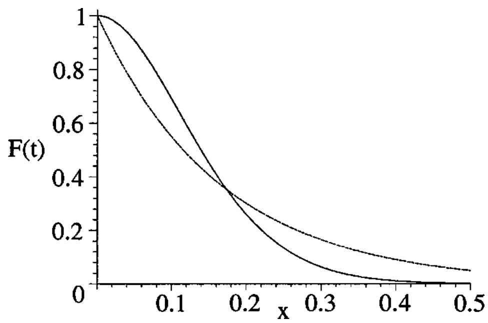

FIG. 3. Diagrams of the time evolution of $\tilde{F}^{(cs)}(t)$ (solid line) and $\tilde{F}^{(Fs)}(t)$ (dashed line). The time is in units of $x=\gamma_1 t=\gamma_2 t$. For simplicity, we have set $\gamma_1=\gamma_2$. The total number of atoms in the two wells is $N=41$ with $|\alpha_1|=5$ and $|\alpha_2|=4$. The Fock state is supposed in $|n,m\rangle=|1,2\rangle$.

are initially in the coherent state $|\alpha_1,\alpha_2\rangle$, the corresponding fidelity of interference is given by

$$\tilde{F}^{(cs)}=\exp[-|\alpha_1|^2(1-e^{-\gamma_1 t})^2-|\alpha_2|^2(1-e^{-\gamma_2 t})^2]. \tag{38}$$

Similarly, for the initial Fock state, one has

$$\tilde{F}^{(Fs)}=\exp[-2n\gamma_1 t-2m\gamma_2 t]. \tag{39}$$

For diagrams of the time evolution of $\tilde{F}^{(cs)}(t)$ and $\tilde{F}^{(Fs)}(t)$, see Fig. 3.

Since the large particle number of the two condensates implies small $\gamma_j\tau_{Fid}$, we can set $1-\exp[-\gamma_j\tau_{Fid}]\cong\gamma_j\tau_{Fid}$, such that

$$\tilde{F}^{(cs)}\cong\exp[-(|\alpha_1|^2\gamma_1^2+|\alpha_2|^2\gamma_2^2)t^2], \tag{40}$$

for short time. The resulting fidelity times are then

$$\tau_{Fid}^{(cs)}\cong[|\alpha_1|^2\gamma_1^2+|\alpha_2|^2\gamma_2^2]^{-1/2}, \quad \tau_{Fid}^{(Fs)}=(2n\gamma_1+2m\gamma_2)^{-1}, \tag{41}$$

which show that the fidelity time is not only related to the initial state of the system, but also to the dissipation parameters.

Furthermore, we can get the fidelity loss rates

$$\tilde{L}^{(cs)}=0, \quad \tilde{L}^{(Fs)}=2n\gamma_1+2m\gamma_2, \tag{42}$$

which indicate that when the atoms in the two wells are initially in the coherent state, the fidelity loss rate of interference is zero; but for the initial Fock state, $\tilde{L}^{(Fs)}$ is related to the initial particle number of the system and the dissipation parameters, but not to the collision parameters.

V. CONCLUSIONS

We have studied interference between the halves of a double-well trap containing a BEC. It is found that when the atoms in the two wells are initially in the coherent state, the intensity exhibits collapses and revivals, but it does not for the initial Fock states. The interference intensity is affected by the collision and dissipation, but for the initial Fock state, it is only related to the dissipation. Whether the initial states are in the coherent states or in a Fock state, the fidelity time has nothing to do with collision. For the initial coherent states, the fidelity loss rate is zero, but for the initial Fock states, it is determined by the initial particle number of the system and dissipation. This shows that interference and its fidelity can be adjusted experimentally by properly preparing the number and initial states of the system.

It is pointed out that the recent realization of a superfluid-Mott-insulator phase transition in a gas of ultracold atoms in an optical lattice [15] is very similar to the state preparation assumed in this paper, we hope our results obtained above will be useful to study Mott-insulator phase transition in the future.

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