# Loss of coherence due to bremsstrahlung

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It is shown that bremsstrahlung can give rise to a loss of coherence for quantum systems. Here a particular mechanism is analyzed, which makes use of the formal properties of the infrared radiation in QED. A charged particle goes through a scattering process into a superposition of two kinematically distinct states. The scattering also gives rise to the emission of a cloud of soft photons, and it is precisely this process that washes out the interference term between the two final states.

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## I. STATEMENT OF THE PROBLEM

The fact that the bremsstrahlung process can lead to the loss of quantum coherence has been studied in detail [1], and one of the relevant results is that this kind of mechanism is at work even when the environment is at zero temperature, so that the relevant process is pure emission of photons. In this article a simple, definite case is examined in terms of the usual formalism of QED: infrared emission by a charged particle undergoing a scattering process in vacuum with the condition that it is found in a superposition of two well separated eigenstates of the momentum.

While the most usual treatment of the infrared radiation goes through an intermediate procedure of regularization by the introduction of an energy cutoff or equivalently of a mass for the photon [2,3], it is also possible to follow the time evolution of the state: the finite time provides the infrared cutoff. This procedure, proposed and discussed by Steinmann [4], is particularly well suited in this case because it allows us to study the time evolution of the decoherence.

The problem is reduced to this form: there is a particle, say an electron, at rest, it suffers a sudden hit, and it becomes relativistic in a very short time, i.e., the momentum transfer is large with respect to its mass, but its final state is a superposition of two different velocities, which, for computational simplicity, will be chosen of the same absolute value. Due to its acceleration the electron radiates; the infrared tail of this radiation cannot be resolved completely and is therefore summed up in defining the transition probability. The question is, how does this fact reflect on the superposition of the charged states?

## II. THE SCATTERING PROCESS AND THE RADIATION

### A. Standard tools

There are standard tools of scattering theory and of QED of soft photons that will be used; they are certainly well known but since some of their features will be essential for the following discussion a short summary of their properties will be presented [3]. The units $c=\hbar=1$ and the standard symbols $\alpha=e^2/(4\pi)$ and $\gamma=1/\sqrt{1-v^2}$ are used.

In scattering theory one considers some total Hamiltonian $H=H_0+V$ and a state evolving with it: $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$; this state is then analyzed in terms of states evolving with the free Hamiltonian $|\Phi_n(t)\rangle = e^{-iH_0t}|\Phi_n(0)\rangle$.

This procedure defines the evolution operator $\mathcal{U}(t) = e^{iH_0t}e^{-iHt}$, which may also be expressed by means of the interaction term in Dirac or interaction representation:

$$\mathcal{U}(t)=\mathcal{P}\exp\int_0^t[-i\tilde{V}(\tau)d\tau],\quad \tilde{V}(t)=e^{iH_0t}Ve^{-iH_0t}. \tag{1}$$

$\mathcal{P}$ is the time ordering and it must be remembered that in the interaction terms the field operators evolve according to $H_0$.

In QED with soft photons the charged particle is comparatively very heavy, there is no pair production, and moreover the spin is not very relevant because the dipole (and multipole) emission is suppressed for very large wavelengths. So the charged particle is treated in first quantization, the spin variables are suppressed, and the velocity, not the three-momentum, is taken as a fixed vector with absolute value near 1; this results in the Hamiltonian [3]

$$H=H_0+V, \tag{2a}$$

$$H_0=-i\vec{v}\cdot\vec{\partial}+\frac{m}{\gamma}+\sum_\nu\int d^3k\,\omega a^\dagger_{k,\nu}a_{k,\nu}, \tag{2b}$$

$$V=-\frac{e}{(2\pi)^{3/2}}\sum_\nu\int\frac{d^3k}{\sqrt{2\omega}}[a_{k,\nu}\vec{v}\cdot\vec{\epsilon}_{k,\nu}e^{i\vec{k}\cdot\vec{r}} + a^\dagger_{k,\nu}\vec{v}\cdot\vec{\epsilon}_{k,\nu}e^{-i\vec{k}\cdot\vec{r}}]+\Delta. \tag{2c}$$

The constant $\Delta$ is an energy renormalization term, having the role that in a covariant treatment is played by the mass counterterm. The quantized electromagnetic field is in the Coulomb gauge with linear polarizations; the fundamental commutation relations are given in noncovariant form:

$$[a_{k,\nu},a^\dagger_{k',\nu'}]=\delta_{\nu,\nu'}\delta(\vec{k}-\vec{k}'). \tag{3}$$

The use of Eq. (1) then gives

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$$\tilde{V}(t)=-\frac{e}{(2\pi)^{3/2}}\sum_{\nu}\int\frac{d^3k}{\sqrt{2\omega}}[a_{k,\nu}\vec{v}\cdot\vec{\epsilon}_{k,\nu}e^{i\vec{k}\cdot\vec{r}+i(\vec{k}\cdot\vec{v}-\omega)t} + a^{\dagger}_{k,\nu}\vec{v}\cdot\vec{\epsilon}_{k,\nu}e^{-i\vec{k}\cdot\vec{r}-i(\vec{k}\cdot\vec{v}-\omega)t}]+\Delta. \tag{4}$$

From these expressions it follows that $[\tilde{V}(t),\tilde{V}(t')]$ is a number and so the application of the Wick theorem to Eq. (1) can be performed in a very compact and efficient way, in a form due originally to Hori [5], with the result

$$\mathcal{U}(t)=\exp\left[\frac{1}{2}\int_0^t\int_0^t\frac{\delta}{\delta\tilde{V}(\tau)}D(\tau,\tau')\frac{\delta}{\delta\tilde{V}(\tau')}\right] \times\mathcal{N}\exp\int_0^t[-i\tilde{V}(\tau)d\tau]; \tag{5a}$$

here $D(\tau,\tau')$ is the contraction of two interaction operators,

$$D(\tau,\tau')=\langle|\mathcal{P}(\tilde{V}(\tau)\tilde{V}(\tau'))|\rangle, \tag{5b}$$

where $|\rangle$ is the photon vacuum.

### B. Finite time evolution

Normal ordering according to Eq. (5) gives rise to a numerical factor in $\mathcal{U}$, i.e.,

$$\exp\left[-\frac{1}{2}\int_0^t\int_0^t D(\tau,\tau')d\tau\,d\tau'\right]=\exp\left[ia-\frac{1}{2}b\right], \tag{6}$$

and a straightforward calculation gives

$$D(\tau,\tau')=\frac{e^2}{(2\pi)^3}\int d\Omega\,\frac{1}{2}\omega\,d\omega\,v^2 \times\sin^2\theta_k[e^{i(\vec{k}\cdot\vec{v}-\omega)(\tau-\tau')}\vartheta(\tau-\tau')+\tau\leftrightarrow\tau'], \tag{7}$$

where $\vec{k}\cdot\vec{v}=\omega v\sin\theta_k$.

In calculating the exponents $a$ and $b$, an ultraviolet divergence is found. This is not unexpected because no renormalization has been performed; moreover, the divergence is different from the correct covariant one because in the ultraviolet region treating the velocities of the electron as constant vectors is kinematically inconsistent. Since we are interested in the infrared behavior, this divergence will simply be cut off by inserting an upper limit $\omega_M$ whenever needed. In this way the result is

$$a=\frac{e^2}{(2\pi)^2}\int\frac{1}{2}dz\,v^2(1-z^2)\left[-\frac{\omega_M t}{1-vz} +\frac{1}{(1-vz)^2}\mathrm{Si}[\omega_M t(1-vz)]\right], \tag{8a}$$

$$b=\frac{e^2}{(2\pi)^2}\int dz\,v^2\frac{(1-z^2)}{(1-vz)^2}\{C+\ln[\omega_M t(1-vz)] -\mathrm{Ci}[\omega_M t(1-vz)]\}. \tag{8b}$$

In these expressions the position $z=\cos\theta_k$ has been used, $C$ is the Euler-Mascheroni constant, and Si and Ci are the sine-integral and cosine-integral functions [6]. The term linear in time that appears in the imaginary part $a$ could be eliminated by suitably defining the counterterm $\Delta$ into $V$; the ultraviolet cutoff is, however, essential in the logarithmic term in $b$. For large values of $t$ the relevant limits are

$$\mathrm{Si}(mt)\to\frac{1}{2}\pi,\quad \mathrm{Ci}(mt)\to 0\quad \text{for } t\to\infty.$$

The most important term is the logarithm appearing in $b$, so it is necessary to calculate its coefficient $B$:

$$B=\frac{e^2}{(2\pi)^2}v^2\int dz\,\frac{1-z^2}{(1-vz)^2}=\frac{4\alpha}{\pi}\left[\frac{1}{v}\ln[\gamma(1+v)]-1\right]>0. \tag{9}$$

Although the complete expression contains the arbitrary parameter $\omega_M$ the coefficient $B$ is independent of $\omega_M$. If, as a particular case, the final state $|\Phi_0(t)\rangle$ contains no photon at all, then the operator term $\mathcal{N}\exp\int_0^t[-i\tilde{V}(\tau)d\tau]$ gives simply 1 and the result for the transition probability is

$$\Pi_0=|\langle\Phi_0(t)|\Psi(t)\rangle|^2=e^{-b}\approx\mathrm{const}\times(\omega_M t)^{-B}. \tag{10}$$

So the probability goes to zero as the time grows.

Acting on a state $|\Phi_n(t)\rangle$ containing $n$ photons of momenta $(\vec{k}_1,\ldots,\vec{k}_n)$, the operator $\mathcal{N}\exp\int_0^t[-i\tilde{V}(\tau)d\tau]$ yields a factor

$$F_n=\frac{1}{\sqrt{n!}}\left[\frac{e}{(2\pi)^{3/2}}\right]^n(\vec{v}\cdot\vec{\epsilon})_1\cdots(\vec{v}\cdot\vec{\epsilon})_n e^{i(\vec{k}_1+\cdots+\vec{k}_n)\cdot\vec{r}} \times\frac{1}{2^{n/2}\sqrt{\omega_1\cdots\omega_n}}\frac{1}{(\vec{k}\cdot\vec{v}-\omega)_1}\cdots\frac{1}{(\vec{k}\cdot\vec{v}-\omega)_n}[1 -e^{-i(\vec{k}\cdot\vec{v}-\omega)_1}]\cdots[1-e^{-i(\vec{k}\cdot\vec{v}-\omega)_n}]. \tag{11}$$

In calculating the absolute square, one gets the additional factor

$$G_n=[g(\omega_R)]^n/n! \tag{12a}$$

with the position[1]

[1]The parameter $\omega_R$ is an ultraviolet cutoff having the same role as $\omega_M$. It is introduced in order to make clear that the final result concerning the time evolution of the interference term [see Eq. (20)] is independent of the ratio $\omega_M/\omega_R$ between the virtual and real cutoffs.

$$g(\omega_R)=\frac{e^2}{(2\pi)^2}\int dz\, v^2\frac{(1-z^2)}{(1-vz)^2}\,\frac{d\omega}{\omega}\{1-\cos[\omega t(1-vz)]\}, \tag{12b}$$

and it is easily verified that $g(\omega_R)$ has the same form as $b$, with the unique substitution $\omega_M\to\omega_R$.

The behavior for large $t$ of $|\langle\Phi_n(t)|\Psi(t)\rangle|^2$ is clear: it goes to zero for every value of $n$ as $(\ln\omega_R t)^{2n}(\omega_M t)^{-B}/n!$, so in the photon Fock space the evolution operator $\mathcal{U}$ converges weakly to zero. If a sum over $n$ is performed, then the exponential of $g(\omega_R)$ compensate the factor $e^{-b}$ and the sum converges to a finite limit, which of course depends on $\omega_R$ and $\omega_M$. Instead of Eq. (10) it results in the fact

$$\sum_n |\langle\Phi_n(t)|\Psi(t)\rangle|^2=e^{-b+g}\approx\text{const}\times\left[\frac{\omega_R}{\omega_M}\right]^B \quad \text{for } t\to\infty. \tag{13}$$

The actual expressions for $b$ and $g$ show also that the small-$t$ behavior of the probabilities is essentially proportional to $t^{2n}$. This is useful to notice since the $t=0$ emission is not very well defined. In this simplified dynamics, in fact, there is a short time in which the primary interaction is active, the one that transfers a large momentum. Within this period the calculation of the accompanying soft radiation is very difficult, but the total emission is certainly not very relevant.

For comparison with the conclusions of the next section we stress this result: the virtual corrections give rise to a decreasing factor, the real corrections yield an increasing factor, and the two compensate for $t\to\infty$. For future convenience it is useful to recall that these results are usually obtained in a slightly different way, there is no mention of finite time, there is an explicit infrared cutoff, say λ, and it is verified that the dependences on λ lead to divergences for every $n$, but they compensate in the sum over $n$ [2,3].

### C. Superposition of two states of motion

With the same technique now it is possible to investigate the behavior of the soft radiation in the case in which the charged particle after the scattering ends in a superposition of two well distinct states of motion.

The kind of scattering process is not relevant, as can be learned from [2]. What really matters is that the particle suffers a spacelike momentum transfer $q^2$, which is much larger than its mass $-q^2\gg m^2$.

The possible origin of the splitting is not analyzed, only a formal dynamical model is built up: the initial particle has a conserved internal variable, say $\tau_3$, such that its eigenvalues affect the velocity after the scattering, e.g., $\vec{v}_\pm=\vec{v}+\tau_3\vec{w}$. The effective Hamiltonian after the scattering, which is linear in $v$, has, therefore, the same structure in $\tau_3$ and, in turn, the same structure is reproduced in the evolution operator $\mathcal{U}$. If the initial state was, e.g., an eigenstate of $\tau_1$, the evolution gives rise to $|\Psi(t)\rangle|=(1/\sqrt{2})[\mathcal{U}_+(t)|\Psi_+(0)\rangle+\mathcal{U}_-(t)|\Psi_-(0)\rangle]$ and the transition probability, in the hypothesis that $\tau_3$ is strictly conserved, takes the form

$$\Pi=\frac{1}{2}|\langle\Phi_+|\mathcal{U}_+|\Psi_+\rangle|^2+\frac{1}{2}|\langle\Phi_-|\mathcal{U}_-|\Psi_-\rangle|^2+\text{Re}[\langle\Phi_-|\mathcal{U}_-|\Psi_-\rangle^*\langle\Phi_+|\mathcal{U}_+|\Psi_+\rangle]. \tag{14}$$

For the absolute squares there is nothing new to say; for every definite $n$ they go to zero for $t\to\infty$ and the sum over $n$ yields a finite result. Something new could happen, after resummation over $n$, in the interference term

$$\mathcal{I}(t)=\text{Re}[\langle\Phi_-|\mathcal{U}_-(t)|\Psi_-\rangle^*\langle\Phi_+|\mathcal{U}_+(t)|\Psi_+\rangle].$$

Here in fact we get two different factors of type $F_n$, Eq. (11), the former with $v_1$ and the latter with $v_2$. They build up terms like $G_n$, again say,

$$Q_n=[q(\omega_R)]^n/n! \tag{15a}$$

with

$$q(\omega_R)=\frac{e^2}{(2\pi)^3}\int d\Omega\,\frac{\sum_{\text{pol}}(\vec{v}_1\cdot\vec{\epsilon})(\vec{v}_1\cdot\vec{\epsilon})}{(\vec{k}\cdot\vec{v}_1-\omega)(\vec{k}\cdot\vec{v}_2-\omega)}\,\omega\,d\omega \times\{1-\exp[-i(\vec{k}\cdot\vec{v}_1-\omega)t]-\exp[i(\vec{k}\cdot\vec{v}_2-\omega)t] +\exp[i(\vec{k}\cdot\vec{v}_2-\vec{k}\cdot\vec{v}_1)t]\}. \tag{15b}$$

So we get again an exponential form, but with a more complicated exponent $q(\omega_R)$, which is now complex. Writing $q(\omega_R)=x+iy$ so that $\text{Re}^{q(\omega_R)}=e^x\cos y$, the time evolution is essentially governed by $e^x$, and the term in $y$ produces only oscillations.

For computational simplicity from now on the calculations are made in the particular case $\vec{v}\cdot\vec{w}=0$ so the two speeds are equal, $v_+=v_-=v$. The term $x$ can be computed and the result is

$$x=\int d\Omega\,\xi\{[C+\ln(\omega_R t s_1)-\text{Ci}(\omega_R t s_1)]+[C+\ln(\omega_R t s_2)-\text{Ci}(\omega_R t s_2)]-[C+\ln(\omega_R t s_3)-\text{Ci}(\omega_R t s_3)]\} \tag{16}$$

with

$$s_1=1-v\cos\theta_+,\quad s_1=1-v\cos\theta_-,$$

$$s_3=|v\cos\theta_+-v\cos\theta_-|,\quad k\cdot v_\pm=v\omega\cos\theta_\pm,\ \text{and}$$

$$\xi=\frac{\alpha}{2\pi^2}v^2\frac{\cos\delta-\cos\theta_+\cos\theta_-}{(1-v\cos\theta_+)(1-v\cos\theta_-)}. \tag{17}$$

The terms $s_i$ are positive by construction so that here also for large values of $t$ only the logarithmic term and the constants survive in Eq. (16):

$$x=\int d\Omega\,\xi\left[C+\ln(\omega_R t)+\ln\left(\frac{s_1 s_2}{s_3}\right)\right],$$

and one must then calculate explicitly the coefficient of the dominant logarithmic term $X=\int d\Omega\,\xi$. Here the angle between the two velocities is $\delta$. It is convenient to take the

polar axis along one of the velocities so that $\theta_+ = \theta$; $\cos\theta_- = \cos\theta\cos\delta - \sin\theta\sin\delta\cos\phi$. The integration is lengthy but quite standard and yields

$$X = \frac{2\alpha}{\pi}\left\{\frac{1}{v}\ln\frac{1+v}{1-v} - 1 - \mathcal{J}\right\} \tag{18a}$$

with

$$\mathcal{J} = \frac{1-v^2\cos\delta}{2}[2(1-\cos\delta)-v^2\sin^2\delta]^{-1/2} \times\left[\arcsin\frac{(1+v)(1-\cos\delta)-v^2\sin^2\delta}{(1-v)\sin\delta}\gamma - \arcsin\frac{(1-v)(1-\cos\delta)-v^2\sin^2\delta}{(1+v)\sin\delta}\gamma\right]. \tag{18b}$$

The expression is not very transparent, since the dependence on the two parameters $\delta$ and $v$, or $\gamma$, is interwoven. It becomes much clearer in some limits. For $\delta\to 0$ it results in $\mathcal{J}\to 1$ so we get for $X$ the limiting form of $B$, Eq. (9), as must happen. In the limit $\gamma\to\infty$ at fixed $\delta\neq 0$ such that $\gamma\delta\gg 1$ results in $\mathcal{J}\to 2\ln(\gamma\delta)$ so that in this limit

$$X = \frac{4\alpha}{\pi}\left\{\ln(2\gamma) - \frac{1}{2} - \ln(\gamma\delta)\right\}. \tag{19a}$$

Comparing this result with the limiting form of Eq. (9),

$$B = \frac{4\alpha}{\pi}[\ln(2\gamma) - 1], \tag{19b}$$

we see that, in this case, at variance with the standard result shown in the previous section, the effect of $X$ does not completely compensate the effect of $B$. If we consider the ratio of the interference term at different times we have

$$\frac{\mathcal{I}(t_1)}{\mathcal{I}(t_2)} = \left[\frac{t_1}{t_2}\right]^{-\nu} \quad \text{with} \quad \nu = \frac{4\alpha}{\pi}\left[\ln(\gamma\delta) - \frac{1}{2}\right]. \tag{20}$$

This result could also be obtained in a different way, without use of finite time, but with an explicit infrared cutoff, say λ, and it is verified that in this case the dependences on λ do not compensate.

### III. PERTURBATIVE TREATMENT

In the previous section the problem was treated in a way that is not very usual in quantum field theories; moreover, some approximations justified by the kinematical restrictions were used. Really, a more general result could be attained by the use of a covariant formulation of the infrared emission, which was elaborated and published long ago [7], but perhaps it may be useful to present a treatment according to the most standard procedures, i.e., a perturbative treatment in terms of Feynman graphs where only the kinematical restrictions inherent to the problem, the consideration of very soft photons, appear. There is a price to pay: the actual calculations are limited to the first nontrivial perturbative order, no finite time effects are considered, but the infrared cutoff is introduced and used for further discussion. The treatment is very standard, so it will just be outlined as rapidly as possible.

The process is the same previously considered: at least three terms must be taken into account. In every term the superposition of two states of motion, now denoted as $i$ and $j$, is considered.

(1). A perturbative scattering amplitude without any radiative correction:

$$M_0 = M_0^{(i)} + M_0^{(j)} = c_i\overline{u}(q_i)Q(q_i,p)u(p) + c_j\overline{u}(q_j)Q(q_j,p)u(p). \tag{21a}$$

(2) A perturbative scattering amplitude with real radiative corrections, describing the emission of one photon:

$$\begin{aligned} M_r &= M_r^{(i)} + M_r^{(j)} \\ &= ie(2\pi)^{-3/2}\{c_i[\overline{u}(q_i)Q(q_i,p-k)S(p-k)\hat{\epsilon}(k)u(p) \\ &\quad + \overline{u}(q_i)\hat{\epsilon}(k)S(q_i+k)Q(q_i+k,p)u(p)] \\ &\quad + c_j[\overline{u}(q_j)Q(q_j,p-k)S(p-k)\hat{\epsilon}(k)u(p) \\ &\quad + \overline{u}(q_j)\hat{\epsilon}(k)S(q_j+k)Q(q_j+k,p)u(p)]\}. \end{aligned} \tag{21b}$$

(3) A perturbative scattering amplitude with virtual radiative corrections, describing the emission and reabsorption of one photon:

$$\begin{aligned} M_v &= M_v^{(i)} + M_v^{(j)} \\ &= i\frac{e^2}{(2\pi)^4}\int\{c_i[\overline{u}(q_i)\gamma_\mu S(q_i-k)Q(q_i-k,p-k) \\ &\quad \times S(p-k)\gamma^\mu u(p)] + c_j[\overline{u}(q_j)\gamma_\mu S(q_j-k)Q(q_j-k,p \\ &\quad -k)S(p-k)\gamma^\mu u(p)]\}D(k)d^4k. \end{aligned} \tag{21c}$$

$S(l)$ and $D(l)$ are clearly the free electron and photon propagators, and $Q(q,p)$ is the perturbative form of the vertex responsible for the scattering with large momentum transfer. Given these definitions, the subsequent calculations strictly follow the procedure exposed in detail in Ref. [1], only the points relevant for the present discussion will be repeated.[2]

In Eq. (21b) the infrared divergent contribution, denoted by a tilde, is

$$\tilde{M}_r = \beta_i(k)M_0^{(i)} + \beta_j(k)M_0^{(j)},$$

$$\beta_s(k) = -\frac{e}{(2\pi)^{3/2}}\left[\frac{q_s\epsilon_\nu}{q_sk} - \frac{p\epsilon_\nu}{pk}\right], \quad s = i,j. \tag{22}$$

In a similar way the infrared contribution of $M_v$ is

$$\tilde{M}_v = \rho_i M_0^{(i)} + \rho_j M_0^{(j)}. \tag{23}$$

[2]In the previous section the choices $p=(m,0)$ and $c_i = c_j = 1/\sqrt{2}$ were made.

The actual form of $\rho_s$ can be given [2], but it is not necessary[3] for the following simple reason: We know that in QED there is a compensation of the infrared divergences order by order in $\alpha$ in the perturbative expansion, so in our case this compensation must happen, when the states are usual, i.e., when only one of the $c_j$ is different from zero at the first order in $\alpha$ between the term $2\,\mathrm{Re}[M_v M_0^*]$ and the term $|M_r|^2$ (integrated over the infrared phase space and summed over the polarizations of the photon). The order $\alpha$ term of the transition probability is

$$\tilde{\Pi} = \sum_{s=i,j} |c_s M_0^{(s)}|^2 \left[ \sum_\nu \int \frac{d^3k}{2\omega} \beta_s^2(k) + 2\rho_s \right] + 2\,\mathrm{Re}(c_i c_j^* M_0^{(i)} M_0^{(j)*}) \times \left[ \sum_\nu \int \frac{d^3k}{2\omega} \beta_i(k)\beta_j(k) + \rho_i + \rho_j \right].$$

The infrared compensation says that

$$\frac{1}{2} \sum_\nu \int \frac{d^3k}{2\omega} \beta_s^2(k) + \rho_s|_{\mathrm{ir}} = 0; \tag{24}$$

the subscript ''ir'' states that the equality is valid for the infrared divergent part.

So we may reexpress the infrared factor which multiplies $\mathrm{Re}(c_i c_j^* M_0^{(i)} M_0^{(j)*})$ in terms of the functions $\beta_s$ with the result

$$J = -\frac{1}{2} \sum_\nu \int \frac{d^3k}{2\omega} [\beta_i(k) - \beta_j(k)]^2, \quad \omega > \lambda. \tag{25}$$

It is explicitly seen that the initial velocity, or momentum, plays no role in the compensation of the $\lambda$ dependence; the relevant effect originates from the difference between $\vec{q}_i$ and $\vec{q}_j$ which appear in the expression of $\beta$. The interference term is affected, therefore, by an infrared factor that diverges logarithmically in the limit $\lambda \to 0$; in fact, from Eqs. (22) and (25) it is seen that the integral separates into an angular factor and a factor $\int d\omega/\omega$ giving rise to the logarithmic divergence. So the divergence exists unless the angular part gives zero, which happens only if $\vec{q}_i = \vec{q}_j$. This kind of divergent behavior is just the logarithm of the zero of $\mathcal{I}(t)$ found in the previous section when $t \to \infty$.

Thus this result does not add anything to what was said in the previous section. It may be interesting, however, that is has been obtained by using standard Feynman rules, standard procedures for the compensation of the infrared terms, and, especially, that no artificial manipulations of the spin variables or of the ultraviolet divergence have been done. The price to pay is that we lose the possibility of summing in closed form all orders in $\alpha$.

[3]In order to be precise it must be recalled that the details of the infrared compensation are gauge dependent, but the final result is not. The most straightforward way to get the final result is the use of the Yennie gauge [8] $D_{\mu\nu} = (i/k^2)[g_{\mu\nu} + 2k_\mu k_\nu/k^2]$, because in this gauge there is no infrared contribution coming from the renormalization term.

## IV. SOME CONCLUSIONS

What has been presented may be considered an extended exercise in QED, because no new idea has been introduced. This exercise could be useful, however, because it contains a description of a standard process that is not completely standard. There is a process of measurement that involves by construction a sum over incoherent contributions so the fact that the interference terms, at the end, disappear, seems quite obvious. Yet the way in which the interference terms go to zero is not simply determined by the measurement process but from the dynamics of the soft radiation; in fact, at fixed choice of the final states, it depends on the time after the primary scattering at which the observation is performed. Although some artificial parameters $\omega_M, \omega_R$ have been introduced, the exponent $\nu$ in Eq. (20) which determines the rate of reduction of the interference terms is completely given by the kinematical conditions $(v, \delta)$ and by the fundamental constant $\alpha$. Looking for some quantitative statement, it must be said that the exponent is very small in the usual conditions. The effect becomes comparatively larger for very energetic electrons: let us assume, for definiteness, that $\gamma = 10^4$ and $\delta = 4°$; then, from Eq. (19), $\nu \approx 0.05$, and from Eq. (20) in going from $t_1$ to $t_2$ the interference term is reduced to one-half if the ratio $t_2/t_1$ is $2^{20} \approx 1.05 \times 10^6$. Perhaps the result is more intuitive in space: if the interference term has a given size at the distance of 100 fm, where the primary hard scattering is certainly no longer active, then it is reduced to one-half at $\Delta\ell = 0.1\ \mu$m, but to have it reduced to one-fifth takes a $\Delta\ell$ of about 10 m.

The results presented here are essentially of the same type as the ones presented in [1]. Some differences can be found. The techniques used are different since here the usual formalism of QED is employed (this is particularly evident in the perturbative treatment), and the whole effect stems from an incomplete compensation between real and virtual emission. The other point is that here the two waves generated in the splitting process are not led to interfere in the same region; in fact, there is a physical parameter that controls the rapidity of the decoherence, which is precisely the angle between the two asymptotic directions of the final states.

From a more formal point of view the result is connected with the presence of infinitely many degrees of freedom, which may cause unitary evolution to take, finally, every physical state outside the original Hilbert space. The true asymptotic states could be described through the infraparticle formulation. In this case there would be no question of suppression of the interference terms, since there would not exist a superposition principle [9]. In the example presented here, in Sec. II, the states are not strictly asymptotic, so the possibility of a superposition is not excluded.

A particular and perhaps accidental property of QED has been used: the fact that the problem of infrared radiation can be controlled in a satisfactory way even when the incoming state is treated as an isolated charged state, without the cloud

of photons. This possibility should be contrasted with other situations found in theories like QCD, where the divergences may be eliminated only after summation over incoming and outgoing infrared particles [10]. However, the infrared structure of QCD is, on the whole, not yet understood, so the comparison cannot be pushed further. One could also think of replacing the infrared electromagnetic radiation with a gravitational radiation, so every microscopic and macroscopic body would experience the effect, but in the usual linearized gravitational theory the radiation is exceedingly tiny.

The effect of the gravitational radiation has an interest in principle: it would require, however, a nontrivial extension of the treatment presented here. A sharp difference is that in QED the relevant physical process is the flow of (Abelian) charge, and it is enough to follow this flow particle by particle; with the gravitational interaction the relevant process is the flow of energy and momentum and so the scattering process must be studied as a whole.

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