Critically bound four-body molecules

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The $(p,d,\overline{p},\overline{d})$ molecule, with a proton, a deuteron, and their antiparticles, is stable againt spontaneous dissociation, but none of its three-body subsystems are stable. This molecule should be built by combining two atoms, for instance a protonium $(p\overline{p})$ and its heavier analog $(d\overline{d})$. Most other four-body molecules have at least one stable three-body subsystem and thus can be built by adding the constituents one by one.

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Bressanini *et al.* [1] have studied the stability of four-charge systems with masses (M^+,m^+,M^-,m^-). For $M=m$, this corresponds to the positronium molecule (Ps_2), whose stability was first demonstrated in 1947 [2]. For $M \gg m$ or $M \ll m$, this is a hydrogen-antihydrogen $H\overline{H}$ system (without annihilation, strong interaction, etc.) which hardly competes with the deeply bound protonium (M^+M^-) involved in the lowest threshold $(M^+M^-)+(m^+m^-)$. Stability is thus restricted to an interval of M/m close to unity. The Monte-Carlo calculation of Ref. [1] leads to an estimate

$$\frac{1}{2.2} \lesssim \frac{M}{m} \lesssim 2.2, \tag{1}$$

which is confirmed by a powerful variational method [3].

The case of three unit charges is well documented [4–6], in particular, for the $(M^{\pm},m^{\mp},m^{\pm})$ configurations. For $M=m$, this is the stable positronium ion Ps^-. For $M \gg m$, we have (p,e^-,e^+), and for $M \ll m$, $(\overline{p},p,e^-)$, both unstable. Mitroy [7], using the same stochastic variational approach as in Ref. [3], found that stability is confined to

$$0.70 \lesssim M/m \lesssim 1.64. \tag{2}$$

Comparing the results (1) and (2) indicates a window for "Borromean" binding. For instance, for $M/m=2$, which is the deuteron-to-proton mass ratio, the (M^+,m^+,M^-,m^-) molecule is bound, but neither $(M^{\pm},m^{\mp},M^{\pm})$ nor $(m^{\pm},m^{\mp},M^{\pm})$ are stable.

The word "Borromean" has been proposed in nuclear physics to identify bound states whose subsystems are unbound [8]. It comes from the Borromean rings, which are interlaced in such a subtle topological way, that if any one of them is removed, the two others become unlocked. For instance, the ^{6}He isotope of ordinary helium is stable, while ^{5}He is not. In a three-body picture, this means that the (α,n,n) system is bound, whereas (α,n) and (n,n) are unbound.

For $N>3$ constituents, one might define Borromean binding as the property of all N'-body subsystems being unstable, with $N'=2$, or $N'=N-1$, or $N'<N$. We propose the following definition: *A bound state is Borromean if there is no path to build the system via a series of stable bound states by adding the constituents one by one.* Then, $(p,d,\overline{p},\overline{d})$ is Borromean. It is truly an atom-atom composite, more representative of larger molecules of ordinary chemistry. The same is true for neighboring systems $(m_1^+,m_2^+,m_3^-,m_4^-)$ with less symmetry. A minimal extension of the domain of stability can be derived using the variational principle [9].

In comparison, H_2 or Ps_2 systems appear to be more robust, with several three-body subsystems being stable, (p,e^+,e^-) or (p,p,e^-) for H_2, and $(e^{\pm},e^{\mp},e^{\mp})$ for Ps_2. The positronium hydride PsH (p,e^+,e^-,e^-) contains the unstable (p,e^+,e^-), but also the stable (p,e^-,e^-) and (e^+,e^-,e^-), and thus is not Borromean.

Note that if the antideuteron is replaced by the celebrated Ω^- hyperon (predicted by Gell-Mann by symmetry considerations which led to the quark model, and discovered by Samios *et al.* [10]), and if the deuteron is replaced by $\overline{\Omega}^+$, the mass ratio $M/m=1.78$ becomes close to one of the critical values of Eq. (2). If $A=(\Omega^-\overline{\Omega}^+)$, we have an effective $(A,p,\overline{p})$ three-body system with both (A,p) and $(A,\overline{p})$ energies vanishing. The Efimov effect [11] survives finite-size effects, since it is governed by the long-range part of the interaction. However, the Coulomb attraction between p and $\overline{p}$ spoils the $-1/\rho^2$ behavior (ρ is the hyper-radius) necessary in the hypercentral potential for Efimov states to appear. See, e.g., the approach by Fedorov and Jensen, in Ref. [11]. Note that a partial and preliminary version of this paper was presented at the Few-Body Conference in Bled [9].

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[1] D. Bressanini, M. Mella, and G. Morosi, Phys. Rev. A **55**, 200 (1997).
[2] E.A. Hylleraas and A. Ore, Phys. Rev. **71**, 493 (1947).
[3] K. Varga, Few-Body Syst., Suppl. **10**, 11 (1999).
[4] A. Martin, J.-M. Richard and T.T. Wu, Phys. Rev. A **46**, 3697 (1992).
[5] E.A.G. Armour and W. Byers Brown, Acc. Chem. Res. 26, 168 (1993).

[6] E.A.G. Armour, J.-M. Richard, and K. Varga, Phys. Rep. (to be published).
[7] J. Mitroy, J. Phys. B **33**, 5307 (2000).
[8] M.V. Zhukov, B.V. Danilin, D.V. Fedorov, J.M. Bang, I.S. Thompson, and J.S. Vaagen, Phys. Rep. **231**, 151 (1993).
[9] J.-M. Richard, Few-Body Syst. Suppl. (to be published).
[10] See, for instance, A. Pais, *Inward Bound, of Matter and Forces in the Physical World* (Clarendon Press, Oxford, 1986).
[11] V. Efimov, Phys. Lett. **33B**, 560 (1970); Yad. Fiz. **12**, 1080 (1970) [Sov. J. Nucl. Phys. **12**, 581 (1971)]; R.N. Amado and R.V. Noble, Phys. Lett. **35B**, 25 (1971); Phys. Rev. D **5**, 1992 (1972); S. Albeverio, R. Høegh-Krohn and T.T. Wu, Phys. Lett. **83A**, 101 (1981); D.V. Fedorov and A.S. Jensen, Phys. Rev. Lett. **71**, 4103 (1993).