Tackling systematic errors in quantum logic gates with composite rotations

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We describe the use of composite rotations to combat systematic errors in single-qubit quantum logic gates and discuss three families of composite rotations which can be used to correct off-resonance and pulse length errors. Although developed and described within the context of nuclear magnetic resonance quantum computing, these sequences should be applicable to any implementation of quantum computation.

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I. INTRODUCTION

Quantum computers [1] are information-processing devices that use quantum-mechanical effects to implement algorithms that are not accessible to classical computers, and thus to tackle otherwise intractable problems [2]. Quantum computers are extremely vulnerable to the effects of errors, and considerable effort has been expended on alleviating the effects of random errors arising from decoherence processes [3–5]. It is, however, also important to consider the effects of systematic errors, which arise from reproducible imperfections in the apparatus used to implement quantum computations.

The effects of systematic errors are clearly visible in nuclear magnetic resonance (NMR) experiments [6], which have been used to implement small quantum computers [7–12]. Implementing complex quantum algorithms requires a network of many quantum logic gates, which for an NMR implementation translates into long cascades of pulses. In these cases small systematic errors in the pulses (which can be ignored in many conventional NMR experiments) accumulate and have significant effects.

It makes sense to consider systematic errors as some of them can be tackled relatively easily. In the Bloch picture, where unitary operations are visualized as rotations of the Bloch vector on the unit sphere, systematic errors are expressed as rotational imperfections. The sensitivity of the final state to these imperfections can be much reduced by replacing single rotations with composed rotations as discussed below.

II. SYSTEMATIC ERRORS IN NMR QUANTUM COMPUTERS

Any implementation of a quantum computer requires quantum bits (qubits) on which the quantum information is stored, and quantum logic gates which act on the qubits to process the quantum information. Fortunately, it is only necessary to implement a small set of quantum logic gates, as more complex operations can be achieved by joining these gates together to form logic circuits. A simple and convenient set comprises a range of single-qubit gates together with one or more two-qubit gates, which implement conditional evolutions and thus logical operations [13].

NMR quantum computers are implemented [11] using the two-spin states of spin-1/2 atomic nuclei in a magnetic field as the qubits. Transitions between these states, and thus single-qubit gates, are achieved by the application of radio frequency (rf) pulses. Two-qubit gates require some sort of spin-spin interaction, which in NMR is provided by the scalar spin-spin coupling (J coupling) interaction. While this does not have quite the form needed for standard two-qubit gates, it can be easily sculpted into the desired form by combining free evolution under the background Hamiltonian (which includes spin-spin coupling terms) with the application of single-qubit gates [11].

As single-qubit gates involve the application of external fields, they are vulnerable to systematic errors in these fields. In the ideal case, the application of a rf field in resonance with the corresponding transition with relative phase ϕ (in the rotating frame [6]) will drive the Bloch vector through some angle about an axis orthogonal to the z axis and at an angle ϕ to the x axis. The rotation angle θ depends on the nutation rate induced by the rf field, usually written ν_1, and the duration of the pulse, τ. In practice, the rf field is not ideal, and this leads to two important types of systematic errors, pulse length errors and off-resonance effects [6,14].

Pulse length errors occur when the duration of the rf pulse is set incorrectly, or (equivalently) when the rf field strength deviates from its nominal value, so that the rotation angle achieved deviates from its theoretical value. Within NMR, this effect is most commonly observed as a result of spatial inhomogeneity in the applied rf field, so that it is impossible for all the spins within a macroscopic sample to experience the same rotation angle. Off-resonance effects arise when the rf field is not quite in resonance with the relevant transition, so that the rotation occurs around some tilted axis.

Composite pulses [6,14,15] are widely used in NMR to minimize the sensitivity of the system to these errors by replacing simple rotations with composite rotations that are less susceptible to such effects. However, conventional composite pulse sequences are rarely appropriate for quantum computation because they usually incorporate assumptions about the initial state of the spins. Such starting states are not known for pulses in the middle of complex quantum computations, and it is therefore necessary to use fully compensating (type-A) composite pulse sequences [15], which work for any initial state. Composite pulses of this kind, which do not offer quite the same degree of compensation as is found

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with more conventional sequences, are of little use in conventional NMR, and have received relatively little study. They are, however, ideally suited to quantum computation.

A distinction should be made between composite pulses, and the more complex approach of shaped pulses [14]. Although this distinction is not absolute (shaped pulses can be considered as extremely complex composite pulses), two differences can be observed. Composite pulses as considered here are assumed to comprise a short train of constant amplitude rf pulses, with only the phase and duration of each subpulse allowed to vary; by contrast shaped pulses comprise long trains of amplitude modulated pulses, with the duration of each subpulse usually held constant. One consequence of this is that composite pulses are usually simpler to implement than shaped pulses; this is especially true when all the pulse durations are integer multiples of some basic time. A second consequence is that composite pulses are usually of short overall duration, and so it is normally permissible to neglect relaxation during composite pulses just as it is usually neglected during simple pulses.

The relative simplicity of composite pulses does result in some limitations. In particular, shaped pulses are suitable for frequency selective excitation [14], while the types of composite pulse discussed here cannot be used for this purpose. Recently, a third hybrid approach, based on "strongly modulated" pulses has been described [16], which is capable of selective excitation. This approach uses a small number of pulses, but allows the amplitude and frequency of each pulse to be varied as well as the initial phase and length, which are the only adjustable parameters in conventional composite pulses. This approach has been demonstrated using NMR [16], although it should be noted that in this case the "composite pulse" was in fact implemented as a shaped pulse, using phase ramping [14] to obtain frequency shifts. In principle, this approach should permit the design of robust selective pulses, but this has not yet been explicitly demonstrated.

III. OFF-RESONANCE ERRORS

The problem of tackling off-resonance errors was initially studied by Tycko [17]; his results were then extended by Cummins and Jones [18,19]. The influence of off-resonance effects on quantum dynamics in extended spin chains has also been considered by Berman *et al.* in the context of selective [20] and nonselective [21] excitation. Here we describe two families of composite pulses which can be used to compensate for off-resonance errors in nonselective excitation, and show how they can be derived using quaternions.

The original method used to develop many type-*A* composite pulse sequences [17–19] was based on dividing the propagator describing the evolution of the quantum system into intended and error components, and then seeking to minimize the error term. While this approach is effective, it is cumbersome, and a much simpler approach can be adopted for single-qubit gates, which are simply rotations on the Bloch sphere and so can be modeled by quaternions. The quaternions corresponding to individual pulses can be multiplied together to give a quaternion description of the composite pulse, which can then be compared with the quaternion of the ideal system.

A quaternion is often thought of as a vector with four coefficients, but when describing a rotation it is more useful to regroup these coefficients as a scalar and a three-vector,

$$\mathsf{q}=\{s,\mathbf{v}\}, \tag{1}$$

where

$$s=\cos(\theta/2) \tag{2}$$

depends solely on the rotation angle θ and

$$\mathbf{v}=\sin(\theta/2)\mathbf{a} \tag{3}$$

depends on both the rotation angle θ and a unit vector along the rotation axis, $\mathbf{a}$. Thus the quaternion describing an on-resonance pulse with phase angle ϕ is

$$\mathsf{q}_{\theta\phi}=\{\cos(\theta/2),\ \sin(\theta/2)\{\cos(\phi),\ \sin(\phi),\ 0\}\}. \tag{4}$$

An off-resonance pulse is conveniently parametrized by its off-resonance fraction $f=\delta/\nu_1$ (where δ is the off-resonance frequency, and ν_1 is the nutation rate), and is described by the quaternion

$$\mathsf{q}_{\theta\phi}=\left\{\cos(\theta'/2),\ \frac{\sin(\theta'/2)}{\sqrt{1+f^2}}\{\cos(\phi),\ \sin(\phi),\ f\}\right\}, \tag{5}$$

where $\theta'=\theta\sqrt{1+f^2}$, and θ is now the nominal rotation angle, that is, the rotation achieved when $f=0$. The quaternion describing a sequence of pulses is obtained by multiplying the quaternions for each pulse according to the rule

$$\mathsf{q}_1\mathsf{q}_2=\{s_1s_2-\mathbf{v}_1\cdot\mathbf{v}_2,\ s_1\mathbf{v}_2+s_2\mathbf{v}_1+\mathbf{v}_1\wedge\mathbf{v}_2\}. \tag{6}$$

Finally, two quaternions can be compared using the quaternion fidelity [15]

$$\mathcal{F}(\mathsf{q}_1,\mathsf{q}_2)=|\mathsf{q}_1\cdot\mathsf{q}_2|=|s_1s_2+\mathbf{v}_1\cdot\mathbf{v}_2| \tag{7}$$

(it is necessary to take the absolute value, as the two quaternions $\{s,\mathbf{v}\}$ and $\{-s,-\mathbf{v}\}$ correspond to equivalent rotations, differing in their rotation angle by integer multiples of 2π).

Following our previous work [18], we seek to tackle off-resonance errors in a θ_x, pulse using a sequence of three pulses applied along the x, $-x$ and x axes; pulses with any other phase angle can then be trivially derived by simply adding the desired value to the phase angles of all the pulses in the sequence. Such sequences can be described completely by the nominal rotation angles of the three pulses, θ_1, θ_2, and θ_3. The composite quaternion for this composite pulse is complicated, but the situation can be greatly simplified by expanding it as a Maclaurin series in f and neglecting all terms above the first power. This gives

$$s=\cos\left(\frac{\theta_1-\theta_2+\theta_3}{2}\right) \tag{8}$$

and

$$\mathbf{v}=\left\{\sin\left(\frac{\theta_1-\theta_2+\theta_3}{2}\right),\quad \sin\frac{\theta_2}{2}\sin\left(\frac{\theta_1-\theta_3}{2}\right),\right.$$
$$\left. f\left(2\cos\left(\frac{\theta_2-\theta_3}{2}\right)\sin\frac{\theta_1}{2}-\sin\left(\frac{\theta_1-\theta_2-\theta_3}{2}\right)\right)\right\},\quad (9)$$

while the ideal quaternion has the form

$$\{\cos(\theta/2),\{\sin(\theta/2),0,0\}\}. \quad (10)$$

It now remains to choose the three nominal rotation angles so that these equations agree.

First we note that in order to achieve the correct rotation angle, $s=\cos(\theta/2)$, we must choose our angles such that $\theta_1-\theta_2+\theta_3=\theta+2a\pi$ (where a is any integer). We also note that the y component of $\mathbf{v}$ should equal zero, and that this can be achieved by choosing $\theta_1=\theta_3+2b\pi$ (where b is any integer). These two choices give

$$\mathbf{v}=\left\{\sin\frac{\theta}{2},\,0,\,f\left(\sin\frac{\theta}{2}-2\sin\left(\frac{\theta}{2}-\theta_1\right)\right)\right\}. \quad (11)$$

Finally, we choose θ_1 such that the z component of $\mathbf{v}$ equals zero; this gives

$$\theta_1=\frac{\theta}{2}-\arcsin\left(\frac{\sin(\theta/2)}{2}\right). \quad (12)$$

Combining this value with our previous relations between the angles gives

$$\theta_1=2n_1\pi+\frac{\theta}{2}-\arcsin\left(\frac{\sin(\theta/2)}{2}\right), \quad (13)$$

$$\theta_2=2n_2\pi-2\arcsin\left(\frac{\sin(\theta/2)}{2}\right), \quad (14)$$

$$\theta_3=2n_3\pi+\frac{\theta}{2}-\arcsin\left(\frac{\sin(\theta/2)}{2}\right), \quad (15)$$

where n_1, n_2, and n_3 are integers subject to the physical restriction that the resulting pulse angles must be positive.

These solutions have the same general form as those found previously [18]. Although they appear to differ in detail, the expressions are, in fact, identical: taking the values $n_1=1$, $n_2=1$ and $n_3=0$ gives our previous family of solutions [18], referred to by the acronym CORPSE (compensation for off-resonance with a pulse sequence). This family is now seen to be just one member of a larger group of families. To choose between these it is necessary to look at higher-order terms, and this is most conveniently achieved using the quaternion fidelity, Eq. (7). As a baseline we take the fidelity of a single off-resonance θ_ϕ pulse compared with its (ideal) on-resonance form,

$$\mathcal{F}\approx 1+f^2\left(\frac{\cos\theta-1}{4}\right), \quad (16)$$

TABLE I. Pulse rotation angles for a CORPSE composite pulse with a target rotation of θ_x; CORPSE pulse phases are $+x$, $-x$, $+x$.

θ	θ_1	θ_2	θ_3
30°	367.6	345.1	7.6
45°	371.5	337.9	11.5
90°	384.3	318.6	24.3
180°	420.0	300.0	60.0

where terms in f^4 and higher have been neglected. Note that the fidelity only contains even order terms in f, as the composite pulse performs symmetrically for positive and negative values of f.

As expected, all members of our general group of solutions result in much better fidelities; in particular, the term in f^2 is always completely removed. The behavior of the term in f^4 is much more complicated, but it can be shown that this term depends *only* on the value of $n=n_1-n_2+n_3$, that is, the total number of *additional* 2π rotations performed by the composite pulse sequence, and has the smallest absolute value when the three integers are chosen so that $n=0$. As our previous values ($n_1=1$, $n_2=1$, and $n_3=0$) are the smallest numbers that fit this criterion, it seems that the CORPSE family of pulse sequences is indeed the best member of this group. The only other family of interest is that with $n_1=0$, $n_2=1$, and $n_3=0$, previously referred to as short CORPSE [19]; while this performs less well than CORPSE, it is somewhat shorter. Numerical values of individual pulse rotation angles for CORPSE sequences with a variety of target angles are given in Table I.

The performance of the CORPSE sequence for a 180° pulse is demonstrated in Fig. 1; the CORPSE pulse performs better than a simple pulse as long as $|f|\leqslant 0.663$. For smaller values of θ the effective range of f is reduced, but not dramatically so: for a 30° pulse the CORPSE pulse outperforms a simple pulse as long as $|f|\leqslant 0.297$. Much better off-resonance compensation can be achieved in conventional NMR experiments by using adiabatic methods [14] such as wideband, uniform rate, smooth truncation (WURST) [22,23], but this approach can only be used for specific applications, such as decoupling, and cannot be applied in most quantum computing experiments.

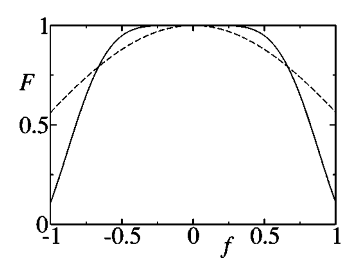

FIG. 1. Fidelity (F) of simple (dashed line) and CORPSE composite pulses (solid line) as a function of the off-resonance fraction f for pulses with a target rotation angle of 180°.

IV. PULSE LENGTH ERRORS

A similar approach can be used to develop composite pulses to tackle pulse length errors. As before we begin with a sequence of three pulses, but the subsequent development is quite different. In particular, we allow the three pulses to have arbitrary phase angles, as well as arbitrary rotation angles, thus giving us six variable parameters, although this number is soon reduced to three.

The quaternion corresponding to each pulse takes the simple form

$$\mathsf{q}_{\theta\phi}=\{\cos(\theta'/2),\sin(\theta'/2)\{\cos(\phi),\sin(\phi),0\}\}, \quad (17)$$

where $\theta'=\theta(1+g)$ is the *actual* rotation angle achieved by a pulse with nominal rotation angle θ, and g is the fractional error in the pulse power. The quaternion describing the composite pulse is very complicated, but can be simplified by restricting attention to the time symmetric case, where $\theta_1=\theta_3$ and $\phi_1=\phi_3$. This automatically ensures that the composite quaternion has no z component, as any time symmetric sequence of rotations about axes in the xy plane is itself a rotation about an axis in the xy plane.

Even after this simplification, the composite quaternion remains extremely complicated. To make further progress we note that a composite pulse of this kind has been previously described for the case of a 180° rotation: the sequence

$$180_{60}\text{-}180_{300}\text{-}180_{60} \quad (18)$$

will perform a 180_x rotation with compensation for pulse length errors (see Ref. [24], but note the corrected phase angles). It seems likely that other members of this family will have *either* $\theta_1=\pi$ or $\theta_2=\pi$; both possibilities were initially explored, but the second choice seemed more productive and forms the basis of our subsequent work.

As before, the composite quaternion can be expanded as a Maclaurin series in g, and it is most useful to concentrate on the first-order error term. This can be set equal to zero by choosing

$$\phi_2=\phi_1\pm\arccos(-\pi/2\theta_1) \quad (19)$$

and, for consistency with Eq. (18), we will use the minus sign in future. Sequences obeying this equation will be insensitive to pulse length errors; the rotation and phase angle can then be adjusted by choosing suitable values for θ_1 and ϕ_1. As before we will derive values for a θ_x pulse; pulses with other phase angles can be obtained by offsetting all the phase angles by the desired amount.

Solving these equations is complex, but the solutions are fairly straightforward:

$$\theta_1=\theta_3=\operatorname{arcsinc}\left(\frac{2\cos(\theta/2)}{\pi}\right), \quad (20)$$

$$\theta_2=\pi, \quad (21)$$

$$\phi_1=\phi_3=\arccos\left(\frac{-\pi\cos\theta_1}{2\theta_1\sin(\theta/2)}\right), \quad (22)$$

$$\phi_2=\phi_1-\arccos(-\pi/2\theta_1), \quad (23)$$

where sinc(x) is defined as sin(x)/x. We refer to this as a short composite rotation for undoing length over and under shoot or SCROFULOUS sequence.

Numerical values of individual pulse rotation and phase angles for a variety of target angles are given in Table II. The performance of SCROFULOUS and plain 180° pulses are compared in Fig. 2.

TABLE II. Pulse rotation and phase angles for a SCROFULOUS composite pulse with a target rotation of θ_x; note that $\theta_3=\theta_1$ and $\phi_3=\phi_1$.

θ	θ_1	ϕ_1	θ_2	ϕ_2
30°	93.0	78.6	180.0	273.3
45°	96.7	73.4	180.0	274.9
90°	115.2	62.0	180.0	280.6
180°	180.0	60.0	180.0	300.0

V. THE BROADBAND NUMBER 1 (BB1) FAMILY

Another approach to composite pulse design has been described by Wimperis [25]. While sequences such as CORPSE and SCROFULOUS seek a single composite pulse that performs the desired rotation with reduced sensitivity to errors, an alternative approach is to combine a naive pulse, which performs the desired rotation, with a sequence of error-correcting pulses, which partially compensate for imperfections. This approach appears to simplify the design of composite pulse sequences, and for the case of pulse length errors produces excellent results.

The form of the error-correcting pulse sequence is quite tightly constrained, as it must have no overall effect in the absence of errors, but it must also retain sufficient flexibility that it can act against errors when they do occur. Here we concentrate on one particular form suggested by Wimperis [25],

$$180_{\phi_1}\text{-}360_{\phi_2}\text{-}180_{\phi_1}, \quad (24)$$

where the values of ϕ_1 and ϕ_2 remain to be determined. When placed in front of a θ_x pulse, Wimperis refers to the entire sequence as BB1 [25], but as we will generalize his approach we refer to the error-correcting sequence [Eq. (24)] as W1.

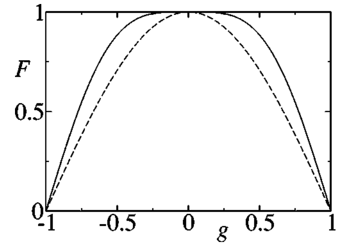

FIG. 2. Fidelity (F) of simple (dashed line) and SCROFULOUS composite pulses (solid line) as a function of the fractional pulse length error g for pulses with a target rotation angle of 180°.

TABLE III. Pulse phase angles for a W1 correction sequence with a target rotation of θ_x; pulse rotation angles are $\theta_1 = 180°$ and $\theta_2 = 360°$.

θ	ϕ_1	ϕ_2
30°	92.4	277.2
45°	93.6	280.8
90°	97.2	291.5
180°	104.5	313.4

As before we evaluate the quaternion for the composite rotation (W1 followed by a θ_x pulse) in the presence of pulse length errors, and then expand this quaternion as a Maclaurin series in g, the fractional error in the pulse power. The y and z components in the first-order error term are easily removed by setting $\phi_2 = 3\phi_1$; the remaining components can then be eliminated by choosing

$$\phi_1 = \pm \arccos\left(-\frac{\theta}{4\pi}\right). \tag{25}$$

The positive solution is then identical to that previously described [25]. Examining higher-order error terms shows that this pulse sequence is even better than it first appears, as these choices also completely remove the second-order error terms. As discussed below, this effect appears to be a property of the W1 sequence and its close relations.

It is easy to imagine a range of variations of the BB1 sequence. Most simply the W1 error correction sequence can be placed *after* the θ_x pulse, instead of before it. Unsurprisingly this has no effect: the solution is the same as before, and the performance of this reversed sequence is identical to that of BB1. More surprisingly, the W1 sequence can be placed in the middle of the θ_x pulse, so that the overall sequence

$$(\theta/2)_x\text{-W1-}(\theta/2)_x \tag{26}$$

is time symmetric. Indeed the W1 pulse can be placed at *any* point within the θ_x pulse, with almost identical effects. The form of the composite quaternion depends slightly on where the W1 pulse is placed, but the fidelity of the pulse sequence is unchanged: all error terms below sixth order are canceled, with the size of the sixth-order term depending on the value of θ.

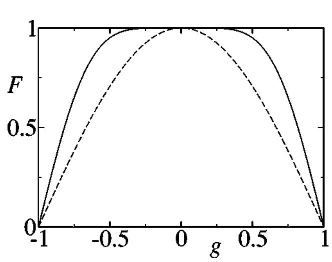

FIG. 3. Fidelity (F) of simple (dashed line) and BB1 composite pulses (solid line) as a function of the fractional pulse length error g for pulses with a target rotation angle of 180°.

Numerical values of pulse phase angles for a variety of target angles are given in Table III. The performance of the BB1 and plain 180° pulses are compared in Fig. 3. For all target angles below 180°, the BB1 composite pulse outperforms a simple pulse when $|g| < 1$.

Another simple variation is to use two or more error-correcting sequences; as before these can be placed at various different points around or within the θ_x pulse. For simplicity we assume that all the error-correcting sequences are identical to one another, and have the same general form as W1. In this case it can be shown that the correction sequences, which we call Wn, have phase angles given by $\phi_2 = 3\phi_1$ and

$$\phi_1 = \pm \arccos\left(-\frac{\theta}{4n\pi}\right), \tag{27}$$

where n is the total number of sequences used. As before the fidelity is independent of where the Wn sequences are placed, but it does depend on the value of n. The second- and fourth-order error terms are canceled in all cases, and the size of the sixth-order error term now depends on both θ and n. The smallest sixth-order term is achieved when $n = 2$, but the term is not completely removed. The gain over $n = 1$ is fairly small, and in practice the simpler composite pulses based on the W1 sequence are likely to be the most effective.

Having varied the position and number of the error-correcting pulse sequences the next logical step is to vary their form. In principle, any sequence that has no overall effect in the absence of errors could be used. In practice we find that many possible sequences allow the second-order error term in the fidelity expression to be removed, but the simultaneous cancellation of second- and fourth-order errors seems to be a special feature of the Wn family of sequences. The performance of the Wn family is remarkably good: indeed the Wn family performs better than many conventional composite pulses do when implementing the specific actions for which they have been designed.

Given the success of this approach to tackling pulse length errors, it seems obvious to apply the method to tackle off-resonance effects. As yet, however, this approach has had no success.

VI. SIMULTANEOUS ERRORS

So far we have only considered the case of *either* off-resonance effects *or* pulse length errors being present. In reality, both problems may well occur simultaneously. It is therefore important to consider how such simultaneous errors might be tackled. Ideally, we would like to design pulse sequences which can compensate for both problems at the same time; this, however, is a complicated and as yet unresolved problem, and here we simply analyze the sensitivity of each of our pulse sequences to the *other* kind of error.

We proceed as before, calculating composite pulse and simple pulse quaternions in the presence of errors, and determining the quaternion fidelity. This fidelity can then be expanded as a Maclaurin series in the error, and the lower-order

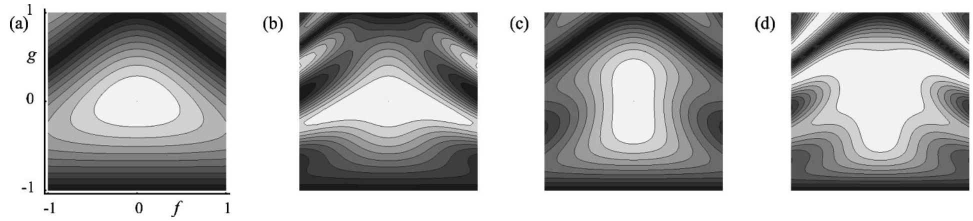

FIG. 4. Fidelity of (a) plain, (b) CORPSE, (c) SCROFULOUS, and (d) BB1 180° pulses as a function of simultaneous off-resonance effects, f, and fractional pulse length error, g. Contours are plotted at 5% intervals.

terms examined. Note that this procedure still assumes that only one type of error is present at a time; in order to detail with the case where both errors are present *simultaneously*, it would be possible to use a Maclaurin expansion in both errors, but this is unlikely to lead to much insight. Instead we will simply plot the fidelity as function of both errors for some chosen target angle.

We begin by considering the response of the CORPSE pulse sequence to pulse length errors. In the absence of off-resonance effects, the behavior of CORPSE is trivial to calculate, as the three pulses are applied along the $+x$, $-x$, and $+x$ axes, so that the behavior is identical to that of a simple pulse. The behavior of a 180° pulse in the presence of simultaneous errors is shown in Fig. 4.

The behavior of the SCROFULOUS pulse sequence is difficult to calculate for general target rotation angles, due to the dependence of θ_1 on the arcsinc function, and so we concentrate on the case of 180° pulses. For this case the dependence of the fidelity on off-resonance effects is given by $\mathcal{F}\approx 1-2f^2$, while a simple pulse has a fidelity $\mathcal{F}\approx 1-f^2/2$ [see Eq. (16)]. In general, SCROFULOUS is considerably more sensitive to off-resonance effects than plain pulses.

Finally, we consider the BB1 family of pulse sequences, taking the time-symmetrized version of BB1, Eq. (26), as our standard. In this case we can solve the problem for any target rotation angle, and up to second order the result is identical to that of a plain pulse, Eq. (16). Thus, unlike SCROFULOUS, the BB1 sequence achieves its impressive tolerance to pulse length errors at little or no cost in sensitivity to off-resonance effects. This is confirmed for simultaneous errors by Fig. 4.

VII. CONCLUSIONS

Composite pulses show great promise for combatting systematic errors in NMR quantum computers. Sequences have now been developed which correct for both off-resonance errors and pulse length errors, although the problem of correcting *simultaneous* errors remains. More generally, any implementation of a quantum computer must ultimately be concerned with rotations on the Bloch sphere, and so composite pulse techniques may find broader application in quantum computing. Composite pulses are not, however, a panacea, and some caution must be exercised in their use.

The CORPSE pulse sequence appears to be the best approach for tackling small off-resonance errors (for large *known* off-resonance effects the resonance offset tailored, or resonance offset tailoring to enhance nutations (ROTTEN), scheme [26] is preferable). Conventional composite pulses with better off-resonance compensation are known, but these cannot be used for quantum computing. For pulse length errors variations on the BB1 scheme of Wimperis [25] give the best results; indeed BB1 performs better than many conventional composite pulses. The SCROFULOUS family of pulses is less effective, but does have the advantage of being considerably shorter. This could prove useful in systems with short relaxation times where relaxation during the pulse cannot be neglected; this effect, however, is relatively unimportant for quantum computing as such systems are poorly suited to this task.

ACKNOWLEDGMENTS

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