PHYSICAL REVIEW A **67**, 042107 (2003)

How one shutter can close N slits

Y. Aharonov[1,2] and L. Vaidman[1]
[1]School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv 69978, Israel
[2]Physics Department, University of South Carolina, Columbia, South Carolina 29208

(Received 29 October 2002; published 22 April 2003)

It is shown that a quantum shutter, preselected and postselected in particular quantum states, can close simultaneously arbitrary number of slits preventing the passage of a single photon in an arbitrary state. A set of K preselected and postselected shutters can close the slits preventing the passage of K or less photons. This result indicates that the surprising properties of preselected and postselected quantum systems are even more robust than previously expected.

DOI: 10.1103/PhysRevA.67.042107 PACS number(s): 03.65.Ta

Probably the most paradoxical claim of quantum theory is that a particle can be in some sense in several places simultaneously. Without this feature one cannot explain the interference picture obtained in multiple-slit experiments performed with one particle at a time. A more robust claim of this type can be made about quantum preselected and postselected particle. With utilization of a particular preselection and postselection, we can claim that the particle should have been found with certainty in each one out of several places given that it was looked for only in that place [1]. Such claims became a subject of a significant controversy [2–10]. Here we discuss another aspect of such preselected and postselected particle which makes the claim that such a particle is simultaneously in several places even more robust.

Consider a photon arriving at a screen with N holes (slits) at time t, Fig. 1. We have a single particle (shutter) which, if placed in a slit, prevents the passage of the photon through this slit. Our task is to close all N slits at time t with this single shutter.

We are allowed to perform preselection and postselection on the shutter: to prepare it at time t_1 in state $|\Psi_1\rangle$ and select it in the state $|\Psi_2\rangle$ at t_2, $t_1<t<t_2$. If the postselection measurement of $|\Psi_2\rangle$ fails, the experiment fails, but if it succeeds, we should be able to claim that all N slits were closed for the photon at time t.

If the photon, bouncing of the shutter causes a measurable recoil, then a postselection can achieve this goal in a trivial way. We just observe the shutter at time t_2. If we find a recoil, we may claim that the slits were closed at time t. Indeed, we know that in this case the photon bounced back from the screen. However, in the present work we do not rely on this effect. The setup is such that there is no measurable recoil of the shutter.

The existence of a solution for this problem is surprising. The probability for the photon to be reflected from the slits with the shutter in one slit, or in an arbitrary superposition in all N slits, is $1/N$. Nevertheless, the preselected and postselected shutter reflects the photon with certainty. (We will see below that the probability of the postselection is smaller than $1/N$.)

To achieve this task we prepare (preselect) the shutter at t_1 in the state

$$|\Psi_1\rangle=\frac{1}{\sqrt{2N-1}}\left(\sum_{i=1}^{N}|i\rangle+\sqrt{N-1}|N+1\rangle\right). \tag{1}$$

We postselect the shutter at t_2 in the state

$$|\Psi_2\rangle=\frac{1}{\sqrt{2N-1}}\left(\sum_{i=1}^{N}|i\rangle-\sqrt{N-1}|N+1\rangle\right), \tag{2}$$

where $|i\rangle$ is a state of a shutter localized in slit i, $i=1,\ldots,N$ and $|N+1\rangle$ is a state of the shutter localized in some specific different location.

In order to prove our claim, let us consider the time evolution of the quantum state of the shutter and the photon during the whole procedure. We assume that the free evolution of the shutter between t_1 and t_2 can be neglected. Initially, the photon moves toward N slits, so its state is

$$|\Psi_{in}\rangle_{\rm ph}=\sum_{i=1}^{N}\alpha_i|i\rangle_{\rm ph}, \tag{3}$$

where $|i\rangle_{\rm ph}$ is the state of a photon moving toward the slit i. Let us signify the state of a photon reflected from slit i as $|\tilde{i}\rangle_{\rm ph}$. Then, after t, the time of the interaction between the shutter and the photon, their joint quantum state is

$$\begin{aligned}|\Psi\rangle_{\rm s,ph}&=\frac{1}{\sqrt{2N-1}}\sum_{i=1}^{N}|i\rangle\left(\alpha_i|\tilde{i}\rangle_{\rm ph}+\sum_{j\neq i}^{N}\alpha_j|j\rangle_{\rm ph}\right)\\&\quad+\sqrt{\frac{N-1}{2N-1}}\;|N+1\rangle\sum_{j=1}^{N}\alpha_j|j\rangle_{\rm ph}\\&=\frac{1}{\sqrt{2N-1}}\sum_{i=1}^{N}\alpha_i|i\rangle|\tilde{i}\rangle_{\rm ph}+\frac{1}{\sqrt{2N-1}}\\&\quad\times\sum_{j=1}^{N}\alpha_j\left(\sum_{i\neq j}^{N}|i\rangle+\sqrt{N-1}\;|N+1\rangle\right)|j\rangle_{\rm ph}.\end{aligned} \tag{4}$$

We can see that all states of the shutter appearing in the second term in the last expression (i.e., all states correlated with a photon which passed through the screen) are orthogonal to the postselected state $|\Psi_2\rangle$. Therefore, after the postselection, the photon state will have only reflected wave components. The screen operates as a perfect mirror; the final state of the photon is

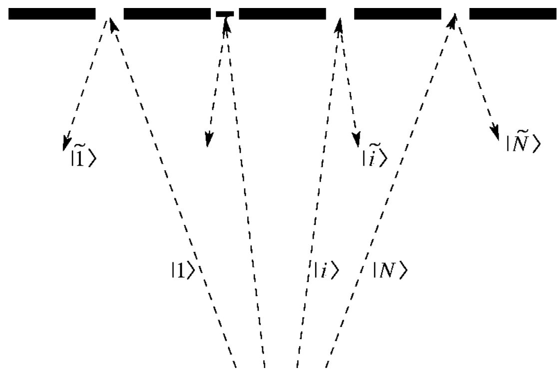

FIG. 1. A single photon arrives at N slits, but a single shutter reflects the photon as if there were shutters in every slit.

$$|\Psi_{fin}\rangle_{\rm ph} = \sum_{i=1}^{N} \alpha_i |\tilde{i}\,\rangle_{\rm ph}. \tag{5}$$

The probability of the postselection of the state $|\Psi_2\rangle$ at t_2, given that a photon arrived at the screen at time t, is

$$\begin{aligned}\mathrm{Prob}(|\Psi_2\rangle) &= ||\mathbf{P}_{\Psi_2}|\Psi\rangle_{\rm p,ph}||^2\\ &= \left\| \frac{1}{2N-1}\sum_{i=1}^{N} \alpha_i |\tilde{i}\rangle_{\rm ph}\right\|^2\\ &= \frac{1}{(2N-1)^2}.\end{aligned} \tag{6}$$

It is important that the probability of the postselection in the case that there are no photons arriving at the screen does not vanish, in fact, it happens to be the same, $|\langle\Psi_2|\Psi_1\rangle|^2 = 1/(2N-1)^2$. The probability for photon reflection irrespectively of the postselection, $1/(2N-1)$, is larger than the probability of the postselection. Otherwise, the method could increase unconditional reflectivity.

We have shown that a single quantum shutter that has been preselected and postselected can close any number of slits. It acts on the single photon exactly in the same way as N shutters. Conceptually, using this method one can build the whole screen out of a single preselected and postselected shutter (particle). This screen will act on a single photon as a real screen made from many particles. In particular, a photon passing through such a screen will follow a corresponding diffraction pattern.

Not less surprising is a ‘‘dual’’ problem which can be solved using our method. We have now N shutters which close at least $N-1$ out of the N slits. Nevertheless, we can preselected and postselect the state of these shutters in such a way that a single photon will ‘‘see’’ N open slits.

Consider the preselected state of N shutters

$$|\Phi_1\rangle = \frac{1}{\sqrt{2N-1}}\left(\sum_{i=1}^{N} |{\rm op}\rangle_i \prod_{j\neq i}^{N} |{\rm cl}\rangle_j + \sqrt{N-1}\prod_{j=1}^{N} |{\rm cl}\rangle_j\right), \tag{7}$$

where $|{\rm op}\rangle_i$ and $|{\rm cl}\rangle_i$ are the states of a shutter corresponding to an open or closed slit i, respectively. If now we test the number of closed slits, we will find with probability $(N-1)/(2N-1)$ that all slits are closed, and with probability $N/(2N-1)$ that all but one slits are closed. However, we do not test the number of closed slits. We send at time t the photon in an arbitrary state (3) toward the screen. Then, at time t_2, we postselect the shutters in the state,

$$|\Phi_2\rangle = \frac{1}{\sqrt{2N-1}}\left(\sum_{1}^{N} |{\rm op}\rangle_i \prod_{j\neq i}^{N} |{\rm cl}\rangle_j - \sqrt{N-1}\prod_{j=1}^{N} |{\rm cl}\rangle_j\right). \tag{8}$$

A calculation, identical to the one performed above, shows that a single photon passes the slits without distortion, as if no shutters were present.

In our method a single (preselected and postselected) shutter closes N slits for a single photon. What will happen if at time t several photons are trying to pass through the slits? If K photons move toward the screen in a particular correlated state

$$|\Psi\rangle_{K{\rm ph}} = \sum_{i=1}^{N} \alpha_i \prod_{k=1}^{K} |i\rangle_k, \tag{9}$$

then the shutter will reflect with certainty all the photons as it reflected one. However, when the photons arrive in an arbitrary state, we cannot be sure that even one photon will be reflected. Indeed, consider an incoming two-photon state,

$$|\Psi\rangle_{2{\rm ph}} = \frac{1}{\sqrt{2}}(|1\rangle_1|2\rangle_2 + |2\rangle_1|1\rangle_2). \tag{10}$$

After the interaction between the shutter and the photons at time t, the state of the shutter correlated with undisturbed state (10) is

$$\frac{1}{\sqrt{2N-3}}\left(\sum_{i=3}^{N} |i\rangle + \sqrt{N-1}\;\;|N+1\rangle\right). \tag{11}$$

This state is not orthogonal to postselected state (2). Therefore, a successful postselection is possible when both photons pass through the slits undisturbed, i.e., the two photons might pass through the screen with our preselected and postselected shutter.

In order to close N slits for a pair of photons we need *two* preselected and postselected shutters placed one after the other. The first shutter should be preselected at time t_1 in the state

$$|\Psi_1'\rangle = \frac{1}{\sqrt{2N-2}}\left(\sum_{i=1}^{N} |i\rangle + \sqrt{N-2}\;\;|N+1\rangle\right) \tag{12}$$

and postselected at time t_2 in the state

$$|\Psi_2'\rangle = \frac{1}{\sqrt{2N-2}}\left(\sum_{i=1}^{N} |i\rangle - \sqrt{N-2}\;\;|N+1\rangle\right). \tag{13}$$

The second shutter is preselected and postselected as in the previous example, in states (1) and (2).

If the two photons pass through two different slits without disturbance, then the state of the shutter will be orthogonal to $|\Psi'_2\rangle$. Therefore, given a successful postselection, one photon should be reflected by the first shutter. The second photon is reflected by the second shutter as explained above.

If the pair of photons pass through the same slit, then the state of the first shutter will not be orthogonal to $|\Psi'_2\rangle$. Therefore, the photons in such a pair might both pass through. But, in this case, the second shutter will reflect both photons with certainty, since it stops any number of photons arriving together as in correlated state (9).

Note that there is no possibility of ''trapping'' the photon(s) between the two shutters. The photon(s) reflected by the second shutter in a particular slit cannot be reflected back by the other side of the first shutter, because the arrival of the photon(s) to the second shutter ensures that the first shutter is absent at the slit.

In order to stop three photons we have to add another shutter in front of the two described above. The additional shutter should reflect one photon any time three photons arrive at different slits. To this end, the shutter should be preselected and postselected in the states $|\Psi''_1\rangle$ and $|\Psi''_2\rangle$,

$$|\Psi''_1\rangle = \frac{1}{\sqrt{2N-3}}\left(\sum_{i=1}^{N} |i\rangle + \sqrt{N-3}\,|N+1\rangle\right), \qquad (14)$$

$$|\Psi''_2\rangle = \frac{1}{\sqrt{2N-3}}\left(\sum_{i=1}^{N} |i\rangle - \sqrt{N-3}\,|N+1\rangle\right). \qquad (15)$$

The generalization for larger number of photons is obvious. In this way K preselected and postselected shutters close an arbitrarily large number of slits N for passing K or less photons in an arbitrary state.

In this paper we have shown a surprising feature of preselected and postselected shutters. A single shutter can close an arbitrary number of slits preventing the passage of a single photon in an arbitrary state, while K shutters can close the slits preventing passage of any number of photons $n \leqslant K$. On the other hand, N shutters which close at least $N-1$ slits can leave all slits open for a single photon.

For a preselected and postselected state of a single shutter which closes N slits, it was known before [1] that the outcomes of *weak measurements* performed in all slits correspond to one shutter being in every slit. The present result shows that a measuring device, namely, the photon, performing strong measurement while being in a superposition in different slits also indicates the presence of the shutter in every slit.

This paper considers gedanken experiments which shed a new light on the problem of nonlocality in quantum mechanics. The main problem for practical realization of such an experiment is that it requires a gate between two quantum objects: the photon passing through slits and the quantum shutter. If the shutter is a photon too, then today's technology allows preparation of the initial state. Indeed, there are several techniques for creating a single photon [11] and linear optical elements such as beam splitters allow preparation of an arbitrary superposition. Postselection is even simpler, it is just a ''reversed'' preparation scheme ending with a detector instead of a single-photon source. However, it is very difficult to arrange strong interaction between photons, so the choice of photons for both particle and the shutter is not promising. Atoms do interact efficiently with photons in microwave cavities, so this maybe a basis for a more realistic proposal, but there are several alternatives for a possible efficient quantum gate, and it is not clear when and which of them will be realized first.

It is a pleasure to thank Shmuel Nussinov for helpful discussions. This research was supported in part by Grant No. 62/01 of the Israel Science Foundation, by the Israel MOD Research and Technology Unit, by the NSF Grant No. PHY-9971005, and by the ONR Grant No. N00014-00-0383.

[1] Y. Aharonov and L. Vaidman, J. Phys. A **24**, 2315 (1991).
[2] W.D. Sharp and N. Shanks, Philos. Sci. **60**, 488 (1993).
[3] L. Vaidman, Stud. Hist. Philos. Mod. Phys. **30**, 373 (1999).
[4] R.B. Griffiths, Phys. Rev. A **54**, 2759 (1996); **57**, 1604 (1998).
[5] A. Kent, Phys. Rev. Lett. **78**, 2874 (1997).
[6] R.E. Kastner, Found. Phys. **29**, 851 (1999).
[7] L. Vaidman, Found. Phys. **29**, 865 (1999).
[8] A. Shimony, Erken. **45**, 337 (1997).
[9] O. Cohen, Phys. Rev. A **51**, 4373 (1995).
[10] L. Vaidman, Phys. Rev. A **57**, 2251 (1998).
[11] T.B. Pittman, B.C. Jacobs, and J.D. Franson, Phys. Rev. A **66**, 042303 (2002).